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Tripled fixed point theorems for contractions in partially ordered $\mathcal{L}$-fuzzy normed spaces

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Abstract

Recently, Kumam, et al. in [P. Kumam, J. Martinez-Moreno, A. Roldán, C. Roldán, J. Inequal. Appl., **2014** (2014), 7 pages] proved some tripled fixed point theorems in fuzzy normed spaces. In this paper, we give a new version of the result of Kumam, et al. by removing some restrictions. In our result, the t-norms are not required to be the minimum ones.

Keywords: Fuzzy normed space, tripled fixed point, fixed point, t-norm.
2010 MSC: 47H10, 54H25.

1. Introduction

The tripled fixed point theorem and its applications in metric spaces are firstly obtained by Berinde and Borcut [10]. Recently, some authors considered tripled fixed point (or coincidence point) theorems in fuzzy metric spaces; see [1, 2, 3, 4, 5, 6, 7, 8, 9, 11, 16, 17, 18, 20].

In [15], the authors gave the following result,

Theorem 1.1. *Let $(X, \sqsubseteq)$ be a partially ordered set and $(X, \mathcal{P}, \mathcal{T})$ be a complete FNS such that $\mathcal{T}$ is of H-type and $\mathcal{T}(a, a) \geq a$ for all $a \in [0, 1]$. Let $k \in (0, 1)$ be a number and $F : X \times X \times X \to X$ and $g : X \to X$ be two mappings such that F has the mixed g-monotone property and*

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$$\mathcal{P}(F(x,y,z)-F(u,v,w),kt) \geq_L \mathcal{T}^3\left(\mathcal{P}(x-u,t),\mathcal{P}(y-v,t),\mathcal{P}(z-w,t)\right),$$

for which $gx \sqsubseteq gu$ *and* $gy \sqsupseteq gv$ *and* $gz \sqsubseteq gw$*, where* $0<k<1$*. Suppose either*

(a) F *is continuous or*

(b) X *has the sequential* g*-monotone property.*

If there exist $x_0, y_0, z_0 \in X$ *such that* $gx_0 \sqsubseteq F(x_0,y_0,z_0)$*,* $gy_0 \sqsupseteq F(y_0,x_0,y_0)$ *and* $gz_0 \sqsubseteq F(z_0,y_0,x_0)$*, then* F *and* g *have a tripled coincidence point.*

The hypothesis $\mathcal{T}(a,a) \geq a$, for all $a \in X$ (which we can find in the previous theorem) is a very restrictive hypothesis, because there is an unique example of t-norm verifying this property.

Lemma 1.2 ([21])**.** *The only* t*-norm* $\mathcal{T}$ *satisfying* $\mathcal{T}(a,a) \geq a$*, for all* $a \in X$*, is the minimum* t*-norm.*

Therefore, the Theorem 1.1 is very restrictive because it is only valid in fuzzy metric spaces under the minimum t-norm. In this paper, by modifying the conditions on the result of Kumam et al. [15], we give a new tripled coincidence point (or fixed point) theorem in partial order fuzzy metric spaces. In our result, we do not require that the t-norm $\mathcal{T}$ satisfies that $\mathcal{T}(a,a) \geq a$, for $a \in X$.

2. Preliminaries

Definition 2.1 ([12])**.** Let $\mathcal{L} = (L, \leq_L)$ be a complete lattice and U a non-empty set called universe. An $\mathcal{L}$-fuzzy set on U is defined as a mapping $\mathcal{A} : U \to L$. For each u in U, $\mathcal{A}(u)$ represents the degree (in L) to which u satisfies $\mathcal{A}$.

Lemma 2.2 ([12])**.** *Consider the set* L^* *and operation* $\leq_{L^*}$ *defined by*

$$L^* = \left\{(x_1,x_2) \in [0,1]^2 \ s.t. \ x_1^2 + x_2^2 \leq 1\right\},$$

$(x_1,x_2) \leq (y_1,y_2) \Leftrightarrow x_1 \leq y_1$ *and* $x_2 \geq y_2$*, for all* $(x_1,x_2),(y_1,y_2) \in L^*$*. Then* $(L^*, \leq_{L^*})$ *is a complete lattice.*

Classically, a triangular norm (t-norm) T on $([0,1],\leq)$ is defined as an increasing, commutative, associative mapping $T:[0,1]^2 \to [0,1]$ satisfying $T(1,x)=x$, for all $x \in [0,1]$. These definitions can be straightforwardly extended to any lattice $\mathcal{L} = (L,\leq_L)$. Define first $0_{\mathcal{L}} = \inf L$ and $1_{\mathcal{L}} = \sup L$.

