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The form of the solutions of nonlinear difference equations systems

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Abstract

In this paper, we deal with the form of the solutions of the following nonlinear difference equations systems

$$x_{n+1} = \frac{x_{n-7}}{1 + x_{n-7}y_{n-3}}, \qquad y_{n+1} = \frac{y_{n-7}}{\pm 1 \pm x_{n-3}y_{n-7}},$$

where the initial conditions $x_{-7},\ x_{-6},\ x_{-5},\ x_{-4},\ x_{-3},\ x_{-2},\ x_{-1},\ x_0,\ y_{-7},\ y_{-6},\ y_{-5},\ y_{-4},\ y_{-3},\ y_{-2},\ y_{-1},\ y_0$ are real numbers.

Keywords: Difference equations, system of difference equations, solution of difference equation.
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1. Introduction

It is very rare that a sequence of events is determined by the state of one entity. Thus describing a sequence of events with a reasonable amount of accuracy is rarely effective with a single difference equation. However, describing such a sequence generally requires more than just several difference equations. Just as separate entities influence and are influenced by one another, a collection of difference equations that describe several entities often depend on each other.

The increasing study of realistic mathematical models is a reflection of their use in helping to understand the dynamic processes involved in areas such as population dynamics, biology, epidemiology, ecology, and

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economy. More realistic models should include some of the past states of these systems; that is, ideally, a real system should be modeled by difference equations with time delays. Most of these models are described by nonlinear delay difference equations; see, for example, [4]–[20]. The subject of the qualitative study of the nonlinear delay population models is very extensive, and the current research work tends to center around the relevant global dynamics of the considered systems of difference equations such as oscillation, boundedness of solutions, persistence, global stability of positive steady states, permanence, and global existence of periodic solutions. See [1], [19], [22]-[50] and the references therein. In particular, Cinar [2] studied the periodicity of the positive solutions of the system of the difference equations

$$x_{n+1} = \frac{1}{y_n}, \qquad y_{n+1} = \frac{y_n}{x_{n-1}y_{n-1}}.$$

In [4] Clark and Kulenovic investigated the global asymptotic stability of the following system

$$x_{n+1} = \frac{x_n}{a + cy_n}, \qquad y_{n+1} = \frac{y_n}{b + dx_n}.$$

In [6] Din deal with the boundedness character, steady-states, the local asymptotic stability of equilibrium points, and global behavior of the unique positive equilibrium point of a discrete predator-prey model given by

$$x_{n+1} = \frac{\alpha x_n - \beta x_n y_n}{1 + \gamma x_n}, \qquad y_{n+1} = \frac{\delta x_n y_n}{x_n + \eta y_n}.$$

Elsayed and El-Metwally [25] studied the periodic nature and the form of the solutions of the nonlinear difference equations systems

$$x_{n+1} = \frac{x_n y_{n-2}}{y_{n-1}\left(\pm 1 \pm x_n y_{n-2}\right)}, \qquad y_{n+1} = \frac{y_n x_{n-2}}{x_{n-1}\left(\pm 1 \pm y_n x_{n-2}\right)}.$$

Gelisken and Kara [31] investigated some behavior of solutions of some systems of rational difference equations of higher order and they showed that every solution is periodic with a period which depends on the order.

In [38] Kurbanli studied a three-dimensional system of rational difference equations

$$x_{n+1} = \frac{x_{n-1}}{x_{n-1}y_n - 1}, \; y_{n+1} = \frac{y_{n-1}}{y_{n-1}x_n - 1}, \; z_{n+1} = \frac{x_n}{z_{n-1}y_n}.$$

Touafek et al. [45] investigated the boundedness, and the form of the solutions of the following systems of rational difference equations

$$x_{n+1} = \frac{x_{n-3}}{\pm 1 \pm x_{n-3}y_{n-1}}, \qquad y_{n+1} = \frac{y_{n-3}}{\pm 1 \pm y_{n-3}x_{n-1}}.$$

Yalçınkaya [48] obtained the sufficient conditions for the global asymptotic stability of the system of two nonlinear difference equations

$$x_{n+1} = \frac{x_n + y_{n-1}}{x_n y_{n-1} - 1}, \qquad y_{n+1} = \frac{y_n + x_{n-1}}{y_n x_{n-1} - 1}.$$

Our goal in this paper is to investigate the form of the solutions of the following nonlinear difference equations systems

$$x_{n+1} = \frac{x_{n-7}}{1 + x_{n-7}y_{n-3}}, \qquad y_{n+1} = \frac{y_{n-7}}{\pm 1 \pm x_{n-3}y_{n-7}},$$

with real number's initial conditions x_{-7}, $x_{-6}, x_{-5}, x_{-4}, x_{-3}, x_{-2}$, x_{-1}, $x_0, y_{-7}, y_{-6}, y_{-5}, y_{-4}$, y_{-3}, y_{-2}, y_{-1}, y_0.

2. Main Results

2.1. The First System: $x_{n+1} = \frac{x_{n-7}}{1+x_{n-7}y_{n-3}}$, $y_{n+1} = \frac{y_{n-7}}{1+y_{n-7}x_{n-3}}$.

In this subsection we study the solutions of the system of two difference equations

$$x_{n+1} = \frac{x_{n-7}}{1+x_{n-7}y_{n-3}}, \qquad y_{n+1} = \frac{y_{n-7}}{1+y_{n-7}x_{n-3}}, \tag{2.1}$$

with a real number's initial conditions.

Theorem 2.1. *Suppose that $\{x_n, y_n\}$ are solutions of system (2.1). Also, assume that the initial conditions are arbitrary real numbers and let $x_{-7} = a$, $x_{-6} = b$, $x_{-5} = c$, $x_{-4} = d$, $x_{-3} = e$, $x_{-2} = f$, $x_{-1} = g$, $x_0 = h$, $y_{-7} = p$, $y_{-6} = q$, $y_{-5} = r$, $y_{-4} = s$, $y_{-3} = t$, $y_{-2} = u$, $y_{-1} = v$ and $y_0 = w$. Then*

$$x_{8n-7} = a\prod_{i=0}^{n-1}\left[\frac{1+(2i)at}{1+(2i+1)at}\right], \qquad x_{8n-6} = b\prod_{i=0}^{n-1}\left[\frac{1+(2i)bu}{1+(2i+1)bu}\right],$$

$$x_{8n-5} = c\prod_{i=0}^{n-1}\left[\frac{1+(2i)cv}{1+(2i+1)cv}\right], \qquad x_{8n-4} = d\prod_{i=0}^{n-1}\left[\frac{1+(2i)dw}{1+(2i+1)dw}\right],$$

$$x_{8n-3} = e\prod_{i=0}^{n-1}\left[\frac{1+(2i+1)pe}{1+(2i+2)pe}\right], \qquad x_{8n-2} = f\prod_{i=0}^{n-1}\left[\frac{1+(2i+1)fq}{1+(2i+2)fq}\right],$$

$$x_{8n-1} = g\prod_{i=0}^{n-1}\left[\frac{1+(2i+1)rg}{1+(2i+2)rg}\right], \qquad x_{8n} = h\prod_{i=0}^{n-1}\left[\frac{1+(2i+1)sh}{1+(2i+2)sh}\right],$$

$$y_{8n-7} = p\prod_{i=0}^{n-1}\left[\frac{1+(2i)pe}{1+(2i+1)pe}\right], \qquad y_{8n-6} = q\prod_{i=0}^{n-1}\left[\frac{1+(2i)qf}{1+(2i+1)qf}\right],$$

$$y_{8n-5} = r\prod_{i=0}^{n-1}\left[\frac{1+(2i)rg}{1+(2i+1)rg}\right], \qquad y_{8n-4} = s\prod_{i=0}^{n-1}\left[\frac{1+(2i)sh}{1+(2i+1)sh}\right],$$

$$y_{8n-3} = t\prod_{i=0}^{n-1}\left[\frac{1+(2i+1)at}{1+(2i+2)at}\right], \qquad y_{8n-2} = u\prod_{i=0}^{n-1}\left[\frac{1+(2i+1)bu}{1+(2i+2)bu}\right],$$

$$y_{8n-1} = v\prod_{i=0}^{n-1}\left[\frac{1+(2i+1)cv}{1+(2i+2)cv}\right], \qquad y_{8n} = w\prod_{i=0}^{n-1}\left[\frac{1+(2i+1)dw}{1+(2i+2)dw}\right],$$

where $\prod_{i=0}^{-1}\left[\frac{1+(2i)at}{1+(2i+1)at}\right] = 1$ also for all components.

