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The existence of Bayesian fuzzy equilibrium problems for a new general Bayesian abstract fuzzy economy model with differential private information

Wiyada Kumam

Program in Applied Statistics, Department of Mathematics and Computer Science, Faculty of Science and Technology, Rajamangala University of Technology Thanyaburi (RMUTT), Thanyaburi, Pathumthani 12110, Thailand.

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Abstract

In this work, we introduced a new Bayesian abstract fuzzy economy model with differential private information and the Baysian fuzzy equilibrium problem, and we also prove the existence of the Baysian fuzzy equilibrium problem for this new model. Our main results extended and improved the recent results announced by many authors from the literature. The new concept of idea that the uncertainties characterize the individual attribute of the choice or preference of the agents concerned in different economic actions.

Keywords: Bayesian abstract fuzzy economy model, Bayesian fuzzy equilibrium problem, incomplete information, random fuzzy mappings, fuzzy mappings.
2010 MSC: 58E35, 47H10, 91B50, 91A44.

1. Introduction

The theory of equilibrium problem considers a large spectrum of interesting and important tools used in various fields of optimization, economics, physics, statistics, applied mathematics and engineering sciences. The new direction in mathematical economics and game theory concerns the fact that the irresolutions and uncertainties which identify the separate characteristic of agent's decisions, concerned in different economic actions, must be included in the mathematical models. The first proved the existence of equilibrium models for the n-person games where the player's preferences are representable by continuous quasi-concave utilities

Email address: wiyada.kum@mail.rmutt.ac.th (Wiyada Kumam)

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and the strategy sets are simplexes by John Nash [10]. Later, in 1952 Debreu [5] and Arrow-Debreu [2] proved the existence of the social equilibrium and an equilibrium for a competitive economy, respectively. The model of abstract economy model introduced by Shafer and Sonnenschein [17] in 1975, (see also Borglin and Keiding [3]) consists of a finite set of agents, each characterized by certain constraints and preferences, described by correspondences. Many authors use this idea studied the existence of equilibrium model for generalized fuzzy games. For example, in 1998, Huang [7] studied some equilibrium problems for abstract economies. In 2011 Wang, Cho and Huang [18] studied the robustness of a generalized abstract fuzzy economie models in generalized convex spaces, etc. The techniques to prove the existence of equilibrium problems for the abstract economy model are often used to solve some problems related to this field, principally the variational inequality problems, minimization and maximization problems, some classes of equilibrium problems and the existence of equilibrium for the exchange economies.

In 1965, Zadeh [21] introduced the notion of a fuzzy subset of a nonempty set X, as a function from X to $[0, 1]$. Fuzzy set theory has been shown to be a useful tool to describe situations in which the data are imprecise or blurry. The theory of fuzzy sets, has become a good framework for studying results concerning existence of fuzzy equilibrium for abstract fuzzy economies. The study of a fuzzy abstract economy (or a fuzzy game) has been introduced by Kim and Lee [9], they proved the existence of equilibrium problems for 1-person fuzzy game. This type of game is a generalization of classical abstract economies. They also proved the existence of equilibrium problems in the case of generalized games when the constraints or preferences are imprecise due to the agent's behaviour. Later, in 2009, Patriche [12] studied Bayesian abstract economy model and she also proved the existence of equilibrium problems for an abstract economy model with differential information and a measure space of agents. Although, the existence of random fuzzy equilibrium problems has not been studied so far. In 2013, Patriche [14] introduced the new concept of Bayesian abstract economy model and she proved the existence of the Bayesian fuzzy equilibrium problems. She also [15] introduced the new model of Bayesian abstract fuzzy economy model, and consider the existence of the Bayesian fuzzy equilibrium problem. This model is characterized by a private information set, an the set of action fuzzy mapping, a random fuzzy constraint one and a random fuzzy preference mapping. In 2014, Patriche [16] studied the fuzzy games with a countable space of actions and applications to systems of generalized quasi-variational inequalities. The Bayesian fuzzy equilibrium problem concept is an expansion of the deterministic equilibrium problem. She also generalized and extend the former deterministic models introduced by Debreu [5], Shafer and Sonnenschein [17] and Patriche [13].

The purpose of this work, we will study and obtain the existence of a Baysian fuzzy equilibrium problem for a general Bayesian abstract fuzzy economy model with differential private information and countable set of actions. This paper is orderly as follows: Sections 1 and 2 consist of introduction and mathematical model and preliminaries, respectively. The model with a new general Bayesian abstract fuzzy economies with differential private data and a countable set of actions is accounted in Section 3. In the last section, Section 4, we stat and prove the existence of a Baysian fuzzy equilibrium problem for a general Bayesian abstract fuzzy economy model with differential private data or information and countable set of actions.

