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Best proximity points for cyclic Kannan-Chatterjea-Ćirić type contractions on metric-like spaces

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Abstract

In this paper, we establish some best proximity results for Kannan-Chatterjea-Ćirić type contractions in the setting of metric-like spaces. We also provide some concrete examples illustrating the obtained results.

Keywords: Metric-like, best proximity point, Kannan-Chatterjea-Ćirić contraction.
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1. Introduction and preliminaries

The existence and approximation of best proximity points is an interesting topic in optimization theory. In 2003, Kirk et al. [21] introduced the notion of cyclical contractive mappings, and generalized Banach fixed point result [5] to the class of cyclic mappings.

Theorem 1.1 ([21]). *Let A and B be nonempty closed subsets of a complete metric space (X, d) and let $T : A \cup B \to A \cup B$ be such that*

$$T(A) \subset B \quad \text{and} \quad T(B) \subset A. \tag{1.1}$$

Assume that, for all $x \in A$ and $y \in B$

$$d(Tx, Ty) \le \alpha\, d(x, y), \tag{1.2}$$

where $\alpha \in (0, 1)$. Then, T has a unique fixed point $u \in A \cap B$.

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A mapping satisfying (1.1) is called cyclic. In [10], Eldred and Veeramani are concerned with the case when $A \cap B = \emptyset$, and in this case they didn't seek for the existence of a fixed point of T but for the existence of a best proximity point. For instance, they [10] presented the following existence best proximity point result for cyclic contractions.

Theorem 1.2 ([10])**.** *Let A and B be nonempty closed and convex subsets of a complete metric space (X, d) and let $T : A \cup B \to A \cup B$ be cyclic. Assume that, for all $x \in A$ and $y \in B$*

$$d(Tx, Ty) \leq \alpha\, d(x, y) + (1 - \alpha)d(A, B), \tag{1.3}$$

where $\alpha \in (0, 1)$ and $d(A, B) = \inf\{d(x, y),\ x \in A,\ y \in B\}$. For $x_0 \in A$, define $x_{n+1} = Tx_n$ for each $n \geq 0$. Then, there exists a unique $x \in A$ such that $x_{2n} \to x$ and

$$d(x, Tx) = d(A, B).$$

Here, x is called a best proximity point of T.

In [27], Thagafi and Shahzad introduced a new class of mappings known as cyclic φ-contraction and proved some convergence and existence results for best proximity points. In 2011, Sadiq Basha [6] stated the best proximity points theorems for proximal contractions. For other best proximity point results, see [1, 7, 8, 14, 15, 16, 17, 18, 19, 20, 22, 23, 25, 26, 28]. In this paper, we are concerned with the existence of best proximity points for cyclic Kannan-Chatterjea-Ćirić type contractions in the class of metric-like spaces.

On the other hand, metric-like spaces were considered by Hitzler and Seda [13] under the name of dislocated metric spaces. In what follows, we recall some notations and definitions we will need in the sequel.

Definition 1.3. Let X be a nonempty set. A function $\sigma : X \times X \to \mathbb{R}^+$ is said to be a b-metric-like (or a dislocated b-metric) on X if for any $x, y, z \in X$, the following conditions hold:

(σ_1) $\sigma(x, y) = 0 \Longrightarrow x = y$;

(σ_2) $\sigma(x, y) = \sigma(y, x)$;

(σ_3) $\sigma(x, z) \leq \sigma(x, y) + \sigma(y, z)$.

The pair (X, σ) is then called a metric-like space. For (common) fixed point results on metric-like spaces, see [2, 3, 4, 11, 12].

Let (X, σ) be a metric-like space. A sequence $\{x_n\}$ in X converges to $x \in X$ if and only if

$$\lim_{n\to\infty} \sigma(x_n, x) = \sigma(x, x). \tag{1.4}$$

$\{x_n\}$ is Cauchy in (X, σ) if and only if $\lim\limits_{n,m\to\infty} \sigma(x_n, x_m)$ exists and is finite. Moreover, (X, σ) is complete if and only if each Cauchy sequence in X is convergent. For A and B two nonempty subsets of a metric-like space (X, σ), define

$$\sigma(A, B) = \inf\{\sigma(a, b) :\ a \in A,\ b \in B\}.$$

Again, the definition of a best proximity point is as follows.