Proof. For $n = 0$ the result holds. Now suppose that $n > 0$ and that our assumption holds for $n-1$. That is

$$x_{8n-15} = a\prod_{i=0}^{n-2}\left[\frac{1+(2i)at}{1+(2i+1)at}\right], \qquad x_{8n-14} = b\prod_{i=0}^{n-2}\left[\frac{1+(2i)bu}{1+(2i+1)bu}\right],$$

$$x_{8n-13} = c\prod_{i=0}^{n-2}\left[\frac{1+(2i)cv}{1+(2i+1)cv}\right], \qquad x_{8n-12} = d\prod_{i=0}^{n-2}\left[\frac{1+(2i)dw}{1+(2i+1)dw}\right],$$

$$x_{8n-11} = e\prod_{i=0}^{n-2}\left[\frac{1+(2i+1)pe}{1+(2i+2)pe}\right], \qquad x_{8n-10} = f\prod_{i=0}^{n-2}\left[\frac{1+(2i+1)fq}{1+(2i+2)fq}\right],$$

$$x_{8n-9} = g\prod_{i=0}^{n-2}\left[\frac{1+(2i+1)rg}{1+(2i+2)rg}\right], \qquad x_{8n-8} = h\prod_{i=0}^{n-2}\left[\frac{1+(2i+1)sh}{1+(2i+2)sh}\right],$$

$$y_{8n-15} = p\prod_{i=0}^{n-2}\left[\frac{1+(2i)pe}{1+(2i+1)pe}\right], \qquad y_{8n-14} = q\prod_{i=0}^{n-2}\left[\frac{1+(2i)qf}{1+(2i+1)qf}\right],$$

$$y_{8n-13} = r\prod_{i=0}^{n-2}\left[\frac{1+(2i)rg}{1+(2i+1)rg}\right], \qquad y_{8n-12} = s\prod_{i=0}^{n-2}\left[\frac{1+(2i)sh}{1+(2i+1)sh}\right],$$

$$y_{8n-11} = t\prod_{i=0}^{n-2}\left[\frac{1+(2i+1)at}{1+(2i+2)at}\right], \qquad y_{8n-10} = u\prod_{i=0}^{n-2}\left[\frac{1+(2i+1)bu}{1+(2i+2)bu}\right],$$

$$y_{8n-9} = v\prod_{i=0}^{n-2}\left[\frac{1+(2i+1)cv}{1+(2i+2)cv}\right], \qquad y_{8n-8} = w\prod_{i=0}^{n-2}\left[\frac{1+(2i+1)dw}{1+(2i+2)dw}\right].$$

Now, it follows from system (2.1) that

$$\begin{aligned}
x_{8n-7} &= \frac{x_{8n-15}}{1+x_{8n-15}y_{8n-11}} \\
&= \frac{a\prod_{i=0}^{n-2}\left[\frac{1+(2i)at}{1+(2i+1)at}\right]}{1+a\prod_{i=0}^{n-2}\left[\frac{1+(2i)at}{1+(2i+1)at}\right] t\prod_{i=0}^{n-2}\left[\frac{1+(2i+1)at}{1+(2i+2)at}\right]} \\
&= \frac{a\prod_{i=0}^{n-2}\left[\frac{1+(2i)at}{1+(2i+1)at}\right]}{1+at\prod_{i=0}^{n-2}\left[\frac{1+(2i)at}{1+(2i+2)at}\right]} \\
&= \frac{a\prod_{i=0}^{n-2}\left[\frac{1+(2i)at}{1+(2i+1)at}\right]}{1+\frac{at}{1+(2n-2)at}} = \frac{a\prod_{i=0}^{n-2}\left[\frac{1+(2i)at}{1+(2i+1)at}\right]}{\frac{1+(2n-2)at+at}{1+(2n-2)at}} \\
&= a\prod_{i=0}^{n-2}\left[\frac{1+(2i)at}{1+(2i+1)at}\right]\left[\frac{1+(2n-2)at}{1+(2n-1)at}\right].
\end{aligned}$$

Therefore, we have

$$x_{8n-7} = a\prod_{i=0}^{n-1}\left[\frac{1+(2i)at}{1+(2i+1)at}\right],$$

and

$$\begin{aligned}
y_{8n-7} &= \frac{y_{8n-15}}{1+y_{8n-15}x_{8n-11}} \\
&= \frac{p\prod_{i=0}^{n-2}\left[\frac{1+(2i)pe}{1+(2i+1)pe}\right]}{1+p\prod_{i=0}^{n-2}\left[\frac{1+(2i)pe}{1+(2i+1)pe}\right] e\prod_{i=0}^{n-2}\left[\frac{1+(2i+1)pe}{1+(2i+2)pe}\right]} \\
&= \frac{p\prod_{i=0}^{n-2}\left[\frac{1+(2i)pe}{1+(2i+1)pe}\right]}{1+pe\prod_{i=0}^{n-2}\left[\frac{1+(2i)pe}{1+(2i+2)pe}\right]} = \frac{p\prod_{i=0}^{n-2}\left[\frac{1+(2i)pe}{1+(2i+1)pe}\right]}{1+\frac{pe}{1+(2n-2)pe}}
\end{aligned}$$

$$= \frac{p\prod_{i=0}^{n-2}\left[\frac{1+(2i)pe}{1+(2i+1)pe}\right]}{\frac{1+(2n-2)pe+pe}{1+(2n-2)pe}} = p\prod_{i=0}^{n-2}\left[\frac{1+(2i)pe}{1+(2i+1)pe}\right]\left[\frac{1+(2n-2)pe}{1+(2n-1)pe}\right]$$
$$= p\prod_{i=0}^{n-1}\left[\frac{1+(2i)pe}{1+(2i+1)pe}\right].$$

Similarly we can prove the other relations. The proof is complete. □

Definition 2.2. (Equilibrium point)

Let $I,\ J$ be some intervals of real numbers and let

$$f,g : I^{k+1}\times J^{k+1}\to I,$$

be continuously differentiable functions. Let us consider the following system of the form

$$x_{n+1} = f(x_n, x_{n-1}, ..., x_{n-k}, y_n, y_{n-1}, ..., y_{n-k}),$$
$$y_{n+1} = g(x_n, x_{n-1}, ..., x_{n-k}, y_n, y_{n-1}, ..., y_{n-k}), \quad n = 0, 1, 2, ...$$

An equilibrium point of this system is a point $(\overline{x}, \overline{y})$ that satisfies

$$\overline{x} = f(\overline{x}, \overline{x}, ..., \overline{x}, \overline{y}, \overline{y}, ..., \overline{y}),$$
$$\overline{y} = g(\overline{x}, \overline{x}, ..., \overline{x}, \overline{y}, \overline{y}, ..., \overline{y}).$$

Lemma 2.3. *The equilibrium points of system* (2.1) *are* $(0, \alpha)$ *and* $(\gamma, 0)$ *where* $\alpha, \gamma \in [0,\infty)$.

Proof. For the equilibrium points of system (2.1), we can write

$$\bar{x} = \frac{\bar{x}}{1+\bar{x}\bar{y}}, \qquad \bar{y} = \frac{\bar{y}}{1+\bar{x}\bar{y}}.$$

Then

$$\bar{x}(1+\bar{x}\bar{y}) = \bar{x}, \qquad \bar{y}(1+\bar{x}\bar{y}) = \bar{y},$$

we have

$$\bar{x}(1+\bar{x}\bar{y}-1) = 0, \qquad \bar{y}(1+\bar{x}\bar{y}-1) = 0.$$

Therefore every $(0, \alpha)$ and $(\gamma, 0)$ are solutions. Thus the equilibrium points of system (2.1) are $(0, \alpha)$ and $(\gamma, 0)$. □

Lemma 2.4. *Every positive solution of the system* (2.1) *is bounded and convergent.*