2. Mathematical model and Preliminaries

Let X be a topological space and A be a subset of a topological space X. Next, we will use the following notation for this paper. We denoted 2^A the family of all subset of A and clA denotes the closure of A in X. If a mapping $T : A \to 2^X$ is correspondences, then clT is correspondence defined by $(clT(x)) = cl(T(x))$ for each $x \in A$.

We will review a basic definitions and lemmas from continuity and measurability of correspondences as follows.

Let Z, Y be two topological spaces and $P : Z \to 2^Y$ be a correspondence. P is said to be *upper semicontinuous*, if for each $z \in Z$ and each open set V in Y with $P(z) \subset V$, there exists an open neighborhood $\mathcal{U}$ of $z \in Z$ such that $P(y) \subset V$ for each $y \in Y$. P is said to be *lower semicontinuous*, if for each $z \in Z$ and open set V in Y with $P(z) \cap V \neq \emptyset$, there exists an open neighborhood $\mathcal{U}$ of z in Z such that $P(y) \cap V \neq \emptyset$ for each $y \in \mathcal{U}$.

Lemma 2.1 ([9]). *Let Z and Y be two topological spaces, and let D be an open subset of Z. Suppose $P_1 : Z \to 2^Y, P_2 : Z \to 2^Y$ are upper semicontinuous correspondences such that $P_2(z) \subset P_1(z)$ for all $z \in D$. Then the correspondence $P : Z \to 2^Y$ defined by*

$$P(z) = \begin{cases} P_1(z), & \text{if} \quad z \notin D \\ P_2(z), & \text{if} \quad z \in D \end{cases}$$

is also upper semicontinuous.

We now present below some notions concerning the fuzzy sets and the fuzzy mappings.

Definition 2.2 ([4]). If Y is a topological space, then a function A , from Y into $[0, 1]$, is called a fuzzy set on Y. The family of all fuzzy sets on Y is denoted by $\mathcal{F}(Y)$.

(1) If X and Y are topological spaces, then a mapping $P : X \to \mathcal{F}(Y)$ is called a *fuzzy mapping*.

(2) If P is a fuzzy mapping, then, for each $x \in X, P(x)$ is a fuzzy set on Y and $P(x)(y) \in [0, 1], y \in Y$ is called the *degree of membership of* y in $P(x)$.

(3) Let $A \in \mathcal{F}(x), a \in [0, 1]$. Then the set $(A)_a = \{y \in Y : A(y) > a\}$ is called a *strong a-cut set* of the fuzzy set A.

The random fuzzy mappings have been defined in order to model random structures generating imprecisely-valued data which can be correctly related by using fuzzy sets and fuzzy logic.

Let Y be a topological space, let $\mathcal{F}(Y)$ be a collection of all fuzzy sets over Y.

Definition 2.3 ([11]). A fuzzy mapping $P : \Omega \to \mathcal{F}(Y)$ is said to be measurable, if for any given $a \in [0, 1], (P(\cdot))_a : \Omega \to 2^Y$ is a measurable set-valued mapping.

(1) A fuzzy mapping $P : \Omega \to \mathcal{F}(Y)$ is said to *have a measurable graph*, if for any given $a \in [0, 1]$, the set-valued mapping $(P(\cdot))_a : \Omega \to 2^Y$ has a measurable graph.

(2) A fuzzy mapping $P : \Omega \times X \to \mathcal{F}(Y)$ is called a *random fuzzy mapping*, if for any given $x \in X, P(\cdot, x) : \Omega \to \mathcal{F}(Y)$ is a measurable fuzzy mapping.

The following properties are essential tools used to prove the existence of fuzzy equilibrium problem for a general Bayesian abstract fuzzy economy model as follows:

Theorem 2.4 ([19]). *(Aumann measurable selection theorem) Let $(\Omega, \mathcal{F}, \mu)$ be a complete finite measure space, let Y be a complete, separable metric space, and let $T : \Omega \to 2^Y$ be a non-empty valued correspondence with a measurable graph, i.e., $G_T \in \mathcal{F} \otimes \mathcal{B}(Y)$. Then there is a measurable function $t : \Omega \to Y$ such that $f(\omega) \in T(\omega)\mu$-a.e..*

Theorem 2.5 ([1]). *(Kuratowski-Ryll-Nardzewski selection theorem) A weakly measurable correspondence with non-empty closed values from a measurable space into a Polish space admits a measurable selector.*

Theorem 2.6 ([8]). *Let L be a locally convex topological linear space and K a compact convex set on L. Let $\mathfrak{R}(K)$ be a family of all closed convex (non-empty) subset of K. Then for any upper semicontinuous point to set transformation f from K into $\mathfrak{R}(K)$, there exists a point $x_0 \in K$ such that $x_0 \in f(x_0)$.*