Definition 1.4. Let (X, σ) be a metric-like space. Consider A and B two nonempty subsets of X. An element $a \in X$ is said to be a best proximity point of $T : A \to B$ if

$$\sigma(a, Ta) = \sigma(A, B).$$

Now, we introduce different type contractions.

Definition 1.5. Let (X,σ) be a metric-like space. Let A and B be nonempty subsets of X. Take the cyclic mapping $T : A \cup B \to A \cup B$.

(i) T is said to be a cyclic Kannan type contraction if

$$\sigma(Tx,Ty) \leq k(\sigma(x,Tx) + \sigma(y,Ty)) + (1-2k)\sigma(A,B) \tag{1.5}$$

for all $x \in A$ and $y \in B$, where $k \in (0, \frac{1}{2})$.

(ii) T is said to be a cyclic Chatterjee type contraction if

$$\sigma(Tx,Ty) \leq k(\sigma(Tx,y) + \sigma(Ty,x)) + (1-4k)\sigma(A,B) \tag{1.6}$$

for all $x \in A$ and $y \in B$, where $k \in (0, \frac{1}{4})$.

(iii) T is said to be a cyclic Ćirić type contraction

$$\sigma(Tx,Ty) \leq k \max\{\sigma(x,y), \sigma(Tx,x), \sigma(Ty,y)\} + (1-k)\sigma(A,B) \tag{1.7}$$

for all $x \in A$ and $y \in B$, where $k \in (0,1)$.

In this paper, we establish some existence results on best proximity points for various α-proximal contractions in the setting of metric-like spaces. We will support the obtained theorems by some concrete examples where some known results in literature are not applicable.

2. Main results

The first main result is

Theorem 2.1. *Let A and B be nonempty closed subsets of a complete metric-like space (X,σ). Let $T : A \cup B \to A \cup B$ be a cyclic Kannan type mapping. For $x_0 \in A \cup B$, define $x_{n+1} = Tx_n$ for each $n \geq 0$. Then*

$$\sigma(x_n, x_{n+1}) \to \sigma(A,B) \quad \text{as} \quad n \to \infty. \tag{2.1}$$

We have:

(*a*) *If $x_0 \in A$ and $\{x_{2n}\}$ has a subsequence $\{x_{2n_i}\}$ converging to $u \in A$ with $\sigma(u,u) = 0$, then $u \in A$ is a best proximity point of T, that is,*

$$\sigma(u,Tu) = \sigma(A,B). \tag{2.2}$$

(*b*) *If $x_0 \in B$ and $\{x_{2n-1}\}$ has a subsequence $\{x_{2n_i-1}\}$ converging to $v \in B$ with $\sigma(v,v) = 0$, then $v \in B$ is a best proximity point of T, that is,*

$$\sigma(v,Tv) = \sigma(A,B). \tag{2.3}$$

Proof. Let $x_0 \in A \cup B$. Define $x_{n+1} = Tx_n$ for all $n \geq 0$. By (1.5), we have

$$\begin{aligned}
\sigma(x_{n+2},x_{n+1}) =& \sigma(Tx_{n+1},Tx_n) \leq k(\sigma(x_{n+1},Tx_{n+1}) + \sigma(x_n,Tx_n)) + (1-2k)\sigma(A,B)\\
=& k(\sigma(x_{n+1},x_{n+2}) + \sigma(x_n,x_{n+1})) + (1-2k)\sigma(A,B)\\
\leq& k(\sigma(x_{n+1},x_{n+2}) + \sigma(x_n,x_{n+1})) + (1-2k)\sigma(x_n,x_{n+1})\\
=& k\sigma(x_{n+1},x_{n+2}) + (1-k)\sigma(x_n,x_{n+1}).
\end{aligned}$$