Definition 2.3 ([19])**.** A *t-norm* on $\mathcal{L}$ is a mapping $\mathcal{T} : L^2 \longrightarrow L$ satisfying the following conditions:

1. $\mathcal{T}(x, 1_{\mathcal{L}}) = x$,
2. $\mathcal{T}(x,y) = \mathcal{T}(y,x)$,
3. $\mathcal{T}(x,\mathcal{T}(y,z)) = \mathcal{T}(\mathcal{T}(x,y),z)$,
4. If $x \leq_L x'$ and $y \leq_L y'$, then $\mathcal{T}(x,y) \leq_L \mathcal{T}(x',y')$.

A t-norm can also be defined recursively as an $(n+1)$-ary operation ($n \in \mathbb{N}$) by $\mathcal{T}^1 = \mathcal{T}$ and

$$\mathcal{T}^n(x_1,...,x_{n+1}) = \mathcal{T}(\mathcal{T}^{n-1}(x_1,...,x_n),x_{n+1}).$$

Definition 2.4 ([19])**.** A negation on $\mathcal{L}$ is any strictly decreasing mapping $\mathcal{N} : L \to L$ satisfying $\mathcal{N}(0_{\mathcal{L}}) = 1_{\mathcal{L}}$ and $\mathcal{N}(1_{\mathcal{L}}) = 0_{\mathcal{L}}$. If $\mathcal{N}(\mathcal{N}(x)) = x$, for all $x \in L$, then N is called an involute negation.

In this paper $\mathcal{N} : L \to N$ is fixed. The negation $\mathcal{N}_s$ on $([0,1],\leq)$, defined, for all $x \in [0,1]$ by $\mathcal{N}_s(x) = 1-x$, is called the standard negation on $([0,1],\leq)$.

Definition 2.5 ([13])**.** For any $a \in [0,1]$, let the sequence $\{\mathcal{T}^n a\}_{n=1}^{\infty}$ be defined by $\mathcal{T}^1 a = a$ and $\mathcal{T}^n a = \mathcal{T}(\mathcal{T}^{n-1}a, a)$. Then A t-norm $\mathcal{T}$ is said to be *of H-type* if the sequence $\{\mathcal{T}^n a\}_{n=1}^{\infty}$ is equicontinuous at $a = 1$.

Definition 2.6 ([19])**.** An *$\mathcal{L}$-fuzzy normed space* is a triple $(X, \mathcal{P}, \mathcal{T})$, where X is a vector space, $\mathcal{T}$ is a continuous t-norm and $\mathcal{P} : X \times (0, \infty) \to \mathcal{L}$ is a $\mathcal{L}$-fuzzy set such that, for all $x, y \in X$ and $t, s > 0$,

(F1) $\mathcal{P}(x,t) >_L 0_{\mathcal{L}}$;
(F2) $\mathcal{P}(x,t) = 1_{\mathcal{L}}$ for all $t > 0$ if and only if $x = 0$;
(F3) $\mathcal{P}(\alpha x, t) = \mathcal{P}(x, \frac{t}{|\alpha|})$ for all $\alpha \neq 0$;
(F4) $\mathcal{T}(\mathcal{P}(x,t), \mathcal{P}(y,s)) \leq_L \mathcal{P}(x+y, t+s)$;
(F5) $\mathcal{P}(x, \cdot) : (0,\infty) \to \mathcal{L}$ is continuous;
(F6) $\lim_{t\to\infty} \mathcal{P}(x,t) = 1_{\mathcal{L}}$ and $\lim_{t\to 0} \mathcal{P}(x,t) = 0_{\mathcal{L}}$.

Lemma 2.7 ([19])**.** *Let $\mathcal{P}$ be an $\mathcal{L}$-fuzzy norm on X. Then*

1. *$\mathcal{P}(x, \cdot)$ is a non-decreasing function on $(0, \infty)$;*
2. *$\mathcal{P}(x - y, t) = \mathcal{P}(y - x, t)$, for all $x, y \in X$ and $t \in (0, +\infty)$.*

Definition 2.8 ([19])**.** Let $(X, \mathcal{P}, \mathcal{T})$ be an $\mathcal{L}$-fuzzy normed space.