Lemma 2.7 ([20]). *Let Y be a countable complete metric space, $(T, \mathcal{T}, \lambda)$ be an atomless probability space and $F : T \to 2^Y$ be a measurable correspondence. Then $\mathcal{D}_F$ is non-empty and convex in the space $\mathcal{M}(Y)$ - the space of probability measures on Y, endowed with the topology of weak convergence.*

Lemma 2.8 ([20])**.** *Let Y be a countable complete metric space, $(T, \mathcal{T}, \lambda)$ be an atomless probability space and $F : T \to 2^Y$ be a measurable correspondence. If F is compact valued, then $\mathcal{D}_F$ is compact in $\mathcal{M}(Y)$.*

Lemma 2.9 ([20])**.** *Let X be a metric space, $(T, \mathcal{T}, \widehat{h}_g)$ be an atomless probability space, Y be a countable complete metric space and $F : T \times X \to 2^Y$ be a correspondence. Let us assume that for any fixed x in $X, F(\cdot, x)$ (also denoted by F_x) is a compact-valued measurable correspondence, and for each fixed $t \in T, F(t, \cdot)$ is upper semicontinuous on X. Also, let us assume that there exists a compact-valued correspondence $H : T \times X \to 2^Y$ such that $F(t, x)H(t)$ for all t and x. Then $\mathcal{D}_{F_x}$ is upper semicontinuous on X.*

3. A generalized fuzzy game model

In this section, we present the model of a general Bayesian abstract fuzzy economy model and the Bayesian fuzzy equilibrium problem for model. Recently, a general Bayesian abstract fuzzy economy model was studied and considered by Patrich [14] as the following.

Let $(\Omega, \mathcal{F}, \mu)$ be a complete finite measure space, where Ω denotes the set of states of nature of the world, and the σ -algebra $\mathcal{F}$ denotes the set of events. Let Y denote the strategy or commodity space, where is a separable Banach space. Let I be a countable or uncountable set (the set of agents). For each $i \in I$, let $X_i : \Omega \to \mathcal{F}$ be a fuzzy mapping, and let $z_i \in (0, 1]$.

It is known that if $x : \Omega \to Y$ is a μ-measurable function, then x is the Bochner integrable if and only if $\int_\Omega \|x(\omega)\| d\mu(\omega) < \infty$ (see in [9]). It is denoted by $L_1(\mu, Y)$, the space of equivalence classes of Y-valued Bochner integrable functions $x : \Omega \to Y$ normed by $\|x\| = \int_\Omega \|x(\omega)\| d\mu(\omega)$. So, $L_1(\mu, Y)$ is a Banach space (see in [6]).

If there exists a mapping $h \in L_1(\mu, Y)$ such that $\sup\{x : x \in (X_i(\cdot))_{z_i}(\omega)\} \leq h(\omega)\mu$ -a.e then the correspondence $(X_i(\cdot))_{z_i} : \Omega \to 2^Y$ is said to be *integrably bounded*.

We denote by $S_{(X_i(\cdot))_{z_i}}$ the set of all selections of the correspondence $(X_i(\cdot))_{z_i} : \Omega \to 2^Y$ that belong to the space $L_1(\mu, Y)$, where

$$S_{(X_i(\cdot))_{z_i}} = \{x_i \in L_1(\mu, Y) : x_i(\omega) \in (X_i(\omega))_{z_i} \mu - a.e.\}.$$

Let $L_{X_i} = \{x_i \in S_{(X_i(\cdot))_{z_i}} : x_i$ is $\mathcal{F}_i$-measurable $\}$. We let $L_X := \prod_{i \in I} L_{X_i}$ and we denote $L_{X_{-i}}$ by the set $\prod_{i \neq j} L_{X_j}$. An element x_i of L_{X_i} is called a strategy for agent i. The typical element of L_{X_i} is denoted by x_i and that of $(X_i(\omega))_{x_i}$ by $x_i(\omega)$ (or x_i). We can see a general Bayesian abstract fuzzy economy from the definition as the following.

Definition 3.1. *A general Bayesian abstract fuzzy economy model* is the following family

$$G = \{(\Omega, \mathcal{F}, \mu), (X_i, \mathcal{F}_i, A_i, P_i, a_i, p_i, z_i)_{i \in I}\},$$

where

(a) $X_i : \Omega \to \mathcal{F}(Y)$ is the *action(strategy) fuzzy mapping* of agent i;

(b) $\mathcal{F}_i$ is a sub σ -algebra of $\mathcal{F}$, which denote the *private information* of agent i;

(c) for each $\omega \in \Omega, A_i(\omega_i, \cdot) : L_X \to \mathcal{F}(Y)$ is the *random fuzzy constraint mapping* of agent i;

(d) for each $\omega \in \Omega, P_i(\omega_i, \cdot) : L_X \to \mathcal{F}(Y)$ is the *random fuzzy preference mapping* of agent i;

(e) $a_i : L_X \to (0, 1]$ is a *random fuzzy constraint function*, and $p_i : L_X \to (0, 1]$ is a *random fuzzy preference function* of agent i;

(f) $z_i \in (0, 1]$ is such that for all $(\omega, x) \in \Omega \times L_X, (A_i(\omega, \widetilde{x}))_{a_i(\widetilde{x})} \subset (X_i(\omega))_{z_i}$ and $(P_i(\omega, \widetilde{x}))_{p_i(\widetilde{x})} \subset (X_i(\omega))_{z_i}$.