Thus,

$$\sigma(x_{n+2},x_{n+1}) \leq \sigma(x_{n+1},x_n) \quad \text{for all} \quad n \geq 0,$$

that is, $\{\sigma(x_{n+1}, x_n)\}$ is nonincreasing and is bounded below, so there exists $t \geq 0$ such that $\lim_{n\to\infty} \sigma(x_{n+1}, x_n) = t$. We know that

$$\sigma(A, B) \leq \sigma(x_{n+2}, x_{n+1}) \leq k(\sigma(x_{n+1}, x_{n+2}) + \sigma(x_n, x_{n+1})) + (1 - 2k)\sigma(A, B),$$

so letting $n \to \infty$, we deduce that $t = \sigma(A, B)$, i.e., $\lim_{n\to\infty} \sigma(x_{n+1}, x_n) = \sigma(A, B)$.

Assume that $x_0 \in A$. Since T is cyclic, so $\{x_{2n}\} \in A$ and $\{x_{2n+1}\} \in B$ for all $n \geq 0$. Now, if $\{x_{2n}\}$ has a subsequence $\{x_{2n_i}\}$ converging to $u \in A$ with $\sigma(u, u) = 0$, then

$$\lim_{i\to\infty} \sigma(x_{2n_i}, u) = \sigma(u, u) = 0.$$

We have

$$\begin{aligned}\sigma(A, B) \leq \sigma(u, Tu) &\leq \sigma(u, x_{2n_i}) + \sigma(x_{2n_i}, Tu)\\ &= \sigma(u, x_{2n_i}) + \sigma(Tx_{2n_i-1}, Tu)\\ &\leq \sigma(u, x_{2n_i}) + k[\sigma(x_{2n_i}, x_{2n_i-1}) + \sigma(Tu, u)] + (1 - 2k)\sigma(A, B).\end{aligned}$$

Letting $i \to \infty$, using (2.1) we obtain

$$\sigma(A, B) \leq \sigma(u, Tu) \leq k\sigma(u, Tu) + (1 - k)\,\sigma(A, B).$$

Thus, $\sigma(u, Tu) = \sigma(A, B)$, that is, u is best proximity of T.

The proof of case (b) is similar to above case. □

The following example makes effective Theorem 2.1.

Example 2.2. Let $X = \{0, 1, 2, 3\}$ be endowed with the metric-like σ

$$\sigma(x, y) = x + y \quad \text{for all} \quad x, y \in X.$$

(X, σ) is a complete metric-like space. Take $A = \{0\}$ and $B = \{1, 2\}$. We have $\sigma(A, B) = 1$. Choose $T : A \cup B \to A \cup B$ as

$$T0 = 1 \quad \text{and} \quad T1 = T2 = 0.$$

We have $T(A) = \{1\} \subset B$ and $T(B) = \{0\} = A$. Let $k \in (0, \frac{1}{2})$.
Let $x \in A$ and $y \in B$, then $x = 0$ and $y \in \{1, 2\}$. In this case, we have

$$\begin{aligned}\sigma(Tx, Ty) = \sigma(1, 0) &= 1 = 2k + 1 - 2k \leq k(y + 1) + (1 - 2k)\\ &= k(0 + 1 + y + 0) + (1 - 2k)\sigma(A, B)\\ &= k(\sigma(x, Tx) + \sigma(y, Ty)) + (1 - 2k)\sigma(A, B).\end{aligned}$$

Thus (1.5) holds for all $x \in A$ and $y \in B$.

Now, choose $x_0 \in A \cup B$ such that $x_{n+1} = Tx_n$ for all $n \geq 0$. If $x_0 \in A$, then $x_{2n} = 0$ and $x_{2n+1} = 1$ for all $n \geq 0$. While, if $x_0 \in B$, then $x_{2n} = 1$ for all $n \geq 1$ and $x_{2n+1} = 0$ for all $n \geq 0$. We conclude that, for all $n \geq 1$

$$\sigma(x_n, x_{n+1}) = \sigma(1, 0) = 1 = \sigma(A, B),$$

that is, (2.1) is verified.