1. A sequence $\{x_n\} \subset X$ is called a *Cauchy sequence* if, for any $\epsilon \in L \setminus \{0_{\mathcal{L}}\}$ and $t > 0$, there exists $n_0 \in \mathbb{N}$ such that $\mathcal{P}(x_n - x_m, t) >_L \mathcal{N}(\epsilon)$ for all $n, m \geq n_0$.
2. A sequence $\{x_n\} \subset X$ is said to be *convergent to* a point $x \in X$, denoted by $x_n \to x$ or by $\lim_{n\to\infty} x_n = x$, if, for any $\epsilon \in L \setminus \{0_{\mathcal{L}}\}$ and $t > 0$, there exists $n_0 \in \mathbb{N}$ such that $\mathcal{P}(x_n - x, t) >_L \mathcal{N}(\epsilon)$ for all $n \geq n_0$.
3. An $\mathcal{L}$-fuzzy normed space in which every Cauchy sequence is convergent is said to be *complete*.

Lemma 2.9 ([13])**.** *Let $(X, \mathcal{P}, \mathcal{T})$ be an $\mathcal{L}$-fuzzy normed space such that $\mathcal{T}$ is of H-type. Let $\{x_n\}$ be a sequence in X. If*

$$\mathcal{P}(x_{n+1} - x_n, kt) \geq_L \mathcal{P}(x_n - x_{n-1}, t)$$

for some $k > 1$, $n \in \mathbb{N}$ and $t > 0$, then the sequence $\{x_n\}$ is Cauchy.

Definition 2.10 ([10])**.** Let $F : X^3 \to X$ and $g : X \to X$ be two mappings.

- We say that F and g are *commuting* if $gF(x,y,z) = F(gx, gy, gz)$ for all $x, y, z \in X$.
- A point $(x,y,z) \in X^3$ is called a *tripled coincidence point of the mappings F and g* if $F(x,y,z) = gx$, $F(y,x,y) = gy$ and $F(z,y,x) = gz$. If g is the identity, (x,y,z) is called a tripled fixed point of F.
- If $(X, \sqsubseteq)$ is a partially ordered set, then F is said to have the *mixed g-monotone property* if it verifies the following properties:

$$\begin{aligned}
x_1, x_2 \in X, \ \ gx_1 \sqsubseteq gx_2 \quad &\Longrightarrow \quad F(x_1, y, z) \sqsubseteq F(x_2, y, z), \ \ \forall y \in X,\\
y_1, y_2 \in X, \ \ gy_1 \sqsubseteq gy_2 \quad &\Longrightarrow \quad F(x, y_1, z) \sqsupseteq F(x, y_2, z), \ \ \forall x \in X,\\
z_1, z_2 \in X, \ \ gz_1 \sqsubseteq gz_2 \quad &\Longrightarrow \quad F(x, y, z_1) \sqsubseteq F(x, y, z_2), \ \ \forall x \in X.
\end{aligned}$$

  If g is the identity mapping, then F is said to have the *mixed monotone property.*

- If $(X, \sqsubseteq)$ is a partially ordered set, then X is said to have the *sequential g-monotone property* if it verifies the following properties:

  (B1) If $\{x_n\}$ is a non-decreasing sequence and $\lim_{n\to\infty} x_n = x$, then $gx_n \sqsubseteq gx$ for all $n \in \mathbb{N}$.
  (B2) If $\{x_n\}$ is a non-increasing sequence and $\lim_{n\to\infty} y_n = y$, then $gy_n \sqsupseteq gy$ for all $n \in \mathbb{N}$.

  If g is the identity mapping, then X is said to have the sequential *monotone property.*

Definition 2.11 ([19])**.** Let X and Y be two $\mathcal{L}$-fuzzy normed spaces. A function $f : X \to Y$ is said to be *continuous at a point* $x_0 \in X$ if, for any sequence $\{x_n\}$ in X converging to x_0, the sequence $\{f(x_n)\}$ in Y converges to $f(x_0)$. If f is continuous at each $x \in X$, then f is said to be *continuous on* X.