The definition of equilibrium is a Bayesian fuzzy equilibrium problem for G as follows.

Definition 3.2. *A Bayesian fuzzy equilibrium problem for G* is a strategy profile $\widetilde{x}^* \in L_X$ such that for all $i \in I$,

(i) $\widetilde{x}^*(\omega) \in cl(A_i(\omega, \widetilde{x}^*))_{a_i(\widetilde{x}^*)} \quad \mu - a.e.$;

(ii) $(A_i(\omega, \widetilde{x}^*))_{a_i(\widetilde{x}^*)} \cap (P_i(\omega, \widetilde{x}^*))_{p_i(\widetilde{x}^*)} = \emptyset \quad \mu - a.e.$.

In addition, [16] defined and considered a model of an abstract fuzzy economy model with private information and a countable set of actions as follows.

Let I be a nonempty finite set (the set of agents). For each $i \in I$, the space of actions S_i is a countable complete metric space and $(\Omega_i, \mathcal{Z}_i)$ is a measurable space. $(\Omega, \mathcal{F})$ is the product measurable space ($\Omega = \prod_{i\in I} \Omega_i, \mathcal{F} = \bigotimes_{i\in I} \mathcal{Z}_i$), and μ is a probability measure on $(\Omega, \mathcal{F})$. We can see an abstract fuzzy economy (or a generalized fuzzy game) with private information and a countable space of actions model from the next definition.

Definition 3.3. *An abstract fuzzy economy model (or a generalized fuzzy game) with private information and a countable space of actions* is defined as follows:

$$\Delta = (((\Omega_i, \mathcal{Z}_i)_{i\in I}, \mu), (S_i, X_i, (A_i, a_i), (P_i, p_i), z_i)_{i\in I}),$$

where I is a non-empty finite set (the set of agents) and

(a) $X_i : \Omega_i \to \mathcal{F}(S_i)$ is *the action (strategy) fuzzy mapping* of agent i;

(b) $A_i : \Omega_i \times \mathcal{D}_{X,z} \to \mathcal{F}(S_i)$ is *a random fuzzy mapping* (*the constraint mapping* of agent i);

(c) $P_i : \Omega_i \times \mathcal{D}_{X,z} \to \mathcal{F}(S_i)$ is *a random fuzzy mapping* (*the preference mapping* of agent i);

(d) $a_i : \mathcal{D}_{X,z} \to (0, 1]$ is *a random fuzzy constraint function* and $p_i : \mathcal{D}_{X,z} \to (0, 1]$ is *a random fuzzy preference function*;

(e) $z_i \in (0, 1]$ is such that for all $(\omega_i, h_g) \in \Omega_i \times \mathcal{D}_{x,Z}$, $(A_i(\omega_i, h_g))_{a_i(h_g)} \subset (X_i(\omega_i))_{z_i}$ and $(P_i(\omega_i, h_g))_{p_i(h_g)} \subset (X_i(\omega_i))_{z_i}$.

The deterministic definition of equilibrium problem we owe to Shafer and Sonnenschein [17] and the stochastic one proposed by Patrich in [15]. The notion of fuzzy equilibrium problem for Γ was introduced as follow.

Definition 3.4. *A fuzzy equilibrium problem* for Γ is defined as a strategy profile $g^* = (g_1^*, g_2^*, ..., g_n^*) \in \prod_{i\in I} Meas(\Omega_i, S_i)$ such that for each $i \in I$:

(i) $g_i^*(\omega_i), \in (A_i(\omega_i, h_{g^*}))_{a_i(h_{g^*})}$ for each $\omega_i \in \Omega_i$;

(ii) $(A_i(\omega_i, h_{g^*}))_{a_i(h_{g^*})} \cap (P_i(\omega_i, h_{g^*}))_{p_i(h_{g^*})} = \emptyset$ for each $\omega_i \in \Omega_i$.

4. The Bayesian abstract fuzzy economy model

Now, we introduce a general Bayesian abstract fuzzy economy model with differential private information and a countable space of actions and proved the existence of the Bayesian fuzzy equilibrium for a general Bayesian abstract fuzzy economy model with private information and a countable set of actions as follows.

4.1. A general Bayesian abstract fuzzy economy model with differential private information

The idea of my work is motivated and inspired by Definitions 3.1 and 3.3. we determined a new general Bayesian abstract fuzzy economy model with differential private information and a countable space of actions as follows.