Proof. Let $\{x_n, y_n\}$ be a positive solution of system (2.1). It follows from system (2.1) that

$$x_{n+1} = \frac{x_{n-7}}{1+x_{n-7}y_{n-3}} \le x_{n-7},$$

and

$$y_{n+1} = \frac{y_{n-7}}{1+y_{n-7}x_{n-3}} \le y_{n-7}.$$

Then the subsequences

$$\{x_{8n-7}\}_{n=0}^{\infty},\ \{x_{8n-6}\}_{n=0}^{\infty},\ \{x_{8n-5}\}_{n=0}^{\infty},\ \{x_{8n-4}\}_{n=0}^{\infty},\{x_{8n-3}\}_{n=0}^{\infty},\ \{x_{8n-2}\}_{n=0}^{\infty},\ \{x_{8n-1}\}_{n=0}^{\infty}$$

and $\{x_{8n}\}_{n=0}^{\infty}$ are decreasing and so are bounded from above by

$$M = \max\{x_{-7}, x_{-6}, x_{-5}, x_{-4}, x_{-3}, x_{-2}, x_{-1}, x_0\}.$$

Similarly the subsequences $\{y_{8n-7}\}_{n=0}^{\infty}$, $\{y_{8n-6}\}_{n=0}^{\infty}$, $\{y_{8n-5}\}_{n=0}^{\infty}$, $\{y_{8n-4}\}_{n=0}^{\infty}$, $\{y_{8n-3}\}_{n=0}^{\infty}$, $\{y_{8n-2}\}_{n=0}^{\infty}$, $\{y_{8n-1}\}_{n=0}^{\infty}$ and $\{y_{8n}\}_{n=0}^{\infty}$ are decreasing and so are bounded from above by

$$N = \max\{y_{-7}, y_{-6}, y_{-5}, y_{-4}, y_{-3}, y_{-2}, y_{-1}, y_0\}.$$

□

Lemma 2.5. *If $a, b, c, d, e, f, g, h, p, q, r, s, t, u, v$ and w be arbitrary real numbers and let $\{x_n, y_n\}$ be a solution of system* (2.1) *then the following statements are true:*

(i) If $a = 0,\ t \neq 0,$ (or $t = 0,\ a \neq 0$), then $x_{8n-7} = 0$ and $y_{8n-3} = t$ (or $x_{8n-7} = a$ and $y_{8n-3} = 0$).

(ii) If $b = 0,\ u \neq 0,$ (or $u = 0,\ b \neq 0$), then $x_{8n-6} = 0$ and $y_{8n-2} = u$ (or $x_{8n-6} = b$ and $y_{8n-2} = 0$).

(iii) If $c = 0,\ v \neq 0,$ (or $v = 0,\ c \neq 0$), then $x_{8n-5} = 0$ and $y_{8n-1} = v$ (or $x_{8n-5} = c$ and $y_{8n-1} = 0$).

(iv) If $d = 0,\ w \neq 0,$ (or $w = 0,\ d \neq 0$), then $x_{8n-4} = 0$ and $y_{8n} = w$ (or $x_{8n-4} = d$ and $y_{8n} = 0$).

(v) If $e = 0,\ p \neq 0,$ (or $p = 0,\ e \neq 0$), then $x_{8n-3} = 0$ and $y_{8n-7} = p$ (or $x_{8n-3} = e$ and $y_{8n-7} = 0$).

(vi) If $f = 0,\ q \neq 0,$ (or $q = 0,\ f \neq 0$), then $x_{8n-2} = 0$ and $y_{8n-6} = q$ (or $x_{8n-2} = f$ and $y_{8n-6} = 0$).

(vii) If $g = 0,\ r \neq 0,$ (or $r = 0,\ g \neq 0$), then $x_{8n-1} = 0$ and $y_{8n-5} = r$ (or $x_{8n-1} = g$ and $y_{8n-5} = 0$).

(viii) If $h = 0,\ s \neq 0,$ (or $s = 0,\ h \neq 0$), then $x_{8n} = 0$ and $y_{8n-4} = s$ (or $x_{8n} = h$ and $y_{8n-4} = 0$).

Proof. The proof follows from the form of the solutions of system (2.1). □

2.2. The Second System: $x_{n+1} = \frac{x_{n-7}}{1+x_{n-7}y_{n-3}},\ y_{n+1} = \frac{y_{n-7}}{1-y_{n-7}x_{n-3}}$.

In this subsection, we obtain the form of the solution of the following system of the difference equations:

$$x_{n+1} = \frac{x_{n-7}}{1 + x_{n-7}y_{n-3}}, \qquad y_{n+1} = \frac{y_{n-7}}{1 - y_{n-7}x_{n-3}}, \tag{2.2}$$

where the initial conditions are arbitrary real numbers with $x_{-7}y_{-3},\ x_{-6}y_{-2},\ x_{-5}y_{-1},\ x_{-4}y_0 \neq -1$ and $x_{-3}y_{-7},\ x_{-2}y_{-6},\ x_{-1}y_{-5},\ x_0y_{-4} \neq 1$.

The following theorem is devoted to the form of the solution of system (2.2).

Theorem 2.6. *Suppose that $\{x_n, y_n\}$ are solutions of system* (2.2)*. Then for $n = 0, 1, 2, ...,$*

$$\begin{aligned}
x_{8n-7} &= \frac{a}{(1+at)^n}, & x_{8n-6} &= \frac{b}{(1+bu)^n},\\
x_{8n-5} &= \frac{c}{(1+cv)^n}, & x_{8n-4} &= \frac{d}{(1+dw)^n},\\
x_{8n-3} &= e\,(1-pe)^n, & x_{8n-2} &= f\,(1-qf)^n,\\
x_{8n-1} &= g\,(1-rg)^n, & x_{8n} &= h\,(1-sh)^n,\\
y_{8n-7} &= \frac{p}{(1-pe)^n}, & y_{8n-6} &= \frac{q}{(1-qf)^n},\\
y_{8n-5} &= \frac{r}{(1-rg)^n}, & y_{8n-4} &= \frac{s}{(1-sh)^n},\\
y_{8n-3} &= t\,(1+at)^n, & y_{8n-2} &= u\,(1+bu)^n,\\
y_{8n-1} &= v\,(1+cv)^n, & y_{8n} &= w\,(1+dw)^n.
\end{aligned}$$

Proof. For $n=0$ the result holds. Now suppose that $n>0$ and that our assumption holds for $n-1$. That is,

$$x_{8n-15}=\frac{a}{(1+at)^{n-1}},\qquad x_{8n-14}=\frac{b}{(1+bu)^{n-1}},$$
$$x_{8n-13}=\frac{c}{(1+cv)^{n-1}},\qquad x_{8n-12}=\frac{d}{(1+dw)^{n-1}},$$
$$x_{8n-11}=e\left(1-pe\right)^{n-1},\qquad x_{8n-10}=f\left(1-qf\right)^{n-1},$$
$$x_{8n-9}=g\left(1-rg\right)^{n-1},\qquad x_{8n-8}=h\left(1-sh\right)^{n-1},$$
$$y_{8n-15}=\frac{p}{(1-pe)^{n-1}},\qquad y_{8n-14}=\frac{q}{(1-qf)^{n-1}},$$
$$y_{8n-13}=\frac{r}{(1-rg)^{n-1}},\qquad y_{8n-12}=\frac{s}{(1-sh)^{n-1}},$$
$$y_{8n-11}=t\left(1+at\right)^{n-1},\qquad y_{8n-10}=u\left(1+bu\right)^{n-1},$$
$$y_{8n-9}=v\left(1+cv\right)^{n-1},\qquad y_{8n-8}=w\left(1+dw\right)^{n-1}.$$

Now, it follows from system (2.2) that

$$x_{8n-7}=\frac{x_{8n-15}}{1+x_{8n-15}y_{8n-11}}=\frac{\frac{a}{(1+at)^{n-1}}}{1+\frac{at(1+at)^{n-1}}{(1+at)^{n-1}}}=\frac{a}{(1+at)^{n-1}(1+at)}=\frac{a}{(1+at)^{n}}.$$

$$y_{8n-7}=\frac{y_{8n-15}}{1-y_{8n-15}x_{8n-11}}=\frac{\frac{p}{(1-pe)^{n-1}}}{1-\frac{pe(1-pe)^{n-1}}{(1-pe)^{n-1}}}=\frac{p}{(1-pe)^{n-1}(1-pe)}=\frac{p}{(1-pe)^{n}}.$$

Also, we see from system (2.2) that

$$\begin{aligned}x_{8n-3}&=\frac{x_{8n-11}}{1+x_{8n-11}y_{8n-7}}=\frac{e\left(1-pe\right)^{n-1}}{1+\dfrac{pe\left(1-pe\right)^{n-1}}{(1-pe)^{n}}}\\&=\frac{e\left(1-pe\right)^{n-1}}{1+\frac{pe}{(1-pe)}}\left(\frac{1-pe}{1-pe}\right)=\frac{e\left(1-pe\right)^{n}}{1-pe+pe}=e\left(1-pe\right)^{n}.\end{aligned}$$

$$\begin{aligned}y_{8n-3}&=\frac{y_{8n-11}}{1-y_{8n-11}x_{8n-7}}=\frac{t\left(1+at\right)^{n-1}}{1-\dfrac{at\left(1+at\right)^{n-1}}{(1+at)^{n}}}\\&=\frac{t\left(1+at\right)^{n-1}}{1-\frac{at}{(1+at)}}\left(\frac{1+at}{1+at}\right)=\frac{t\left(1+at\right)^{n}}{1+at-at}=t\left(1+at\right)^{n}.\end{aligned}$$