The following lemma proved by Haghi et al. [14] is useful for our main results:

Lemma 2.12. *Let X be a nonempty set and $g : X \to X$ be a mapping. Then there exists a subset $E \subset X$ such that $g(E) = g(X)$ and $g : E \to X$ is one-to-one.*

3. Main results

Theorem 3.1. *Let $(X, \sqsubseteq)$ be a partially ordered set and $(X, \mathcal{P}, \mathcal{T})$ be a complete $\mathcal{L}$-fuzzy normed space, such that $\mathcal{T}$ is of H-type. Let $k \in (0, 1)$ be a number and $F : X \times X \times X \to X$ be mapping such that F has the mixed monotone property and*

$$\mathcal{P}(F(x, y, z) - F(u, v, w), kt) \geq_L \mathcal{T}^3 \left(\mathcal{P}(x - u, t), \mathcal{P}(y - v, t), \mathcal{P}(z - w, t)\right), \tag{3.1}$$

for which $x \sqsubseteq u$, $y \sqsupseteq v$ and $z \sqsubseteq w$. Suppose that either:

(a) *F is continuous or*
(b) *X has the sequential monotone property.*

If there exist $x_0, y_0, z_0 \in X$ such that $x_0 \sqsubseteq F(x_0, y_0, z_0)$, $y_0 \sqsupseteq F(y_0, x_0, y_0)$ and $z_0 \sqsubseteq F(z_0, y_0, x_0)$, then F has a tripled fixed point. Furthermore, if x_0 and y_0 are comparable, then $x = y$, that is, $x = F(x, x)$.

Proof. As in [1] starting with $x_0, y_0, z_0 \in X$ such that $x_0 \sqsubseteq F(x_0, y_0, z_0)$, $y_0 \sqsupseteq F(y_0, x_0, y_0)$ and $z_0 \sqsubseteq F(z_0, y_0, x_0)$, one can define inductively three sequences $\{x_n\}, \{y_n\}, \{z_n\} \subset X$ such that $x_{n+1} = F(x_n, y_n, z_n)$, $y_{n+1} = F(y_n, x_n, y_n)$ and $z_{n+1} = F(z_n, y_n, x_n)$.

Define

$$\delta_n(t) = \mathcal{T}^3 \left(\mathcal{P}(x_n - x_{n+1}, t), \mathcal{P}(y_n - y_{n+1}, t), \mathcal{P}(z_n - z_{n+1}, t)\right).$$

Continuing as in [1], we have

$$\mathcal{P}(x_n - x_{n+1}, kt) \geq_L \delta_{n-1}(t), \quad \mathcal{P}(y_n - y_{n+1}, kt) \geq_L \delta_{n-1}(t) \tag{3.2}$$

and

$$\mathcal{P}(z_n - z_{n+1}, kt) \geq_L \mathcal{T}^2 \left(\delta_{n-1}(t), \delta_{n-1}(t)\right). \tag{3.3}$$

It follows that

$$\delta_n(kt) = \mathcal{T}^3 \left(\mathcal{P}(x_n - x_{n+1}, kt), \mathcal{P}(y_n - y_{n+1}, kt), \mathcal{P}(z_n - z_{n+1}, kt)\right) \geq_L \mathcal{T}^4 \left(\delta_{n-1}(t)\right)$$

and so

$$1 \geq_L \delta_n(t) \geq_L \mathcal{T}^4 \left(\delta_{n-1}\left(\frac{t}{k}\right)\right) \geq_L \cdots \geq_L \mathcal{T}^{4^n} \left(\delta_0\left(\frac{t}{k^n}\right)\right). \tag{3.4}$$

Since $\lim_{n\to\infty} \delta_0\left(\frac{t}{k^n}\right) = 1$ for all $t > 0$, we have $\lim_{n\to\infty} \delta_n(t) = 1$ for all $t > 0$.

On the other hand, we have

$$t(1 - k)(1 + k + \cdots + k^{m-n-1}) < t, \qquad \forall m > n, 0 < k < 1.$$

By Definition 2.6, we get

$$\begin{aligned} \mathcal{P}(x_n - x_m, t) &\geq_L \mathcal{P}\left(x_n - x_m, t(1 - k)(1 + k + \cdots + k^{m-n-1})\right) \\ &\geq_L \mathcal{T}^2 \left(\mathcal{P}(x_n - x_{n+1}, t(1 - k)), \mathcal{P}\left(x_{n+1} - x_m, t(1 - k)(k + \cdots + k^{m-n-1})\right)\right) \\ &\geq_L \mathcal{T}^{m-n} \left(\mathcal{P}(x_n - x_{n+1}, t(1 - k)), \mathcal{P}\left(x_{n+1} - x_{n+2}, t(1 - k)k\right),\right. \\ &\qquad \left. \cdots, \mathcal{P}\left(x_{m-1} - x_m, t(1 - k)k^{m-n-1}\right)\right). \end{aligned} \tag{3.5}$$