Definition 4.1. *A general Bayesian abstract fuzzy economy model with differential private information and a countable space of actions* is defined as follows:

$$\Delta = (((\Omega_i, \mathcal{Z}_i)_{i\in I}, \mu), (X_i, \mathcal{F}_i, (A_i, a_i), (P_i, p_i), z_i)_{i\in I}),$$

where I is non-empty finite set (the set of agents) and,

(a) $X_i : \Omega_i \to \mathcal{F}(Y)$ is a *action (strategy) fuzzy mapping* of agent i;

(b) $\mathcal{F}_i$ is a sub σ-algebra of $\mathcal{Z} := \bigotimes_{i\in I} \mathcal{Z}_i$, which denotes the *differential private information* of agent i;

(c) $A_i : \Omega_i \times \mathcal{D}_{X,z} \to \mathcal{F}(Y)$ is a *random fuzzy constraint mapping* of agent i;

(d) $P_i : \Omega_i \times \mathcal{D}_{X,z} \to \mathcal{F}(Y)$ is a *random fuzzy preference mapping* of agent i;

(e) $a_i : \mathcal{D}_{X,z} \to (0, 1]$ is a *random fuzzy constraint function*, and $p_i : \mathcal{D}_{X,z} \to (0, 1]$ is a *random fuzzy preference function* of agent i;

(f) $z_i \in (0, 1]$ is such that for all $(\omega_i, h_g) \in \Omega_i \times \mathcal{D}_{X,z}$, $(A_i(\omega_i, h_g))_{a_i(h_g)} \subset (X_i(\omega_i))_{z_i}$ and $(P_i(\omega_i, h_g))_{p_i(h_g)} \subset (X_i(\omega_i))_{z_i}$.

Next, we will illustrate some elements of the model and also give some interpretations.

Let I be a non-empty finite set (the set of agents). The space of actions Y is a countable complete metric space and $(\Omega_i, \mathcal{Z}_i)$ is a measurable space for all $i \in I$. Let $(\Omega, \mathcal{Z})$ be a product measurable space $(\Omega = \prod_{i\in I} \Omega_i, \mathcal{Z} = \bigotimes_{i\in I} \mathcal{Z}_i)$, and μ be a probability measure on $(\Omega, \mathcal{Z})$. For a point $\omega = (\omega_1, \omega_2, ..., \omega_n) \in \Omega$, we define the coordinate projections $\tau_i(\omega) = \omega_i$. A random mapping $\tau_i(\omega)$ is interpreted as player $i's$ private information related to his action. $Meas(\Omega_i, \mathcal{F}_i)$ the set of measurable mappings from $(\Omega_i, \mathcal{Z}_i)$ to S_i. An element g_i of $Meas(\Omega_i, \mathcal{F}_i)$ is called *a pure strategy* for player i. A *pure strategy* profile g is an n-vector function $(g_1, g_2, ..., g_n)$ that specifies a pure strategy for each player.

We suppose that there exists a fuzzy mapping $X_i : \Omega_i \to \mathcal{F}(Y)$ such that each agent i can choose an action from $(X_i(\omega_i))_{z_i} \subset \mathcal{F}_i$ for each $\omega_i \in \Omega_i$. A measurable function $g_i : \Omega_i \to \mathcal{F}_i$ is said to be a measurable selection of $(X_i(\cdot))_{z_i}$ if $g_i(\omega_i) \in (X_i(\omega_i))_{z_i}$ for every $\omega_i \in \Omega_i$. $\mathcal{D}_{(X_i(\cdot))_{z_i}}$ is the set $\{(\mu\tau_i^{-1})g_i^{-1} : g_i$ is a measurable selection of $(X_i(\cdot))_{z_i}\}$ and $\mathcal{D}_{X,z} := \prod_{i\in I} \mathcal{D}_{(X_i(\cdot))_{z_i}}$.

We denote $hg_i = (\mu\tau_i^{-1})g_i^{-1}$, where g_i is a measurable selection of $(X_i(\cdot))_{z_i}$ and $h_g = (h_{g_1}, h_{g_2}, ...h_{g_n})$ for all $i \in I$. For each agent i, the constraints and the preferences are described by using the random fuzzy mappings A_i and P_i respectively. In the state of the world $\omega \in \Omega = \prod_{i\in I} \Omega_i$, the number $P_i(\omega_i, h_g)(y) \in [0, 1]$ associated to (h_g, y) can be interpreted as the degree of intensity with which y is preferred to $g_i(\omega_i)$ or the degree of truth with which y is preferred to $g_i(\omega_i)$. We also can see the value $A_i(\omega_i, h_g)(y) \in [0, 1]$, associated to (h_g, y), as the belief of the player i that in the state ω_i he can choose $y \in Y$. The element z_i is the action level in each state of the world, $a_i(h_g)$ expresses the perceived degree of feasibility of the strategy g and $p_i(h_g)$ represents the preference level of the strategy g.