Similarly, we can prove the other relations. The proof is complete. □

Lemma 2.7. *Let $\{x_n,y_n\}$ be a positive solution of system (2.2), then $\{x_n\}$ is bounded and converges to zero.*

Proof. It follows from system (2.2) that

$$x_{n+1}=\frac{x_{n-7}}{1+x_{n-7}y_{n-3}}<x_{n-7}.$$

Then the subsequences

$$\{x_{8n-7}\}_{n=0}^{\infty}, \{x_{8n-6}\}_{n=0}^{\infty}, \{x_{8n-5}\}_{n=0}^{\infty}, \{x_{8n-4}\}_{n=0}^{\infty}, \{x_{8n-3}\}_{n=0}^{\infty}, \{x_{8n-2}\}_{n=0}^{\infty}, \{x_{8n-1}\}_{n=0}^{\infty}$$

and $\{x_{8n}\}_{n=0}^{\infty}$ are decreasing and so are bounded from above by

$$M = \max\{x_{-7}, x_{-6}, x_{-5}, x_{-4}, x_{-3}, x_{-2}, x_{-1}, x_0\}.$$

□

Lemma 2.8. *If $a, b, c, d, e, f, g, h, p, q, r, s, t, u, v$ and w be arbitrary real numbers and let $\{x_n, y_n\}$ be a solution of system (2.2) then the following statements are true:*

(i) If $a = 0,\ t \neq 0$, (or $a \neq 0,\ t = 0$) then $x_{8n-7} = 0$ and $y_{8n-3} = t$ (or $x_{8n-7} = a$ and $y_{8n-3} = 0$).

(ii) If $b = 0,\ u \neq 0$, (or $u = 0,\ b \neq 0$), then $x_{8n-6} = 0$ and $y_{8n-2} = u$ (or $x_{8n-6} = b$ and $y_{8n-2} = 0$).

(iii) If $c = 0,\ v \neq 0$, (or $v = 0,\ c \neq 0$), then $x_{8n-5} = 0$ and $y_{8n-1} = v$ (or $x_{8n-5} = c$ and $y_{8n-1} = 0$).

(iv) If $d = 0,\ w \neq 0$, (or $w = 0,\ d \neq 0$), then $x_{8n-4} = 0$ and $y_{8n} = w$ (or $x_{8n-4} = d$ and $y_{8n} = 0$).

(v) If $e = 0,\ p \neq 0$, (or $p = 0,\ e \neq 0$), then $x_{8n-3} = 0$ and $y_{8n-7} = p$ (or $x_{8n-3} = e$ and $y_{8n-7} = 0$).

(vi) If $f = 0,\ q \neq 0$, (or $q = 0,\ f \neq 0$), then $x_{8n-2} = 0$ and $y_{8n-6} = q$ (or $x_{8n-2} = f$ and $y_{8n-6} = 0$).

(vii) If $g = 0,\ r \neq 0$, (or $r = 0,\ g \neq 0$), then $x_{8n-1} = 0$ and $y_{8n-5} = r$ (or $x_{8n-1} = g$ and $y_{8n-5} = 0$).

(viii) If $h = 0,\ s \neq 0$, (or $s = 0,\ h \neq 0$), then $x_{8n} = 0$ and $y_{8n-4} = s$ (or $x_{8n} = h$ and $y_{8n-4} = 0$).

Proof. The proof follows from the form of the solutions of system (2.2). □

2.3. *The Third System:* $x_{n+1} = \frac{x_{n-7}}{1+x_{n-7}y_{n-3}}$, $y_{n+1} = \frac{y_{n-7}}{-1+y_{n-7}x_{n-3}}$.

In this subsection, we investigate the solutions of the following system of difference equations

$$x_{n+1} = \frac{x_{n-7}}{1 + x_{n-7}y_{n-3}}, \qquad y_{n+1} = \frac{y_{n-7}}{-1 + y_{n-7}x_{n-3}}, \tag{2.3}$$

where the initial conditions are arbitrary real numbers with $x_{-7}y_{-3}$, $x_{-6}y_{-2}$, $x_{-5}y_{-1}$, $x_{-4}y_0 \neq \pm 1$ and $x_{-3}y_{-7}$, $x_{-2}y_{-6}$, $x_{-1}y_{-5}$, $x_0y_{-4} \neq 1, \neq \frac{1}{2}$.

Theorem 2.9. *Suppose that $\{x_n, y_n\}$ are solutions of system (2.3). Then for $n = 0, 1, 2, \ldots$,*

$$x_{16n-7} = \frac{a}{(1+at)^n (1-at)^n}, \qquad x_{16n-6} = \frac{b}{(1+bu)^n (1-bu)^n},$$

$$x_{16n-5} = \frac{c}{(1+cv)^n (1-cv)^n}, \qquad x_{16n-4} = \frac{d}{(1+dw)^n (1-dw)^n},$$

$$x_{16n-3} = \frac{(-1)^n e(-1+pe)^{2n}}{(-1+2pe)^n}, \qquad x_{16n-2} = \frac{(-1)^n f(-1+fq)^{2n}}{(-1+2fq)^n},$$

$$x_{16n-1} = \frac{(-1)^n g(-1+gr)^{2n}}{(-1+2gr)^n}, \qquad x_{16n} = \frac{(-1)^n h(-1+sh)^{2n}}{(-1+2sh)^n},$$

$$x_{16n+1} = \frac{a}{(1+at)^{n+1} (1-at)^n}, \qquad x_{16n+2} = \frac{b}{(1+bu)^{n+1} (1-bu)^n},$$

$$x_{16n+3} = \frac{c}{(1+cv)^{n+1} (1-cv)^n}, \qquad x_{16n+4} = \frac{d}{(1+dw)^{n+1} (1-dw)^n},$$

$$x_{16n+5} = \frac{(-1)^n e(-1+pe)^{2n+1}}{(-1+2pe)^{n+1}}, \qquad x_{16n+6} = \frac{(-1)^n f(-1+fq)^{2n+1}}{(-1+2fq)^{n+1}},$$

$$x_{16n+7} = \frac{(-1)^n g(-1+gr)^{2n+1}}{(-1+2gr)^{n+1}}, \qquad x_{16n+8} = \frac{(-1)^n h(-1+sh)^{2n+1}}{(-1+2sh)^{n+1}},$$

$$y_{16n-7} = \frac{(-1)^n p(-1+2pe)^n}{(-1+pe)^{2n}}, \qquad y_{16n-6} = \frac{(-1)^n q(-1+2fq)^n}{(-1+fq)^{2n}},$$
$$y_{16n-5} = \frac{(-1)^n r(-1+2gr)^n}{(-1+gr)^{2n}}, \qquad y_{16n-4} = \frac{(-1)^n s(-1+2sh)^n}{(-1+sh)^{2n}},$$
$$y_{16n-3} = t(1+at)^n(1-at)^n, \qquad y_{16n-2} = u(1+bu)^n(1-bu)^n,$$
$$y_{16n-1} = v(1+cv)^n(1-cv)^n, \qquad y_{16n} = w(1+dw)^n(1-dw)^n,$$
$$y_{16n+1} = \frac{(-1)^n p(-1+2pe)^n}{(-1+pe)^{2n+1}}, \qquad y_{16n+2} = \frac{(-1)^n q(-1+2fq)^n}{(-1+fq)^{2n+1}},$$
$$y_{16n+3} = \frac{(-1)^n r(-1+2gr)^n}{(-1+gr)^{2n+1}}, \qquad y_{16n+4} = \frac{(-1)^n s(-1+2sh)^n}{(-1+sh)^{2n+1}},$$
$$y_{16n+5} = -t(1+at)^{n+1}(1-at)^n, \qquad y_{16n+6} = -u(1+bu)^{n+1}(1-bu)^n,$$
$$y_{16n+7} = -v(1+cv)^{n+1}(1-cv)^n, \qquad y_{16n+8} = -w(1+dw)^{n+1}(1-dw)^n.$$