In the case $x_0 \in A$, we have $x_{2n} = 0$, so it has a subsequence $\{x_{2n_i}\}$ converging to $u = 0 \in A$. Here, $\sigma(u, u) = 0$ and $u = 0$ is a best proximity point of T, that is,

$$\sigma(0, T0) = 1 = \sigma(A, B).$$

On the other hand, we could not apply Theorem 3.6 of [11]. In fact for $x = 0$ and $y = 2$, we have

$$\sigma(Tx, Ty) = 1 > 3\alpha = \alpha(\sigma(Tx, x) + \sigma(Ty, y)) \quad \text{for all} \quad \alpha \in (0, \frac{1}{3}).$$

Also, Theorem 1.2 (the main result of [10]) is not applicable for the standard metric d. Indeed, for $x = 0 \in A$ and $y = 1 \in B$

$$d(Tx, Ty) = 1 > \alpha = \alpha d(x, y) + (1 - \alpha)d(A, B) \quad \text{for all} \quad \alpha \in (0, 1).$$

The second main result is,

Theorem 2.3. *Let A and B be nonempty closed subsets of a complete metric-like space (X,σ). Let $T : A \cup B \to A \cup B$ be a cyclic Chatterjee type mapping. For $x_0 \in A \cup B$, define $x_{n+1} = Tx_n$ for each $n \geq 0$. Then*

$$\sigma(x_n, x_{n+1}) \to \sigma(A,B) \quad as \quad n \to \infty. \tag{2.4}$$

We have:

(a) If $x_0 \in A$ and $\{x_{2n}\}$ has a subsequence $\{x_{2n_i}\}$ converging to $u \in A$ with $\sigma(u,u) = 0$, then $u \in A$ is a best proximity point of T, that is,

$$\sigma(u, Tu) = \sigma(A,B). \tag{2.5}$$

(b) If $x_0 \in B$ and $\{x_{2n-1}\}$ has a subsequence $\{x_{2n_i-1}\}$ converging to $v \in B$ with $\sigma(v,v) = 0$, then $v \in B$ is a best proximity point of T, that is,

$$\sigma(v, Tv) = \sigma(A,B). \tag{2.6}$$

Proof. Let $x_0 \in A \cup B$. Define $x_{n+1} = Tx_n$ for all $n \geq 0$. By (1.5), we have

$$\begin{aligned}\sigma(x_{n+2}, x_{n+1}) =& \sigma(Tx_{n+1}, Tx_n) \leq k(\sigma(Tx_{n+1}, x_n) + \sigma(Tx_n, x_{n+1})) + (1-4k)\sigma(A,B)\\ =& k(\sigma(x_{n+2}, x_n) + \sigma(x_{n+1}, x_{n+1})) + (1-4k)\sigma(A,B)\\ \leq& k(\sigma(x_{n+2}, x_{n+1}) + \sigma(x_{n+1}, x_n) + 2\sigma(x_n, x_{n+1})) + (1-4k)\sigma(x_n, x_{n+1})\\ =& k\sigma(x_{n+1}, x_{n+2}) + (1-k)\sigma(x_n, x_{n+1}).\end{aligned}$$

Thus,

$$\sigma(x_{n+2}, x_{n+1}) \leq \sigma(x_{n+1}, x_n) \quad \text{for all} \quad n \geq 0.$$

So, there exists $t \geq 0$ such that $\lim_{n\to\infty} \sigma(x_{n+1}, x_n) = t$. We know that

$$\sigma(A,B) \leq \sigma(x_{n+2}, x_{n+1}) \leq k(\sigma(x_{n+2}, x_{n+1}) + 3\sigma(x_n, x_{n+1})) + (1-4k)\sigma(A,B).$$

Letting $n \to \infty$, we deduce that $t = \sigma(A,B)$, i.e., $\lim_{n\to\infty} \sigma(x_{n+1}, x_n) = \sigma(A,B)$.