The Bayesian fuzzy equilibrium problem for the general Bayesian abstract fuzzy economy model with private information and a countable of actions is defined by the following Definitions 3.2 and 3.4.

Definition 4.2. *A Bayesian fuzzy equilibrium problem* for Δ is defined as strategy profile $g^*=(g_1^*, g_2^*, ..., g_n^*) \in \prod_{i\in I} Meas(\Omega_i, S_i)$ such that for all $i \in I$,

(i) $g_i^*(\omega_i), \in cl((A_i(\omega_i, h_{g^*}))_{a_i(h_{g^*})})$ for each $\omega_i \in \Omega_i \quad \mu - a.e.$;

(ii) $(A_i(\omega_i, h_{g^*}))_{a_i(h_{g^*})} \cap (P_i(\omega_i, h_{g^*}))_{p_i(h_{g^*})} = \emptyset$ for each $\omega_i \in \Omega_i \quad \mu - a.e.$.

4.2. A Bayesian fuzzy equilibrium problem for a general Bayesian abstract fuzzy economy model with private information

In this subsection we establish the existence of a Bayesian fuzzy equilibrium problem for a general

Bayesian abstract fuzzy economy model with private information and a countable set of actions. The control condition and preference correspondences, comes from the constraint and preference fuzzy mappings, verifies the assumptions of measurable graph and weakly open lower sections. Our result is a generalization of Theorem 2 in [16].

Theorem 4.3. *Let $\Delta = (((\Omega_i, \mathcal{Z}_i)_{i\in I}, \mu), (X_i, \mathcal{F}_i, (A_i, a_i), (P_i, p_i), z_i)_{i\in I})$ be a general Bayesian abstract fuzzy economy with differential private information and a countable space of actions satisfying* $(a.1) - (a.6)$ *as follows:*

(a.1) $X_i : \Omega_i \to \mathcal{F}(Y)$ *is such that* $(X_i(\omega_i))_{z_i} : \Omega_i \to 2^{\mathcal{F}_i}$ *is a non-empty convex compact-valued;*

(a.2) $X_i : \Omega_i \to \mathcal{F}(Y)$ *is such that* $(X_i(\omega_i))_{z_i} : \Omega_i \to 2^{\mathcal{F}_i}$ *is* $\mathcal{F}_i-$ *lower measurable;*

(a.3) for each $\widehat{h}_g \in \mathcal{D}_{X,z}$*, the correspondence* $(A_i(\cdot, \widehat{h}_g))_{a_i(\widehat{h}_g)} : \Omega_i \to 2^{\mathcal{F}_i}$ *is measurable and, for all* $\omega_i \in \Omega_i$*, the correspondence* $(A_i(\omega_i, \cdot))_{a_i(\cdot)} : \mathcal{D}_{X,z} \to 2^{\mathcal{F}_i}$ *is upper semicontinuous with non-empty compact values;*

(a.4) for each $\widehat{h}_g \in \mathcal{D}_{X,z}$*, the correspondence* $(P_i(\cdot, \widehat{h}_g))_{p_i(\widehat{h}_g)} : \Omega_i \to 2^{\mathcal{F}_i}$ *is measurable and, for all* $\omega_i \in \Omega_i$*, the correspondence* $(P_i(\omega_i, \cdot))_{p_i(\cdot)} : \mathcal{D}_{X,z} \to 2^{\mathcal{F}_i}$ *is upper semicontinuous with non-empty compact values;*

(a.5) for each $\omega_i \in \Omega_i$ *and each* $g \in \prod_{i\in I} Meas(\Omega_i, S_i), g_i(\omega_i) \notin (P_i(\omega_i, h_g))_{p_i(h_g)}$*;*

(a.6) the set $\mathcal{U}_i^{\omega_i} := \{\widehat{h}_g \in \mathcal{D}_{X,z} : (A_i(\omega_i, \widehat{h}_g))_{a_i(\widehat{h}_g)} \cap (P_i(\omega_i, \widehat{h}_g))_{p_i(\widehat{h}_g)} = \emptyset\}$ *is open in* $\mathcal{D}_{X,z}$ *for each* $\omega_i \in \Omega_i$*.*

Then there exists a Bayesian fuzzy equilibrium for Δ*, where* μ *is atom less and for each* $i \in I$*.*

Proof. The fixed point approach can be capable benefited. To do this, we shall construct several correspondences as follows.