It follows from (3.4) and (3.5) that

$$\begin{aligned}\mathcal{P}(x_n - x_m, t) &\geq \mathcal{T}^{m-n}\left(\left[\mathcal{T}^{4^n}\left(\delta_0\left(\frac{t(1-k)}{k^n}\right)\right)\right], \cdots, \left[\mathcal{T}^{4^{m-1}}\left(\delta_0\left(\frac{t(1-k)}{k^n}\right)\right)\right]\right)\\ &= \mathcal{T}^{4^m-4^n}\left(\delta_0\left(\frac{t(1-k)}{k^n}\right)\right).\end{aligned}$$

By hypothesis, since $\mathcal{T}$ is a t-norm of H-type, there exists $0 < \eta < 1$ such that $\mathcal{T}^p(a) > 1 - \epsilon$ for all $a \in (1-\eta, 1]$ and $p \geq 1$. Since

$$\lim_{n\to\infty} \delta_0\left(\frac{t(1-k)}{k^n}\right) = 1,$$

there exists n_0 such that

$$\mathcal{P}(x_n - x_m, t) > 1 - \epsilon, \qquad \forall m > n > n_0.$$

Therefore, $\{x_n\}$ is a Cauchy sequence. Similarly, $\{y_n\}$ and $\{z_n\}$ are Cauchy sequences. We can continue as in [1] to complete the proof. □

Theorem 3.2. *Let $(X, \mathcal{P}, \mathcal{T})$ be an $\mathcal{L}$-fuzzy normed space such that $\mathcal{T}$ is of H-type, $\sqsubseteq$ be a partial order on X. Let $F : X \times X \times X \to X$ and $g : X \to X$ be two mappings such that F has the mixed g-monotone property and*

$$\mathcal{P}(F(x,y,z) - F(u,v,w), kt) \geq_L \mathcal{T}^3\left(\mathcal{P}(gx - gu, t), \mathcal{P}(gy - gv, t), \mathcal{P}(gz - gw, t)\right)$$

for which $gx \sqsubseteq gu$, $gy \sqsupseteq gv$ and $gz \sqsubseteq gw$, where $0 < k < 1$. Assume that $g(X)$ is complete, $F(X^3) \subset g(X)$ and g is continuous. Suppose either

(a) *F is continuous or*
(b) *X has the sequential g-monotone property.*

If there exist $x_0, y_0, z_0 \in X$ such that $gx_0 \sqsubseteq F(x_0, y_0, z_0)$, $gy_0 \sqsupseteq F(y_0, x_0, y_0)$ and $gz_0 \sqsubseteq F(z_0, y_0, x_0)$, then F and g have a tripled coincidence point.

Proof. As in Theorem 2.2 in [1]. □

4. A note on "On the tripled fixed point and tripled coincidence point theorems in fuzzy normed spaces"

Recently, R. Saadati, et al. in [20] have studied, improved and extended results presented by Abbas et al. to $\mathcal{L}$-fuzzy normed spaces (see [1]).

In order to state our final comments, we give the main results given in [20]:

Theorem 4.1 ([20], Theorem 5.2)**.** *Let $(X, \mathcal{P}, \mathcal{T})$ be an $\mathcal{L}$-fuzzy normed space, $\sqsubseteq$ be a partial order on X. Let $F : X \times X \times X \to X$ and $g : X \to X$ be two mappings such that F has the mixed g-monotone property and*

$$\mathcal{P}(F(x,y,z) - F(u,v,w), kt) \geq_L \mathcal{T}^3\left(\mathcal{P}(gx - gu, t), \mathcal{P}(gy - gv, t), \mathcal{P}(gz - gw, t)\right)$$

for which $gx \sqsubseteq gu$, $gy \sqsupseteq gv$ and $gz \sqsubseteq gw$, where $0 < k < 1$. Assume that $g(X)$ is complete, $F(X^3) \subset g(X)$ and g is continuous. Suppose either

(a) *F is continuous or*
(b) *X has the sequential monotone property.*

If there exist $x_0, y_0, z_0 \in X$ such that $gx_0 \sqsubseteq F(x_0, y_0, z_0)$, $gy_0 \sqsupseteq F(y_0, x_0, y_0)$ and $gz_0 \sqsubseteq F(z_0, y_0, x_0)$, then F and g have a tripled coincidence point.

The proof given by the authors is decisively based on Lemma 2.9. However, Lemma 2.9 is incorrectly enunciated in [20]. They omitted that the t-norm has to be of H-type. As a consequence, all results in the mentioned paper were not correctly proved. Moreover, we can find another mistake in the Theorem 4.1, because it is necessary that X has the sequential g-monotone property. But it is only a misprint.

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