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Iterative solution for nonlinear impulsive advection-reaction-diffusion equations

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Abstract

Through solving equations step by step and by using the generalized Banach fixed point theorem, under simple conditions, the authors present the existence and uniqueness theorem of the iterative solution for nonlinear advection-reaction-diffusion equations with impulsive effects. An explicit iterative scheme for the solution is also derived. The results obtained generalize and improve some known results.

Keywords: Iterative solution, nonlinear advection-reaction-diffusion equations, impulse.
2010 MSC: 35K57, 35R12.

1. Introduction

In this paper, we shall investigate the following nonlinear impulsive advection-reaction-diffusion equations

$$u_t(t,x) = F(t,x,u(t,x),\nabla u(t,x),\Delta u(t,x)), \quad 0<t<T<\infty,\ x\in\Omega\subset\mathbb{R}^m,\ t\neq t_k, \tag{1.1}$$

$$\Delta u(t,x)|_{t=t_k} = I_k(u(t_k,x)), \quad x\in\overline{\Omega},\ k=1,2,\cdots,p, \tag{1.2}$$

$$u(0,x)=u_0(x), \quad x\in\overline{\Omega}, \tag{1.3}$$

where $u(t,x)\in\mathbb{R}^N$, $F(t,x,u,\nabla u,\Delta u)\in\mathbb{R}^N$, and $t\in J=[0,T]$ is the time variable, the subscript u_t denotes partial differentiation with respect to t, $u_0(x)\in\mathbb{R}^N$, $u(t,x)\in C^1(0,T)\times C^2(\Omega)$ and continuous in

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$[0,T]$, $u_0(x) \in C^2(\Omega)$, Δ is the Laplacian operator, ∇ is the gradient operator and Ω is a bounded spatial region. $0 < t_1 < t_2 < \cdots < t_k < \cdots < t_p < T$, $F \in C(J \times \mathbb{R}^m \times \mathbb{R}^N \times \mathbb{R}^N \times \mathbb{R}^N, \mathbb{R}^N)$, $I_k \in C(\mathbb{R}^N, \mathbb{R}^N)$ $(k = 1, 2, \cdots, p)$. $\Delta u(t,x)|_{t=t_k}$ denotes the jump of $u(t,x)$ at $t = t_k$, i.e., $\Delta u(t,x)|_{t=t_k} = u(t_k^+, x) - u(t_k^-, x)$, where $u(t_k^+, x)$ and $u(t_k^-, x)$ represent the right and left limits of $u(t,x)$ at $t = t_k$, respectively.

Advection-reaction-diffusion equations are used to simulate a variety of different phenomena, from mathematical biology to physics and engineering. Diffusion, advection and reaction respectively refer to those terms in the partial differential equations involving second, first and zero order derivatives of the unknowns with respect to the spatial variables. Equation (1.1) governs a large number of phenomena arising in chemical engineering, population dynamics, biology, physiology, combustion, ecology, chemotaxis, etc. [2, 5, 9, 21]. For example, in combustion and heat and mass transfer, $u(t,x)$ may represent either the species concentrations or the temperature [21].

Impulsive differential equations arise naturally from a wide variety of applications, such as spacecraft control, inspection processes in operations research, drug administration and threshold theory in biology. Over the past decade, a significant advance in the theory of impulsive systems has been achieved. For the basic theory and recent development, the reader is referred to [7, 11, 12, 18] and the references therein.

Over the last couple of decades, the existence, uniqueness, qualitative properties, and stability properties of solutions have been extensively studied for nonlinear advection-reaction-diffusion equations, see [3, 6, 8, 13, 15, 17, 19, 20, 22, 23]. In the special cases where F does not possess advection term, under several possible assumptions on the nonlinearity, the existence, uniqueness, stability properties of the special solutions and influence on the dynamics of the problems have been investigated in [19, 20, 22].

Recently, Ramos [16] presented an iterative method for solving nonlinear advection-reaction-diffusion equations without impulses and proved its convergence. The method was formulated in terms of a Picard operator and made use of Banach fixed-point theorem. In spite of the abundant literature on initial value problem for nonlinear advection-reaction-diffusion equations, there are few references dealing with this kinds of problems with impulses.

The aim of this paper is to develop some theories of nonlinear advection-reaction-diffusion equations with impulsive terms. Motivated by the works [14], through solving equations step by step and by using the generalized Banach fixed point theorem (see [4]), we prove the existence and uniqueness solution for impulsive problem (1.1)-(1.3), and derive an approximation sequence of the solution which is explicitly expressed. Our results improve and generalize related results in [16] to some degree.