Let $i \in I$ be fixed and let us denote two sets

$$\mathcal{U}_i := \{(\omega_i, \widehat{h}_g) \in \Omega_i \times \mathcal{D}_{X,z} : (A_i(\omega_i, \widehat{h}_g))_{a_i(\widehat{h}_g)} \cap (P_i(\omega_i, \widehat{h}_g))_{p_i(\widehat{h}_g)} = \emptyset\}$$

and

$$\mathcal{U}_i^{\omega_i} := \{\widehat{h}_g \in \mathcal{D}_{X,z} : \widehat{h}_g(\omega_i) \in \mathcal{U}_i \quad \mu - a.e.\}.$$

We define bifunction $F_i : \Omega_i \times \mathcal{D}_{X,z} \to 2^{\mathcal{F}_i}$, for each $i \in I$ by

$$F_i(\omega_i, \widehat{h}_g) = \begin{cases} (A_i(\omega_i, \widehat{h}_g))_{a_i(\widehat{h}_g)} \cap (P_i(\omega_i, \widehat{h}_g))_{p_i(\widehat{h}_g)}, & \text{if} \quad (\omega_i, \widehat{h}_g) \notin \mathcal{U}_i \\ cl(A_i(\omega_i, \widehat{h}_g))_{a_i(\widehat{h}_g)}, & \text{if} \quad (\omega_i, \widehat{h}_g) \in \mathcal{U}_i \end{cases}$$

and

$$\Phi : \mathcal{D}_{X,z} \to 2^{\mathcal{D}_{X,z}}, \Phi(\widehat{h}_g) = \textstyle\prod_{i\in I} \mathcal{D}_{F_i}(\widehat{h}_g)$$

for each $\widehat{h}_g \in \mathcal{D}_{X,z}$, where $\mathcal{D}_{F_i}(\widehat{h}_g) = \{h_{g_i} = (\mu\tau_i^{-1})g_i^{-1} : g_i$ is a measurable selection of $F_i(\cdot, \widehat{h}_g)\}$.

First, we show that $\mathcal{D}_{X,z}$ is a nonempty convex and compact set.

Now we will apply the Ky Fan fixed point theorem to the correspondence Φ and so by Theorem 2.6 we obtain the existence of a fixed point, which becomes the equilibrium point for the abstract economy model Γ. For this objective, we check the properties of the involved sets and the correspondences F_i and Φ. Then, we note that $\mathcal{D}_{(X_i(\cdot))_{z_i}}$ is nonempty and convex. So by Lemma 2.7 and it is compact by Lemma 2.8. Consequently, the set $\mathcal{D}_{X,z}$ is also nonempty, compact and convex satisfy with the assumption *(a.1)*.

Next, since $((\Omega_i, \mathcal{F}_i)_{i\in I}, \mu)$ is a complete finite measure space, $\mathcal{F}_i$ is countable complete metric space, and $X_i : \Omega_i \to 2^{\mathcal{F}_i}$ has a measurable graph, by Aumann's selection theorem, Theorem 2.4, then there exists a Σ_i-measurable function $f_i : \Omega_i \to \mathcal{F}_i$ such that $f_i(\omega_i) \in X_i(\omega_i)$ for each $\omega_i \in \Omega_i$ and $i \in I$ μ-a.e. satisfy with the assumption *(a.2)*.

From the assumptions *(a.3)* and *(a.4)*, the correspondence F_i has nonempty and compact values and it is measurable with respect to Ω_i. The assumption *(a.6)* implies that the set $U_i^{\omega_i}$ is open in $\mathcal{D}_{X,z}$ and the assumptions *(a.3)* and *(a.4)* imply that for all $\omega_i \in \Omega_i$, $(A_i(\omega_i,\cdot))_{a_i(\cdot)}, (P_i(\omega_i,\cdot))_{p_i(\cdot)} : \mathcal{D}_{(X(\cdot))_z} \to 2^{S_i}$ are upper semicontinuous; therefore, we can apply Lemma 2.1 to confirm that F_i is upper semicontinuous with respect to $\widehat{h}_g \in \mathcal{D}_{X,z}$.

Moreover, for each $\widehat{h}_g \in \mathcal{D}_{X,z}, \mathcal{D}_{F_i}(\widehat{h}_g)$ is nonempty, convex and compact.

The nonemptiness of each $\mathcal{D}_{F_i}(\widehat{h}_g)$ by Theorem 2.5 implied by the existence of a measurable selection from the correspondence F_i. By Lemmas 2.7 and 2.8 also guarantee the convexity and compactness of the set $\mathcal{D}_{F_i}(\widehat{h}_g)$,where $\widehat{h}_g \in \mathcal{D}_{X,z}$.