Assume that $x_0 \in A$. Again, T is cyclic, so $\{x_{2n}\} \in A$ and $\{x_{2n+1}\} \in B$ for all $n \geq 0$. Now, if $\{x_{2n}\}$ has a subsequence $\{x_{2n_i}\}$ converging to $u \in A$ with $\sigma(u,u) = 0$, then

$$\lim_{i\to\infty} \sigma(x_{2n_i}, u) = \sigma(u,u) = 0.$$

We have

$$\begin{aligned}\sigma(A,B) \leq \sigma(u, Tu) \leq& \sigma(u, x_{2n_i}) + \sigma(x_{2n_i}, Tu)\\ =& \sigma(u, x_{2n_i}) + \sigma(Tx_{2n_i-1}, Tu)\\ \leq& \sigma(u, x_{2n_i}) + k[\sigma(x_{2n_i+1}, u) + \sigma(Tu, x_{2n_i-1})] + (1-4k)\sigma(A,B).\end{aligned}$$

Letting $i \to \infty$, we obtain using (2.4)

$$\sigma(A,B) \leq \sigma(u,Tu) \leq k\sigma(u,Tu) + (1-4k)\,\sigma(A,B) \leq k\sigma(u,Tu) + (1-k)\,\sigma(A,B).$$

Thus, $\sigma(u, Tu) = \sigma(A,B)$, that is, u is a best proximity of T.

The proof of case (b) is similar to above case. □

We present the following example.

Example 2.4. Let $X = \{0,1\}$ endowed with the metric-like

$$\sigma(0,0) = \sigma(1,1) = 2 \quad \text{and} \quad \sigma(0,1) = \sigma(1,0) = 1.$$

Note that (X, σ) is a complete metric-like space. Take $k \in (0, \frac{1}{4})$. Let $A = \{0\}$ and $B = \{1\}$. Note that $\sigma(A, B) = 1$ and A, B are closed in (X, σ). Consider $T : A \cup B \to A \cup B$ defined by

$$T0 = 1 \quad \text{and} \quad T1 = 0.$$

Clearly, T is cyclic. Let $x \in A$ and $y \in B$, that is, $x = 0$ and $y = 1$. In this case, we have

$$\begin{aligned}\sigma(T0, T1) = \sigma(1, 0) = 1 = 4k + (1 - 4k) &= k\Big(\sigma(0, 0) + \sigma(1, 1)\Big) + (1 - 4k)\sigma(A, B)\\ &= k\Big(\sigma(0, T1) + \sigma(1, T0)\Big) + (1 - 4k)\sigma(A, B),\end{aligned}$$

that is, (1.6) holds, i.e., T is a cyclic Chatterjee type contraction.

Let $x_0 \in A \cup B$ and $x_{n+1} = Tx_n$ for $n \geq 0$. If $x_0 \in A$, then $x_{2n} = 0$ and $x_{2n+1} = 1$ for all $n \geq 0$. While, if $x_0 \in B$, then $x_{2n} = 1$ for all $n \geq 1$ and $x_{2n+1} = 0$ for all $n \geq 0$. We conclude that, for all $n \geq 1$

$$\sigma(x_n, x_{n+1}) = \sigma(0, 1) = 1 = \sigma(A, B),$$

that is, (2.4) is satisfied. Mention that T has two best proximity points. Indeed, we have $\sigma(0, T0) = \sigma(1, T1) = 1 = \sigma(A, B)$.

On the other hand, Corollary 2.2 (with $m = 2$) of Chandok and Postolache [9] is not applicable for the standard metric. Indeed, for $x = 0 \in A$ and $y = 1 \in B$, we have

$$d(T0, T1) = 1 > 0 = \alpha(d(0, T1) + d(1, T0)),$$

for all $\alpha \in (0, \frac{1}{2})$.

The third main result is,

Theorem 2.5. *Let A and B be nonempty closed subsets of a complete metric-like space (X, σ) such that $A \cap B = \emptyset$. Let $T : A \cup B \to A \cup B$ be a cyclic Ćirić type mapping. For $x_0 \in A \cup B$, define $x_{n+1} = Tx_n$ for each $n \geq 0$. Then*

$$\sigma(x_n, x_{n+1}) \to \sigma(A, B) \quad \text{as} \quad n \to \infty. \tag{2.7}$$

We have:

(a) If $x_0 \in A$ and $\{x_{2n}\}$ has a subsequence $\{x_{2n_i}\}$ converging to $u \in A$ with $\sigma(u, u) = 0$, then $u \in A$ is a best proximity point of T, that is,

$$\sigma(u, Tu) = \sigma(A, B). \tag{2.8}$$

(b) If $x_0 \in B$ and $\{x_{2n-1}\}$ has a subsequence $\{x_{2n_i-1}\}$ converging to $v \in B$ with $\sigma(v, v) = 0$, then $v \in B$ is a best proximity point of T, that is,

$$\sigma(v, Tv) = \sigma(A, B). \tag{2.9}$$

Proof. Let $x_0 \in A \cup B$. Define $x_{n+1} = Tx_n$ for all $n \geq 0$. Since $A \cap B = \emptyset$, we have $\sigma(A, B) > 0$. Then, $\sigma(x_{n+2}, x_{n+1}) > 0$ for all $n \geq 0$. By (1.5), we have

$$\begin{aligned}\sigma(x_{n+2}, x_{n+1}) =&\sigma(Tx_{n+1}, Tx_n) \leq k \max\{\sigma(x_{n+1}, x_n), \sigma(Tx_{n+1}, x_{n+1}), \sigma(Tx_n, x_n)\} + (1 - k)\sigma(A, B)\\ =&k \max\{\sigma(x_{n+1}, x_n), \sigma(x_{n+2}, x_{n+1}), \sigma(x_{n+1}, x_n)\} + (1 - k)\sigma(A, B)\\ =&k \max\{\sigma(x_{n+1}, x_n), \sigma(x_{n+2}, x_{n+1})\} + (1 - k)\sigma(x_n, x_{n+1}).\end{aligned}$$

If for some n, we have $\max\{\sigma(x_{n+1}, x_n), \sigma(x_{n+2}, x_{n+1})\} = \sigma(x_{n+2}, x_{n+1})$. Then,

$$0 < \sigma(x_{n+2}, x_{n+1}) \leq k\sigma(x_{n+2}, x_{n+1}) + (1 - 2k)\sigma(A, B) \leq (1 - k)\sigma(x_{n+2}, x_{n+1}).$$

It is a contradiction. Thus,

$$\sigma(x_{n+2}, x_{n+1}) \leq \sigma(x_{n+1}, x_n) \quad \text{for all} \quad n \geq 0.$$

So, there exists $t \geq 0$ such that $\lim_{n\to\infty} \sigma(x_{n+1}, x_n) = t$. We know that

$$\sigma(A, B) \leq \sigma(x_{n+2}, x_{n+1}) \leq k(\sigma(x_{n+1}, x_n) + (1-k)\sigma(A, B),$$

so letting $n \to \infty$, we deduce that $t = \sigma(A, B)$, i.e., $\lim_{n\to\infty} \sigma(x_{n+1}, x_n) = \sigma(A, B)$.

Assume that $x_0 \in A$. Again, T is cyclic, so $\{x_{2n}\} \in A$ and $\{x_{2n+1}\} \in B$ for all $n \geq 0$. Now, if $\{x_{2n}\}$ has a subsequence $\{x_{2n_i}\}$ converging to $u \in A$ with $\sigma(u, u) = 0$, then

$$\lim_{i\to\infty} \sigma(x_{2n_i}, u) = \sigma(u, u) = 0.$$

We have

$$\begin{aligned}\sigma(A, B) \leq& \sigma(u, Tu) \leq \sigma(u, x_{2n_i}) + \sigma(x_{2n_i}, Tu)\\ =&\sigma(u, x_{2n_i}) + \sigma(Tx_{2n_i-1}, Tu)\\ \leq&\sigma(u, x_{2n_i}) + k \max\{\sigma(x_{2n_i-1}, u), \sigma(x_{2n_i}, x_{2n_i-1}), \sigma(Tu, u)] + (1-k)\sigma(A, B).\end{aligned}$$

Letting $i \to \infty$, from (2.7), we get

$$\sigma(A, B) \leq \sigma(u, Tu) \leq k\sigma(u, Tu) + (1-k)\,\sigma(A, B).$$

Thus, $\sigma(u, Tu) = \sigma(A, B)$, that is, u is a best proximity of T.

The proof of case (b) is similar to above case. □

Now, we provide an example illustrating Theorem 2.5.