The rest of the paper is organized as follows: In Section 2, we give some preliminaries to be used in the next section. The main results is formulated and proved in Section 3.

2. Preliminaries

Let $J' = J \setminus \{t_1,\ t_2, \cdots, t_p\}$, $J_0 = [0,\ t_1]$, $J_1 = (t_1,\ t_2], \cdots, J_{p-1} = (t_{p-1},\ t_p]$, $J_p = (t_p,\ T]$. We introduce a Banach space as follows [1, 10]. Let $PC(J) = \{u :$ for $x \in \Omega$, $u(\cdot, x)$ is a map from J into $L^2(\Omega)$ such that $u(t,x)$ is continuous at $t \neq t_k$, left continuous at $t = t_k$ and its right limit at $t = t_k$ exists for $k = 1, 2, \cdots, p\}$, and the vector function spaces

$$\begin{aligned} SPC(J,\Omega) &= \left\{u = (u_1,\ u_2, \cdots, u_N) | u_i \in PC(J) \times L^2(\Omega),\ i = 1, 2, \cdots, N\right\}, \\ H_1(J,\Omega) &= \left\{u = (u_1,\ u_2, \cdots, u_N) | u_i \in C^1(J) \times L^2(\Omega),\ i = 1, 2, \cdots, N\right\}, \\ H_2(J,\Omega) &= \left\{u = (u_1,\ u_2, \cdots, u_N) | u_i \in C(J) \times L^2(\Omega),\ i = 1, 2, \cdots, N\right\}. \end{aligned}$$

As in [10, 11], it is easily shown that SPC is a Banach space with the norm $\|u\|_{SPC} = \sup_{t\in J} \|u(t,x)\|$, where $\|u(t,x)\| = \left[\sum_{i=1}^N \int_\Omega u_i^2(t,x)dx\right]^{\frac{1}{2}}$.

We need the following lemma in this paper.

Lemma 2.1 ([14]). *Suppose* $0 < \theta < 1$, $h > 0$ *are constants, let*

$$S = \theta^n + C_n^1 \theta^{n-1} h + \frac{C_n^2 \theta^2 h^2}{2!} + \cdots + \frac{h^n}{n!}, \quad n \in \mathbb{N},$$

then

$$S \le o\left(\frac{1}{n^{s+1}}\right) \quad (n \to +\infty) \quad \textit{for any real constant} \quad s > 0.$$

3. Main results

In this section, we give the main results of our paper.

Theorem 3.1. *If the following condition* (H) *is satisfied,*

(H) *There exists a Lebesgue integrable nonnegative function* $q \in L^2(J, \mathbb{R}^+)$ *such that*

$$\|F(t, x, u, \nabla u, \Delta u) - F(t, x, v, \nabla v, \Delta v)\| \le q(t)\|u - v\|,$$

for any $u, v \in SPC(J, \Omega)$, $(t, x) \in J \times \Omega$.
Then problem (1.1)-(1.3) *has a unique solution* $\eta(t, x) \in SPC(J, \Omega) \cap H_1(J', \Omega)$ *which can be written by*

$$\eta(t,x) = \begin{cases} \eta_0(t,x), & t \in J_0, \\ \eta_1(t,x), & t \in J_1, \\ \cdots, & \cdots, \\ \eta_p(t,x), & t \in J_p, \ x \in \Omega. \end{cases}$$

Moreover, for any $y_0 \in SPC(J, \Omega)$, *the iterative sequence* $\{y_n\}$ *defined by*

$$y_n(t,x) = (Ay_{n-1})(t,x) = \begin{cases} (A_0 y_{(n-1)0})(t,x), & t \in J_0, \\ (A_1 y_{(n-1)1})(t,x), & t \in J_1, \\ \cdots, & \cdots, \\ (A_p y_{(n-1)p})(t,x), & t \in J_p, \ x \in \Omega, \ n = 1, 2, \cdots \end{cases}$$

converges uniformly to $\eta(t, x)$ *on* $(t, x) \in J \times \Omega$, *where*

$$y_{n-1}(t,x) = \begin{cases} y_{(n-1)0}(t,x), & t \in J_0, \\ y_{(n-1)1}(t,x), & t \in J_1, \\ \cdots, & \cdots, \\ v_{(n-1)p}(t,x), & t \in J_p, \ x \in \Omega, \end{cases} \qquad A = \begin{cases} A_0, \\ A_1, \\ \cdots, \\ A_p, \end{cases}$$

and A_0, A_i $(i = 1, \cdots, p)$ *are defined by*

$$(A_0 y_{(n-1)0})(t,x) = u_0(x) + \int_0^t F\left(s, x, y_{(n-1)0}(s,x), \nabla y_{(n-1)0}(s,x), \Delta y_{(n-1)0}(s,x)\right) ds, \quad t \in J_0,$$

$$\begin{aligned}(A_i y_{(n-1)i})(t,x) =& I_i(\eta_{i-1}(t_i, x)) + \eta_{i-1}(t_i, x) \\ &+ \int_{t_i}^t F\left(s, x, y_{(n-1)i}(s,x), \nabla y_{(n-1)i}(s,x), \Delta y_{(n-1)i}(s,x)\right) ds, \quad t \in J_i.\end{aligned}$$