Proof. For $n = 0$ the result holds. Now suppose that $n > 0$ and that our assumption holds for $n - 1$. That is,

$$x_{16n-23} = \frac{a}{(1+at)^{n-1}(1-at)^{n-1}}, \qquad x_{16n-22} = \frac{b}{(1+bu)^{n-1}(1-bu)^{n-1}},$$
$$x_{16n-21} = \frac{c}{(1+cv)^{n-1}(1-cv)^{n-1}}, \qquad x_{16n-20} = \frac{d}{(1+dw)^{n-1}(1-dw)^{n-1}},$$
$$x_{16n-19} = \frac{(-1)^{n-1} e(-1+pe)^{2n-2}}{(-1+2pe)^{n-1}}, \qquad x_{16n-18} = \frac{(-1)^{n-1} f(-1+fq)^{2n-2}}{(-1+2fq)^{n-1}},$$
$$x_{16n-17} = \frac{(-1)^{n-1} g(-1+gr)^{2n-2}}{(-1+2gr)^{n-1}}, \qquad x_{16n-16} = \frac{(-1)^{n-1} h(-1+sh)^{2n-2}}{(-1+2sh)^{n-1}},$$
$$x_{16n-15} = \frac{a}{(1+at)^{n}(1-at)^{n-1}}, \qquad x_{16n-14} = \frac{b}{(1+bu)^{n}(1-bu)^{n-1}},$$
$$x_{16n-13} = \frac{c}{(1+cv)^{n}(1-cv)^{n-1}}, \qquad x_{16n-12} = \frac{d}{(1+dw)^{n}(1-dw)^{n-1}},$$
$$x_{16n-11} = \frac{(-1)^{n-1} e(-1+pe)^{2n-1}}{(-1+2pe)^{n}}, \qquad x_{16n-10} = \frac{(-1)^{n-1} f(-1+fq)^{2n-1}}{(-1+2fq)^{n}},$$
$$x_{16n-9} = \frac{(-1)^{n-1} g(-1+gr)^{2n-1}}{(-1+2gr)^{n}}, \qquad x_{16n-8} = \frac{(-1)^{n-1} h(-1+sh)^{2n-1}}{(-1+2sh)^{n}},$$
$$y_{16n-23} = \frac{(-1)^{n-1} p(-1+2pe)^{n-1}}{(-1+pe)^{2n-2}}, \qquad y_{16n-22} = \frac{(-1)^{n-1} q(-1+2fq)^{n-1}}{(-1+fq)^{2n-2}},$$
$$y_{16n-21} = \frac{(-1)^{n-1} r(-1+2gr)^{n-1}}{(-1+gr)^{2n-2}}, \qquad y_{16n-20} = \frac{(-1)^{n-1} s(-1+2sh)^{n-1}}{(-1+sh)^{2n-2}},$$
$$y_{16n-19} = t(1+at)^{n-1}(1-at)^{n-1}, \qquad y_{16n-18} = u(1+bu)^{n-1}(1-bu)^{n-1},$$
$$y_{16n-17} = v(1+cv)^{n-1}(1-cv)^{n-1}, \qquad y_{16n-16} = w(1+dw)^{n-1}(1-dw)^{n-1},$$
$$y_{16n-15} = \frac{(-1)^{n-1} p(-1+2pe)^{n-1}}{(-1+pe)^{2n-1}}, \qquad y_{16n-14} = \frac{(-1)^{n-1} q(-1+2fq)^{n-1}}{(-1+fq)^{2n-1}},$$
$$y_{16n-13} = \frac{(-1)^{n-1} r(-1+2gr)^{n-1}}{(-1+gr)^{2n-1}}, \qquad y_{16n-12} = \frac{(-1)^{n-1} s(-1+2sh)^{n-1}}{(-1+sh)^{2n-1}},$$

$$y_{16n-11} = -t\left(1+at\right)^{n}\left(1-at\right)^{n-1}, \qquad y_{16n-10} = -u\left(1+bu\right)^{n}\left(1-bu\right)^{n-1},$$
$$y_{16n-9} = -v\left(1+cv\right)^{n}\left(1-cv\right)^{n-1}, \qquad y_{16n-8} = -w\left(1+dw\right)^{n}\left(1-dw\right)^{n-1}.$$

Now it follows from system (2.3) that

$$\begin{aligned}
x_{16n-7} &= \frac{x_{16n-15}}{1+x_{16n-15}y_{16n-11}} \\
&= \frac{\frac{a}{\left(1+at\right)^{n}\left(1-at\right)^{n-1}}}{1+\left(\frac{a}{(1+at)^{n}(1-at)^{n-1}}\right)\left(-t\left(1+at\right)^{n}\left(1-at\right)^{n-1}\right)} \\
&= \frac{\frac{a}{\left(1+at\right)^{n}\left(1-at\right)^{n-1}}}{\left[1-at\right]} = \frac{a}{\left(1+at\right)^{n}\left(1-at\right)^{n}},
\end{aligned}$$

$$\begin{aligned}
y_{16n-7} &= \frac{y_{16n-15}}{-1+y_{16n-15}x_{16n-11}} \\
&= \frac{\frac{\left(-1\right)^{n-1}p\left(-1+2pe\right)^{n-1}}{\left(-1+pe\right)^{2n-1}}}{-1+\left(\frac{(-1)^{n-1}p(-1+2pe)^{n-1}}{(-1+pe)^{2n-1}}\right)\left(\frac{(-1)^{n-1}e(-1+pe)^{2n-1}}{(-1+2pe)^{n}}\right)} \\
&= \frac{\frac{(-1)^{n-1}p(-1+2pe)^{n-1}}{(-1+pe)^{2n-1}}}{-1+\frac{pe}{(-1+2pe)}} = \frac{(-1)^{n-1}p(-1+2pe)^{n-1}}{(-1+pe)^{2n-1}\left[-1+\frac{pe}{(-1+2pe)}\right]}\frac{\left(-1+2pe\right)}{\left(-1+2pe\right)} \\
&= \frac{(-1)^{n-1}p(-1+2pe)^{n}}{(-1+pe)^{2n-1}[1-2pe+pe]} = \frac{\left(-1\right)^{n-1}p\left(-1+2pe\right)^{n}}{\left(-1+pe\right)^{2n-1}\left[1-pe\right]} \\
&= \frac{\left(-1\right)^{n}p\left(-1+2pe\right)^{n}}{\left(-1+pe\right)^{2n}},
\end{aligned}$$

$$\begin{aligned}
x_{16n-6} &= \frac{x_{16n-14}}{1+x_{16n-14}y_{16n-10}} = \frac{\frac{b}{(1+bu)^{n}(1-bu)^{n-1}}}{1+\left(\frac{b}{(1+bu)^{n}(1-bu)^{n-1}}\right)\left(-u(1+bu)^{n}(1-bu)^{n-1}\right)} \\
&= \frac{\frac{b}{(1+bu)^{n}(1-bu)^{n-1}}}{1-bu} = \frac{b}{\left(1+bu\right)^{n}\left(1-bu\right)^{n}},
\end{aligned}$$

$$\begin{aligned}
y_{16n-6} &= \frac{y_{16n-14}}{-1+y_{16n-14}x_{16n-10}} = \frac{\frac{(-1)^{n-1}q(-1+2fq)^{n-1}}{(-1+fq)^{2n-1}}}{-1+\left(\frac{(-1)^{n-1}q(-1+2fq)^{n-1}}{(-1+fq)^{2n-1}}\right)\left(\frac{(-1)^{n-1}f(-1+fq)^{2n-1}}{(-1+2fq)^{n}}\right)} \\
&= \frac{\frac{(-1)^{n-1}q(-1+2fq)^{n-1}}{(-1+fq)^{2n-1}}}{-1+\frac{fq}{-1+2fq}} = \frac{\frac{(-1)^{n-1}q(-1+2fq)^{n-1}}{(-1+fq)^{2n-1}}}{\frac{1-2fq+fq}{-1+2fq}} \\
&= \frac{\frac{(-1)^{n-1}q(-1+2fq)^{n}}{(-1+fq)^{2n-1}}}{1-fq} = \frac{(-1)^{n-1}q(-1+2fq)^{n}}{(1-fq)(-1+fq)^{2n-1}} = \frac{\left(-1\right)^{n}q\left(-1+2fq\right)^{n}}{\left(-1+fq\right)^{2n}}.
\end{aligned}$$

We can prove the other relations by the same way. The proof is complete. □

Lemma 2.10. *Let $\{x_n, y_n\}$ be a positive solution of system (2.3), then $\{x_n\}$ is bounded and converges to zero.*

Proof. The proof is as in the Lemma 2.7 and so it will be omitted. □

Lemma 2.11. *If $a, b, c, d, e, f, g, h, p, q, r, s, t, u, v$ and w be arbitrary real numbers and let $\{x_n, y_n\}$ be a solution of system (2.3) then the following statements are true:*

(i) $If\ a = 0,\ t \neq 0,$ *then* $x_{16n-7} = x_{16n+1} = 0$ *and* $y_{16n-3} = t,\ y_{16n+5} = -t.$

(ii) $If\ a \neq 0,\ t = 0,$ *then* $x_{16n-7} = x_{16n+1} = a$ *and* $y_{16n-3} = y_{16n+5} = 0.$