Example 2.6. Let $X = [0, \infty) \times [0, \infty)$ endowed with the metric-like $\sigma : X \times X \to [0, \infty)$ given as

$$\sigma((x_1, x_2), (y_1, y_2)) = \begin{cases} |x_1 - y_1| + |x_2 - y_2| & \text{if} \quad (x_1, x_2), (y_1, y_2) \in [0, 1]^2 \\ x_1 + x_2 + y_1 + y_2 & \text{if not.} \end{cases}$$

It is easy to prove that (X, σ) is a complete metric-like space. Take $A = \{0\} \times [0, 1]$ and $B = \{1\} \times [0, 1]$. Remark that $\sigma(A, B) = \sigma((0, 0), (1, 0)) = 1$. Consider the mapping $T : A \cup B \to B \cup A$ defined by

$$T(0, x) = (1, \frac{x}{4}) \quad \forall\, x \in [0, 1],$$

and

$$T(1, x) = (0, \frac{x}{4}) \quad \forall\, x \in [0, 1].$$

We have $T(A) \subset B$ and $T(B) \subset A$. Take $k = \frac{1}{4}$. Now, let $(0, x) \in A$ and $(1, y) \in B$. We have $x, y \in [0, 1]$. In this case, we have

$$\sigma(T(0, x), T(1, y)) = 1 + |\frac{x}{4} - \frac{y}{4}|.$$

Moreover,

$$\begin{aligned}k \max\{\sigma((0, x),&(1, y)), \sigma((0, x), T(0, x)), \sigma((1, y), T(1, y))\} + (1-k)\sigma(A, B)\\ &= k \max\{1 + |x - y|, 1 + |x - \frac{x}{4}|, 1 + |y - \frac{y}{4}\}\Big) + (1-k)\\ &= 1 + k \max\{|x - y|, \frac{3x}{4}, \frac{3y}{4}\}.\end{aligned}$$

It is obvious that (1.7) holds, that is, T is a cyclic Ćirić type contraction. Let $X_0 = (0, x_0) \in A$ and $X_{n+1} = T(X_n)$ for $n \geq 0$. Here, we get

$$X_{2n} = (0, \frac{x_0}{2^{2n}}) \in A \quad \text{and} \quad X_{2n+1} = (1, \frac{x_0}{2^{4n+2}}) \in B \quad \text{for all} \quad n \geq 0.$$

We have, as $n \to \infty$,

$$\sigma(x_{2n}, x_{2n+1}) = 1 + |\frac{x_0}{2^{4n+2}} - \frac{x_0}{2^{2n}}| \to 1 = \sigma(A, B).$$

Moreover, $\sigma(x_{2n-1}, x_{2n}) \to 0$. Thus, from above, (2.7) holds. Now, let $X_0 = (1, x_0) \in B$ and $X_{n+1} = T(X_n)$ for $n \geq 0$. In this case, we have

$$X_{2n} = (1, \frac{x_0}{2^{2n}}) \in B \quad \text{and} \quad X_{2n+1} = (0, \frac{x_0}{2^{4n+2}}) \in A \quad \text{for all} \quad n \geq 0.$$

Similarly, in this case, (2.7) holds.

On the other hand, Theorem 3.10 in [11] is not applicable. Indeed, for $(0, 0) \in A$ and $(1, 0) \in B$, we have

$$\sigma(T(0,0), T(1,0)) = 1 > \alpha = \alpha \max\{\sigma((0,0),(1,0)), \sigma(T(0,0),(0,0)), \sigma(T(1,0),(1,0))\}$$

for all $\alpha \in (0, 1)$.

Remark 2.7. We may state the following remarks:

- Theorem 2.1 is a generalization of Theorem 3.6 of George and Rajagopalan [11] and extends Theorem 4 of Petrić [24] to the class of metric-like spaces.
- Theorem 2.3 is a generalization of Theorem 3.8 of George and Rajagopalan [11].
- Theorem 2.5 is a generalization of Theorem 3.10 of George and Rajagopalan [11] and extends Theorem 1.2 to the class of metric-like spaces.

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