Remark 3.2. Comparing conditions $(H_1)-(H_3)$ of paper [16] with (H) of this paper, we do not require (H_1) and (H_3), our condition (H) is weaker and more general than (H_2), and by means of a completely different method with [16], we have proven the existence of a unique solution for impulsive problem (1.1)-(1.3). Our result in essence improves and generalizes related results in [16] to some degree.

Remark 3.3. It is value to point out that the iterative sequences $\{y_n\}$ are expressed explicitly, which is an important improvement compared with those in the above mentioned papers.

Proof. Our proof is divided into four steps.

For $0 < \varepsilon < \frac{1}{T}$, from the property of Lebesgue integrable functions, there exists a continuous function $\psi(t)$ in $[0,T]$ such that $\int_0^T |q^2(t) - \psi(t)|dt < \varepsilon$. Let $M = \sup_{t\in J} |\psi(t)|$, evidently, $0 \le M < +\infty$.

Step 1. We consider the following initial value problem to nonlinear advection-reaction-diffusion equation:

$$\begin{cases} u_t(t,x) = F(t,x,u(t,x),\nabla u(t,x),\Delta u(t,x)), \quad t\in J_0,\ x\in\Omega, \\ u(0,x) = u_0(x). \end{cases} \tag{3.1}$$

It is well known that $u \in SPC(J_0,\Omega)\cap H_1(J_0,\Omega)$ is a solution of (3.1) if only if $u\in H_2(J_0,\Omega)$ is a solution of the following integral equation:

$$u(t,x) = u_0(x) + \int_0^t F(s,x,u(s,x),\nabla u(s,x),\Delta u(s,x))ds, \quad t\in J_0,\ x\in\Omega.$$

Define operator A_0 by

$$(A_0u)(t,x) = u_0(x) + \int_0^t F(s,x,u(s,x),\nabla u(s,x),\Delta u(s,x))ds, \quad t\in J_0,\ x\in\Omega. \tag{3.2}$$

Clearly, $A_0 : H_2(J_0,\Omega) \to H_2(J_0,\Omega)$. From (H) and (3.2) and Cauchy-Schwarz-Bunyakovski inequality, for $u,v\in H_2(J_0,\Omega),\ t\in J_0,\ x\in\Omega$, we have

$$\begin{aligned}
&\|(A_0u)(t,x) - (A_0v)(t,x)\|^2 \\
&\quad= \left\|\int_0^t (F(s,x,u,\nabla u,\Delta u) - F(s,x,v,\nabla v,\Delta v))ds\right\|^2 \\
&\quad= \sum_{i=1}^N \int_\Omega \left(\int_0^t (F_i(s,x,u,\nabla u,\Delta u) - F_i(s,x,v,\nabla v,\Delta v))ds\right)^2 dx \\
&\quad\le \sum_{i=1}^N \int_\Omega \left(t\int_0^t (F_i(s,x,u,\nabla u,\Delta u) - F_i(s,x,v,\nabla v,\Delta v))^2ds\right) dx \\
&\quad= \sum_{i=1}^N \int_0^t t\left(\int_\Omega (F_i(s,x,u,\nabla u,\Delta u) - F_i(s,x,v,\nabla v,\Delta v))^2dx\right) ds \\
&\quad\le T\sum_{i=1}^N \int_0^t \left(\int_\Omega (F_i(s,x,u,\nabla u,\Delta u) - F_i(s,x,v,\nabla v,\Delta v))^2dx\right) ds \\
&\quad= T\int_0^t \sum_{i=1}^N \left(\int_\Omega (F_i(s,x,u,\nabla u,\Delta u) - F_i(s,x,v,\nabla v,\Delta v))^2dx\right) ds \\
&\quad= T\int_0^t \|F(s,x,u,\nabla u,\Delta u) - F(s,x,v,\nabla v,\Delta v))\|^2ds \\
&\quad\le T\int_0^t q^2(s)\|u(s,x) - v(s,x)\|^2ds \\
&\quad\le T\left(\int_0^t |q^2(s) - \psi(s)|ds + \int_0^t |\psi(s)|ds\right)\|u-v\|_{SPC}^2 \\
&\quad\le T(\varepsilon + Mt)\|u-v\|_{SPC}^2.
\end{aligned} \tag{3.3}$$