From Lemma 2.9, the correspondence $\mathcal{D}_{F_i}$ is upper semicontinuous on $\mathcal{D}_{X_z}$. Then the correspondence Φ is upper semicontinuous and has nonempty, compact and convex values. We have also proved that it is defined on a nonempty, convex and compact set. We can apply the Ky Fan fixed point theorem, Theorem 2.6 to Φ, and we can obtain that there exists a fixed point $\widehat{h}_g^* \in \Phi(\widehat{h}_g^*)$. In particular, for each player i, $\widehat{h}_{g_i}^* \in \mathcal{D}_{F_i}(\widehat{h}_g^*)$. From the definition of $\mathcal{D}_{F_i}(\widehat{h}_g^*)$, we can conclude that for each player i, there exists $g_i^* \in Meas(\Omega_i, S_i)$ such that g_i^* is a selection of $F_i(\cdot, \widehat{h}_g^*)$ and $h_{g_i^*} = (\mu\tau_i^{-1})(g_i^*)^{-1} = \widehat{h}_g^*$.

Let us denote $h_{g^*} = (h_{g_1^*}, h_{g_2^*}, ..., h_{g_n^*})$.

Now we prove that g^* is a Bayesian fuzzy equilibrium problem for Δ.

For each $i \in I$, since g_i^* is a selection of $F_i(\cdot, h_{g_1^*}, ..., h_{g_n^*})$ it follows that:

$$g_i^*(\omega_i) \in (A_i(\omega_i, h_{g^*}))_{a_i(h_{g^*})} \cap (P_i(\omega_i, h_{g^*}))_{p_i(h_{g^*})}$$

if $(\omega_i, h_{g^*}) \notin \mathcal{U}_i$

or

$$g_i^*(\omega_i) \in (A_i(\omega_i, h_{g^*}))_{a_i(h_{g^*})}$$

if $(\omega_i, h_{g^*}) \in \mathcal{U}_i$.

By the assumption *(a.5)*, it follows that $g_i^* \notin (P_i(\omega_i, h_{g^*}))_{p_i(h_{g^*})}$ for each $\omega_i \in \Omega_i$ $\mu - a.e.$.

Then

$$g_i^*(\omega_i) \in cl(A_i(\omega_i, h_{g^*}))_{a_i(h_{g^*})}$$

and

$$(\omega_i, h_{g^*}) \in \mathcal{U}_i \quad \mu - a.e..$$

Consequently, this is equivalent to the fact that

$$g_i^*(\omega_i) \in cl(A_i(\omega_i, h_{g^*}))_{a_i(h_{g^*})}$$

and

$$(A_i(\omega_i, h_{g^*}))_{a_i(h_{g^*})} \cap (P_i(\omega_i, h_{g^*}))_{p_i(h_{g^*})} = \emptyset,$$

for each $\omega_i \in \Omega_i$ and $i \in I$. Therefore, $g^* = (g_1^*, g_2^*, ..., g_n^*)$ is a equilibrium for Δ. □

If index $|I|$ is 1, then we obtain the following corollary of Theorem 4.3 as follows:

Corollary 4.4. *Let $\Delta = ((\Omega, \mathcal{Z}, \mu), \mathcal{F}, X, (A, a), (P, p), z)$ be an abstract fuzzy economy model, where $\mathcal{F}$ is a countable complete metric space and (Ω, Σ) is a measure space and μ is atomless. Suppose that the following conditions are satisfied:*

(a.1) the correspondence $X(\cdot)_z : \Omega \to 2^{\mathcal{F}}$ is compact valued;

(a.2) *for each* $\widehat{h}_g \in \mathcal{D}_{X,z}$, *the correspondence* $(A(\cdot,\widehat{h}_g))_{a(\widehat{h}_g)} : \Omega \to 2^{\mathcal{F}}$ *is measurable and, for all* $\omega \in \Omega$, *the correspondence* $(A(\omega,\cdot))_{a(\cdot)} : \mathcal{D}_{X,z} \to 2^{\mathcal{F}}$ *is upper semicontinuous with non-empty compact values;*

(a.3) *for each* $\widehat{h}_g \in \mathcal{D}_{X,z}$, *the correspondence* $(P(\cdot,\widehat{h}_g))_{p(\widehat{h}_g)} : \Omega \to 2^{\mathcal{F}}$ *is measurable and, for all* $\omega \in \Omega$, *the correspondence* $(P(\omega,\cdot))_{p(\cdot)} : \mathcal{D}_{X,z} \to 2^{\mathcal{F}}$ *is upper semicontinuous with non-empty compact values;*

(a.4) *for each* $\omega \in \Omega$ *and each* $g \in Meas(\Omega,\mathcal{F}), g(\omega) \notin (P(\omega,h_g))_{p(h_g)}$;

(a.5) *the set* $\mathcal{U}^{\omega} := \{\widehat{h}_g \in \mathcal{D}_{X,z} : (A_i(\omega,\widehat{h}_g))_{a(\widehat{h}_g)} \cap (P(\omega,\widehat{h}_g))_{p(\widehat{h}_g)} = \emptyset\}$ *is open in* $\mathcal{D}_{X,z}$ *for each* $\omega \in \Omega$.

Then there exists a fuzzy equilibrium for Δ.

Proof. Set $|I| = 1$, then it follows from Theorem 4.3. □

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