(iii) $If\ b = 0,\ u \neq 0,$ *then* $x_{16n-6} = x_{16n+2} = 0$ *and* $y_{16n-2} = u, y_{16n+6} = -u.$

(iv) $If\ u = 0,\ b \neq 0,$ *then* $x_{16n-6} = x_{16n+2} = b$ *and* $y_{16n-2} = y_{16n+6} = 0.$

(v) $If\ c = 0,\ v \neq 0,$ *then* $x_{16n-5} = x_{16n+3} = 0$ *and* $y_{16n-1} = v, y_{16n+7} = -v.$

(vi) $If\ v = 0,\ c \neq 0,$ *then* $x_{16n-5} = x_{16n+3} = c$ *and* $y_{16n-1} = y_{16n+7} = 0.$

(vii) $If\ d = 0,\ w \neq 0,$ *then* $x_{16n-4} = x_{16n+4} = 0$ *and* $y_{16n} = w, y_{16n+8} = -w.$

(viii) $If\ w = 0,\ d \neq 0,$ *then* $x_{16n-4} = x_{16n+4} = d$ *and* $y_{16n} = y_{16n+8} = 0.$

(ix) $If\ e = 0,\ p \neq 0,$ *then* $x_{16n-3} = x_{16n+5} = 0$ *and* $y_{16n-7} = p, y_{16n+1} = -p.$

(x) $If\ p = 0,\ e \neq 0,$ *then* $x_{16n-3} = x_{16n+5} = e$ *and* $y_{16n-7} = y_{16n+1} = 0.$

(xi) $If\ f = 0,\ q \neq 0,$ *then* $x_{16n-2} = x_{16n+6} = 0$ *and* $y_{16n-6} = q, y_{16n+2} = -q.$

(xii) $If\ q = 0,\ f \neq 0,$*then* $x_{16n-2} = x_{16n+6} = f$ *and* $y_{16n-6} = y_{16n+2} = 0.$

(xiii) $If\ g = 0,\ r \neq 0,$ *then* $x_{16n-1} = x_{16n+7} = 0$ *and* $y_{16n-5} = r, y_{16n+3} = -r.$

(xiv) $If\ r = 0,\ g \neq 0,$ *then* $x_{16n-1} = x_{16n+7} = g$ *and* $y_{16n-5} = y_{16n+3} = 0.$

(xv) $If\ h = 0,\ s \neq 0,$ *then* $x_{16n} = x_{16n+8} = 0$ *and* $y_{16n-4} = s, y_{16n+4} = -s.$

(xvi) $If\ s = 0,\ h \neq 0,$ *then* $x_{16n} = x_{16n+8} = h$ *and* $y_{16n-4} = y_{16n+4} = 0.$

Proof. The proof follows from the form of the solutions of system (2.3). □

2.4. The Fourth System: $x_{n+1} = \frac{x_{n-7}}{1+x_{n-7}y_{n-3}}$, $y_{n+1} = \frac{y_{n-7}}{-1-y_{n-7}x_{n-3}}$.

In this subsection, we study the solutions of the system of the following two difference equations:

$$x_{n+1} = \frac{x_{n-7}}{1 + x_{n-7}y_{n-3}}, \; y_{n+1} = \frac{y_{n-7}}{-1 - y_{n-7}x_{n-3}}, \tag{2.4}$$

where the initial conditions are arbitrary real numbers with $x_{-7}y_{-3}$, $x_{-6}y_{-2}$, $x_{-5}y_{-1}$, $x_{-4}y_0 \neq -1, \neq \frac{-1}{2}$ and $x_{-3}y_{-7}$, $x_{-2}y_{-6}$, $x_{-1}y_{-5}$, $x_0y_{-4} \neq \pm 1$.

Theorem 2.12. *Suppose that* $\{x_n, y_n\}$ *are solutions of system* (2.4)*. Then for* $n = 0, 1, 2, ...,$

$$x_{16n-7} = \frac{a(1+2at)^n}{(1+at)^{2n}}, \qquad x_{16n-6} = \frac{b(1+2bu)^n}{(1+bu)^{2n}},$$

$$x_{16n-5} = \frac{c(1+2cv)^n}{(1+cv)^{2n}}, \qquad x_{16n-4} = \frac{d(1+2dw)^n}{(1+dw)^{2n}},$$

$$x_{16n-3} = e\,(1-pe)^n\,(1+pe)^n\,, \qquad x_{16n-2} = f\,(1-fq)^n\,(1+fq)^n\,,$$

$$x_{16n-1} = g\,(1-gr)^n\,(1+gr)^n\,, \qquad x_{16n} = h\,(1-sh)^n\,(1+sh)^n\,,$$

$$x_{16n+1} = \frac{a(1+2at)^n}{(1+at)^{2n+1}}, \qquad x_{16n+2} = \frac{b(1+2bu)^n}{(1+bu)^{2n+1}},$$

$$x_{16n+3} = \frac{c(1+2cv)^n}{(1+cv)^{2n+1}}, \qquad x_{16n+4} = \frac{d(1+2dw)^n}{(1+dw)^{2n+1}},$$

$$x_{16n+5} = e\,(1+pe)^{n+1}\,(1-pe)^n\,, \qquad x_{16n+6} = f\,(1+fq)^{n+1}\,(1-fq)^n\,,$$

$$x_{16n+7} = g\,(1+gr)^{n+1}\,(1-gr)^n\,, \qquad x_{16n+8} = h\,(1+sh)^{n+1}\,(1-sh)^n\,,$$

$$y_{16n-7} = \frac{p}{(1-pe)^n(1+pe)^n}, \qquad y_{16n-6} = \frac{q}{(1-qf)^n(1+qf)^n},$$

$$y_{16n-5} = \frac{r}{(1-gr)^n(1+gr)^n}, \qquad y_{16n-4} = \frac{s}{(1-sh)^n(1+sh)^n},$$

$$y_{16n-3} = \frac{t(1+at)^{2n}}{(1+2at)^n}, \qquad y_{16n-2} = \frac{u(1+bu)^{2n}}{(1+2bu)^n},$$

$$y_{16n-1} = \frac{v(1+cv)^{2n}}{(1+2cv)^n}, \qquad y_{16n} = \frac{w(1+dw)^{2n}}{(1+2dw)^n},$$

$$y_{16n+1} = \frac{-p}{(1+pe)^{n+1}(1-pe)^n}, \qquad y_{16n+2} = \frac{-q}{(1+qf)^{n+1}(1-qf)^n},$$

$$y_{16n+3} = \frac{-r}{(1+gr)^{n+1}(1-gr)^n}, \qquad y_{16n+4} = \frac{-s}{(1+sh)^{n+1}(1-sh)^n},$$

$$y_{16n+5} = \frac{-t(1+at)^{2n+1}}{(1+2at)^{n+1}}, \qquad y_{16n+6} = \frac{-u(1+bu)^{2n+1}}{(1+2bu)^{n+1}},$$

$$y_{16n+7} = \frac{-v(1+cv)^{2n+1}}{(1+2cv)^{n+1}}, \qquad y_{16n+8} = \frac{-w(1+dw)^{2n+1}}{(1+2dw)^{n+1}}.$$

Proof. For $n = 0$ the result holds. Now suppose that $n > 0$ and our assumption holds for $n - 1$, that is,

$$x_{16n-23} = \frac{a(1+2at)^{n-1}}{(1+at)^{2n-2}}, \qquad x_{16n-22} = \frac{b(1+2bu)^{n-1}}{(1+bu)^{2n-2}},$$

$$x_{16n-21} = \frac{c(1+2cv)^{n-1}}{(1+cv)^{2n-2}}, \qquad x_{16n-20} = \frac{d(1+2dw)^{n-1}}{(1+dw)^{2n-2}},$$

$$x_{16n-19} = e\,(1-pe)^{n-1}\,(1+pe)^{n-1}\,, \qquad x_{16n-18} = f\,(1-fq)^{n-1}\,(1+fq)^{n-1}\,,$$

$$x_{16n-17} = g\,(1-gr)^{n-1}\,(1+gr)^{n-1}\,, \qquad x_{16n-16} = h\,(1-sh)^{n-1}\,(1+sh)^{n-1}\,,$$

$$x_{16n-15} = \frac{a(1+2at)^{n-1}}{(1+at)^{2n-1}}, \qquad x_{16n-14} = \frac{b(1+2bu)^{n-1}}{(1+bu)^{2n-1}},$$