From (3.2) and (3.3), we obtain

$$\begin{aligned}
&\|(A_0^2u)(t,x)-(A_0^2v)(t,x)\|^2\\
&\quad=\left\|\int_0^t (F(s,x,A_0u,\nabla(A_0u),\Delta(A_0u))-F(s,x,A_0v,\nabla(A_0v),\Delta(A_0v))ds\right\|^2\\
&\quad\le T\int_0^t \|F(s,x,A_0u,\nabla(A_0u),\Delta(A_0u))-F(s,x,A_0v,\nabla(A_0v),\Delta(A_0v)\|^2ds\\
&\quad\le T\int_0^t q^2(s)\|(A_0u)(s,x)-(A_0v)(s,x)\|^2ds\\
&\quad\le T^2\int_0^t q^2(s)(\varepsilon+Ms)ds\|u-v\|_{SPC}^2\\
&\quad\le T^2\left(\int_0^t |q^2(s)-\psi(s)|(\varepsilon+Ms)ds+\int_0^t|\psi(s)|(\varepsilon+Ms)ds\right)\|u-v\|_{SPC}^2\\
&\quad\le T^2\left[\varepsilon(\varepsilon+Mt)+M\varepsilon t+\frac{M^2t^2}{2}\right]\|u-v\|_{SPC}^2.
\end{aligned}\tag{3.4}$$

In the following, by the method of mathematical induction, for any positive integer n and $(t,x)\in J_0\times\Omega$, we will prove that

$$\begin{aligned}
&\|(A_0^nu)(t,x)-(A_0^nv)(t,x)\|^2\\
&\quad\le T^n\left(\varepsilon^n+C_n^1\varepsilon^{n-1}(Mt)+\cdots+C_n^j\varepsilon^{n-j}\frac{(Mt)^j}{j!}+\cdots+\frac{(Mt)^n}{n!}\right)\|u-v\|_{SPC}^2,
\end{aligned}\tag{3.5}$$

where $C_n^j=\frac{n!}{j!(n-j)!}$, $\quad j!=j\cdot(j-1)\cdots 3\cdot 2\cdot 1$. When $n=1$, (3.5) holds by (3.3). For $n=2$, (3.5) holds by (3.4). Suppose (3.5) holds for $n=k$, that is, for any $(t,x)\in J_0\times\Omega$,

$$\|(A_0^ku)(t,x)-(A_0^kv)(t,x)\|^2\le T^k\left(\varepsilon^k+C_k^1\varepsilon^{k-1}(Mt)+\cdots+C_k^j\varepsilon^{k-j}\frac{(Mt)^j}{j!}+\cdots+\frac{(Mt)^k}{k!}\right)\|u-v\|_{SPC}^2.$$

Then, by (H), (3.2), (3.3), and applying formula $C_{k+1}^j=C_k^j+C_k^{j-1}$, for any $(t,x)\in J_0\times\Omega$, one has

$$\begin{aligned}
&\|(A_0^{k+1}u)(t,x)-(A_0^{k+1}v)(t,x)\|^2\\
&\quad=\left\|\int_0^t (F(s,x,A_0^ku,\nabla(A_0^ku),\Delta(A_0^ku))-F(s,x,A_0^kv,\nabla(A_0^kv),\Delta(A_0^kv))ds\right\|^2\\
&\quad\le T\int_0^t \|F(s,x,A_0^ku,\nabla(A_0^ku),\Delta(A_0^ku))-F(s,x,A_0^kv,\nabla(A_0^kv),\Delta(A_0^kv))\|^2ds\\
&\quad\le T\int_0^t q^2(s)\|(A_0^ku)(s,x)-(A_0^kv)(s,x)\|^2ds\\
&\quad\le T^{k+1}\int_0^t q^2(s)\left[\varepsilon^k+C_k^1\varepsilon^{k-1}(Ms)+\cdots+C_k^j\varepsilon^{k-j}\frac{(Ms)^j}{j!}+\cdots+\frac{(Ms)^k}{k!}\right]ds\|u-v\|_{SPC}^2\\
&\quad\le T^{k+1}\left[\int_0^t |q^2(s)-\psi(s)|\left(\varepsilon^k+C_k^1\varepsilon^{k-1}(Ms)+\cdots+C_k^j\varepsilon^{k-j}\frac{(Ms)^j}{j!}+\cdots+\frac{(Ms)^k}{k!}\right)ds\right.\\
&\qquad\left.+\int_0^t|\psi(s)|\left(\varepsilon^k+C_k^1\varepsilon^{k-1}(Ms)+\cdots+C_k^j\varepsilon^{k-j}\frac{(Ms)^j}{j!}+\cdots+\frac{(Ms)^k}{k!}\right)ds\right]\|u-v\|_{SPC}^2\\
&\quad\le T^{k+1}\left[\varepsilon\left(\varepsilon^k+C_k^1\varepsilon^{k-1}(Mt)+\cdots+C_k^j\varepsilon^{k-j}\frac{(Mt)^j}{j!}+\cdots+\frac{(Mt)^k}{k!}\right)\right.\\
&\qquad\left.+\left(\varepsilon^kMt+C_k^1\varepsilon^{k-1}\frac{(Mt)^2}{2!}+\cdots+C_k^j\varepsilon^{k-j}\frac{(Mt)^{j+1}}{(j+1)!}+\cdots+\frac{(Mt)^{k+1}}{(k+1)!}\right)\right]\|u-v\|_{SPC}^2\\
&\quad=T^{k+1}\left[\varepsilon^{k+1}+C_{k+1}^1\varepsilon^k(Mt)+\cdots+C_{k+1}^j\varepsilon^{k-j+1}\frac{(Mt)^j}{j!}+\cdots+\frac{(Mt)^{k+1}}{(k+1)!}\right]\|u-v\|_{SPC}^2.
\end{aligned}$$