$$x_{16n-13}=\frac{c(1+2cv)^{n-1}}{(1+cv)^{2n-1}},\qquad x_{16n-12}=\frac{d(1+2dw)^{n-1}}{(1+dw)^{2n-1}},$$
$$x_{16n-11}=e\left(1+pe\right)^{n}\left(1-pe\right)^{n-1},\qquad x_{16n-10}=f\left(1+fq\right)^{n}\left(1-fq\right)^{n-1},$$
$$x_{16n-9}=g\left(1+gr\right)^{n}\left(1-gr\right)^{n-1},\qquad x_{16n-8}=h\left(1+sh\right)^{n}\left(1-sh\right)^{n-1},$$
$$y_{16n-23}=\frac{p}{(1-pe)^{n-1}(1+pe)^{n-1}},\qquad y_{16n-22}=\frac{q}{(1-qf)^{n-1}(1+qf)^{n-1}},$$
$$y_{16n-21}=\frac{r}{(1-gr)^{n-1}(1+gr)^{n-1}},\qquad y_{16n-20}=\frac{s}{(1-sh)^{n-1}(1+sh)^{n-1}},$$
$$y_{16n-17}=\frac{v(1+cv)^{2n-2}}{(1+2cv)^{n-1}},\qquad y_{16n-18}=\frac{u(1+bu)^{2n-2}}{(1+2bu)^{n-1}},$$
$$y_{16n-19}=\frac{t(1+at)^{2n-2}}{(1+2at)^{n-1}},\qquad y_{16n-16}=\frac{w(1+dw)^{2n-2}}{(1+2dw)^{n-1}},$$
$$y_{16n-15}=\frac{-p}{(1+pe)^{n}(1-pe)^{n-1}},\qquad y_{16n-14}=\frac{-q}{(1+qf)^{n}(1-qf)^{n-1}},$$
$$y_{16n-13}=\frac{-r}{(1+gr)^{n}(1-gr)^{n-1}},\qquad y_{16n-12}=\frac{-s}{(1+sh)^{n}(1-sh)^{n-1}},$$
$$y_{16n-11}=\frac{-t(1+at)^{2n-1}}{(1+2at)^{n}},\qquad y_{16n-10}=\frac{-u(1+bu)^{2n-1}}{(1+2bu)^{n}},$$
$$y_{16n-9}=\frac{-v(1+cv)^{2n-1}}{(1+2cv)^{n}},\qquad y_{16n-8}=\frac{-w(1+dw)^{2n-1}}{(1+2dw)^{n}}.$$

Now, it follows from (2.4) that

$$\begin{aligned}
x_{16n-7}&=\frac{x_{16n-15}}{1+x_{16n-15}y_{16n-11}}=\frac{\frac{a(1+2at)^{n-1}}{(1+at)^{2n-1}}}{1+\left(\frac{a(1+2at)^{n-1}}{(1+at)^{2n-1}}\right)\left(\frac{-t(1+at)^{2n-1}}{(1+2at)^{n}}\right)}\\
&=\frac{\frac{a(1+2at)^{n-1}}{(1+at)^{2n-1}}}{1-\left(\frac{at}{1+2at}\right)}=\frac{a(1+2at)^{n-1}}{(1+at)^{2n-1}\left[1-\left(\frac{at}{1+2at}\right)\right]}\frac{1+2at}{1+2at}\\
&=\frac{a(1+2at)^{n}}{(1+at)^{2n-1}\left[1+2at-at\right]}=\frac{a(1+2at)^{n}}{(1+at)^{2n}},
\end{aligned}$$

$$\begin{aligned}
y_{16n-7}&=\frac{y_{16n-15}}{-1-y_{16n-15}x_{16n-11}}=\frac{\frac{-p}{(1+pe)^{n}(1-pe)^{n-1}}}{-1-\left(\frac{-p}{(1+pe)^{n}(1-pe)^{n-1}}\right)\left(e(1+pe)^{n}(1-pe)^{n-1}\right)}\\
&=\frac{\frac{-p}{(1+pe)^{n}(1-pe)^{n-1}}}{-1+pe}=\frac{p}{(1+pe)^{n}(1-pe)^{n-1}}\left[\frac{1}{1-pe}\right]\\
&=\frac{p}{(1-pe)^{n}(1+pe)^{n}},
\end{aligned}$$

$$\begin{aligned}
x_{16n-6}&=\frac{x_{16n-14}}{1+x_{16n-14}y_{16n-10}}=\frac{\frac{b(1+2bu)^{n-1}}{(1+bu)^{2n-1}}}{1+\left(\frac{b(1+2bu)^{n-1}}{(1+bu)^{2n-1}}\right)\left(\frac{-u(1+bu)^{2n-1}}{(1+2bu)^{n}}\right)}\\
&=\frac{\frac{b(1+2bu)^{n-1}}{(1+bu)^{2n-1}}}{1-\left(\frac{bu}{1+2bu}\right)}=\frac{b(1+2bu)^{n-1}}{(1+bu)^{2n-1}\left[1-\left(\frac{bu}{1+2bu}\right)\right]}\frac{1+2bu}{1+2bu}\\
&=\frac{b(1+2bu)^{n}}{(1+bu)^{2n-1}\left[1+2bu-bu\right]}=\frac{b(1+2bu)^{n}}{(1+bu)^{2n}},
\end{aligned}$$

$$\begin{aligned} y_{16n-6} &= \frac{y_{16n-14}}{-1-y_{16n-14}x_{16n-10}} = \frac{\frac{-q}{(1+qf)^n(1-qf)^{n-1}}}{-1-\left(\frac{-q}{(1+qf)^n(1-qf)^{n-1}}\right)\left(f(1+fq)^n(1-fq)^{n-1}\right)} \\ &= \frac{\frac{-q}{(1+qf)^n(1-qf)^{n-1}}}{-1+qf} = \frac{q}{(1+qf)^n(1-qf)^{n-1}}\left[\frac{1}{1-qf}\right] \\ &= \frac{q}{(1+qf)^n(1-qf)^n}. \end{aligned}$$

We can prove the other relations similarly. The proof is complete. □

Lemma 2.13. *Let $\{x_n, y_n\}$ be a positive solution of system* (2.4)*, then $\{x_n\}$ is bounded and converges to zero.*

Proof. The proof is as in the Lemma 2.7, and so it will be omitted. □

Lemma 2.14. *If $a, b, c, d, e, f, g, h, p, q, r, s, t, u, v$ and w be arbitrary real numbers and let $\{x_n, y_n\}$ be a solution of system* (2.4) *then the following statements are true:*

(i) $If\ a = 0,\ t \neq 0,$ *then* $x_{16n-7} = x_{16n+1} = 0$ *and* $y_{16n-3} = t,\ y_{16n+5} = -t.$

(ii) $If\ a \neq 0,\ t = 0,$ *then* $x_{16n-7} = x_{16n+1} = a$ *and* $y_{16n-3} = y_{16n+5} = 0.$

(iii) $If\ b = 0,\ u \neq 0,$ *then* $x_{16n-6} = x_{16n+2} = 0$ *and* $y_{16n-2} = u, y_{16n+6} = -u.$

(iv) $If\ u = 0,\ b \neq 0,$ *then* $x_{16n-6} = x_{16n+2} = b$ *and* $y_{16n-2} = y_{16n+6} = 0.$

(v) $If\ c = 0,\ v \neq 0,$ *then* $x_{16n-5} = x_{16n+3} = 0$ *and* $y_{16n-1} = v, y_{16n+7} = -v.$

(vi) $If\ v = 0,\ c \neq 0,$ *then* $x_{16n-5} = x_{16n+3} = c$ *and* $y_{16n-1} = y_{16n+7} = 0.$

(vii) $If\ d = 0,\ w \neq 0,$ *then* $x_{16n-4} = x_{16n+4} = 0$ *and* $y_{16n} = w, y_{16n+8} = -w.$

(viii) $If\ w = 0,\ d \neq 0,$ *then* $x_{16n-4} = x_{16n+4} = d$ *and* $y_{16n} = y_{16n+8} = 0.$

(ix) $If\ e = 0,\ p \neq 0,$ *then* $x_{16n-3} = x_{16n+5} = 0$ *and* $y_{16n-7} = p, y_{16n+1} = -p.$

(x) $If\ p = 0,\ e \neq 0,$ *then* $x_{16n-3} = x_{16n+5} = e$ *and* $y_{16n-7} = y_{16n+1} = 0.$

(xi) $If\ f = 0,\ q \neq 0,$ *then* $x_{16n-2} = x_{16n+6} = 0$ *and* $y_{16n-6} = q, y_{16n+2} = -q.$

(xii) $If\ q = 0,\ f \neq 0,$ *then* $x_{16n-2} = x_{16n+6} = f$ *and* $y_{16n-6} = y_{16n+2} = 0.$

(xiii) $If\ g = 0,\ r \neq 0,$ *then* $x_{16n-1} = x_{16n+7} = 0$ *and* $y_{16n-5} = r, y_{16n+3} = -r.$

(*xiv*) *If* $r = 0,\ g \neq 0,$ *then* $x_{16n-1} = x_{16n+7} = g$ *and* $y_{16n-5} = y_{16n+3} = 0.$

(*xv*) *If* $h = 0,\ s \neq 0,$ *then* $x_{16n} = x_{16n+8} = 0$ *and* $y_{16n-4} = s, y_{16n+4} = -s.$

(*xvi*) *If* $s = 0,\ h \neq 0,$ *then* $x_{16n} = x_{16n+8} = h$ *and* $y_{16n-4} = y_{16n+4} = 0.$

Proof. The proof follows from the form of the solutions of system (2.4). □

3. Numerical examples

In order to illustrate the results of the previous sections and to support our theoretical discussions, we consider several interesting numerical examples in this section.

Example 3.1. If we consider the difference equation system (2.1) with the initial conditions $x_{-7} = 0.19,\ x_{-6} = 0.13,\ x_{-5} = -0.35,\ x_{-4} = 0.21,\ x_{-3} = -0.11,\ x_{-2} = 0.09,\ x_{-1} = -0.17,\ x_0 = 0.3,\ y_{-7} = -0.16,\ y_{-6} = -0.33,\ y_{-5} = 0.12,\ y_{-4} = 0.04,\ y_{-3} = -0.17,\ y_{-2} = 0.3,\ y_{-1} = -0.17,$ *and* $y_0 = 0.3$ (see Fig.1).

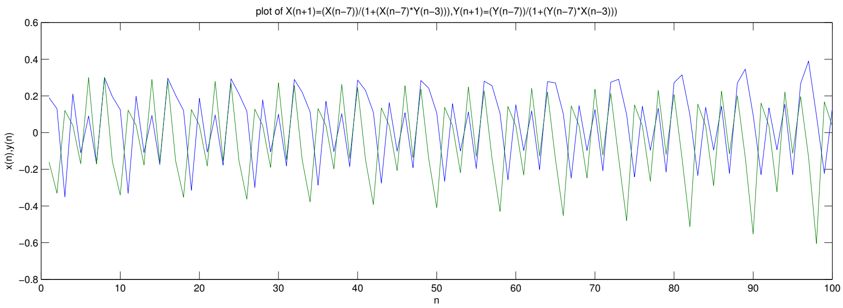

Figure 1: This figure shows the solution of
$x_{n+1}=\frac{x_{n-7}}{1+x_{n-7}y_{n-3}}$, $y_{n+1}=\frac{y_{n-7}}{1+y_{n-7}x_{n-3}}$.

Example 3.2. For the initial conditions $x_{-7} = 0.23,\ x_{-6} = -0.16,\ x_{-5} = -0.07,\ x_{-4} = 0.13,\ x_{-3} = -0.22,\ x_{-2} = 0.15,\ x_{-1} = -0.6,\ x_0 = 0.31,\ y_{-7} = -0.28,\ y_{-6} = -0.14,\ y_{-5} = 0.20,\ y_{-4} = 0.32,\ y_{-3} = -0.07,\ y_{-2} = 0.2,\ y_{-1} = -0.08,$ *and* $y_0 = -0.3$, Figure 2 shows the solution of system (2.2).

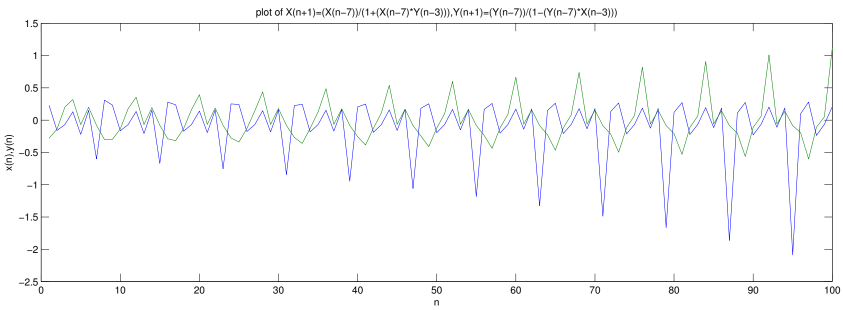

Figure 2: This figure shows the solution of system (2.2).

Example 3.3. Consider the difference equations system (2.3) with the initial conditions $x_{-7} = 0.7,\ x_{-6} = 0.06,\ x_{-5} = 0.17,\ x_{-4} = 0.21,\ x_{-3} = 0.09,\ x_{-2} = 0.14,\ x_{-1} = 0.31,\ x_0 = 0.24,\ y_{-7} = 0.08,\ y_{-6} = 0.24,\ y_{-5} = 0.8,\ y_{-4} = 0.03,\ y_{-3} = 0.17,\ y_{-2} = 0.21,\ y_{-1} = 0.19,\ and\ y_0 = 0.31$. (See Fig. 3).

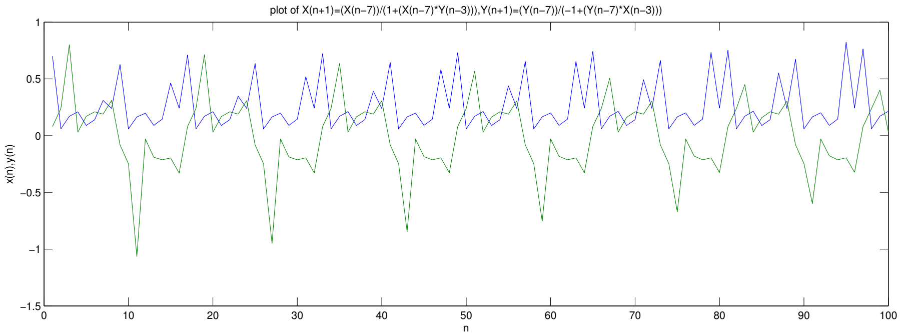

Figure 3: This figure shows the solution of
$x_{n+1} = \frac{x_{n-7}}{1+x_{n-7}y_{n-3}}, y_{n+1} = \frac{y_{n-7}}{-1+y_{n-7}x_{n-3}}$.

Example 3.4. Suppose the difference equations system (2.4) with the initial conditions $x_{-7} = 0.17,\ x_{-6} = 0.2,\ x_{-5} = 0.1,\ x_{-4} = 0.14,\ x_{-3} = 0.31,\ x_{-2} = 0.4,\ x_{-1} = 0.01,\ x_0 = 0.15,\ y_{-7} = -0.12,\ y_{-6} = -0.3,\ y_{-5} = -0.04,\ y_{-4} = -0.18,\ y_{-3} = -0.28,\ y_{-2} = -0.09,\ y_{-1} = -0.13,\ and\ y_0 = -0.34$. (See Fig.4)

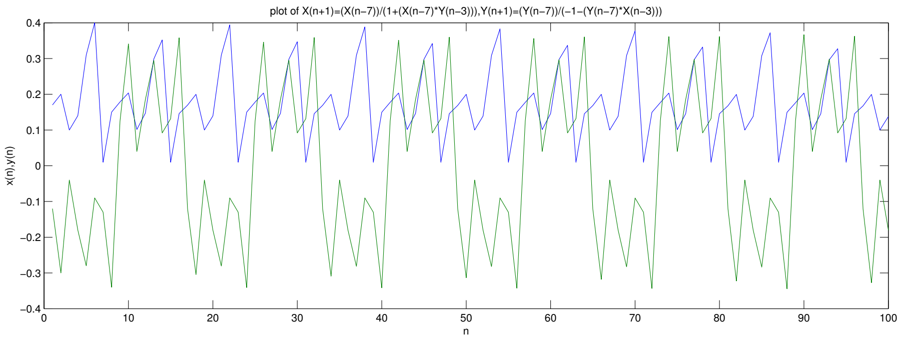

Figure 4: This figure shows the solution of
$x_{n+1} = \frac{x_{n-7}}{1+x_{n-7}y_{n-3}}, y_{n+1} = \frac{y_{n-7}}{-1-y_{n-7}x_{n-3}}$.

4. Conclusion

In this paper, we deal with the form of the solutions of four cases of the difference equations system $x_{n+1} = \frac{x_{n-7}}{1+x_{n-7}y_{n-3}},\ y_{n+1} = \frac{y_{n-7}}{\pm 1 \pm x_{n-3}y_{n-7}}$. Also, we study some behavior of the solutions such as the boundedness in Section 2. Finally, in Section 3, we present some numerical examples by giving some numerical values for the initial values of each case and the figures given to explain the behavior of the obtained solutions in the case of numerical examples by using some mathematical program like Mathematica and Matlab to confirm the obtained results.

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