Hence, (3.5) holds for $n = k + 1$. Therefore, for any positive integer n, denote $\theta = T\varepsilon,\ h = MT^2$, we have

$$\|A_0^n u - A_0^n v\|_{SPC}^2 \le \left(\theta^n + C_n^1\theta^{n-1}h + \cdots + C_k^j\theta^{k-j}\frac{h^j}{j!} + \cdots + \frac{h^n}{n!}\right)\|u - v\|_{SPC}^2. \tag{3.6}$$

Consequently, Lemma 2.1 and (3.6) imply that for any real constant $s > 0$, there exists a positive integer n_0 such that for any $u, v \in H_2(J_0, \Omega)$,

$$\|A_0^n u - A_0^n v\|_{SPC}^2 \le \frac{1}{n^{s+1}}\|u - v\|_{SPC}^2, \quad \forall n > n_0,$$

and

$$\|A_0^n u - A_0^n v\|_{SPC} \le \frac{1}{n^{\frac{s+1}{2}}}\|u - v\|_{SPC}, \quad \forall n > n_0.$$

So A_0^n is a contraction operator on $H_2(J_0, \Omega)$. By the generalized Banach Contraction Theorem, we conclude that A_0 has only one fixed point $\eta_0 \in H_2(J_0, \Omega)$. This implies that (3.1) has a unique solution $\eta_0 \in SPC(J_0, \Omega) \cap H_1(J_0, \Omega)$ such that

$$\begin{cases} (\eta_0)_t(t, x) = F(t, x, \eta_0(t, x), \nabla\eta_0(t, x), \Delta\eta_0(t, x)), & t \in J_0,\ x \in \Omega, \\ \eta_0(0, x) = u_0(x). \end{cases} \tag{3.7}$$

Step 2. We consider the following nonlinear advection-reaction-diffusion equation:

$$\begin{cases} u_t(t, x) = F(t, x, u(t, x), \nabla u(t, x), \Delta u(t, x)), & t \in J_1,\ x \in \Omega, \\ u(t_1^+, x) = I_1(\eta_0(t_1, x)) + \eta_0(t_1, x). \end{cases} \tag{3.8}$$

It is easy to prove that $u \in SPC(J_1, \Omega) \cap H_1(J_1', \Omega)$ is a solution of (3.8) if only if $u \in H_2(J_1', \Omega)$ is a solution of the following integral equation:

$$u(t, x) = I_1(\eta_0(t_1, x)) + \eta_0(t_1, x) + \int_{t_1}^t F(s, x, u(s, x), \nabla u(s, x), \Delta u(s, x))ds, \quad t \in J_1,\ x \in \Omega.$$

Let

$$(A_1 u)(t, x) = I_1(\eta_0(t_1, x)) + \eta_0(t_1, x) + \int_{t_1}^t F(s, x, u(s, x), \nabla u(s, x), \Delta u(s, x))ds, \quad t \in J_1,\ x \in \Omega. \tag{3.9}$$

Clearly, $A_1 : H_2(J_1, \Omega) \to H_2(J_1, \Omega)$. From (H) and (3.9), for $u, v \in H_2(J_1, \Omega)$, we get

$$\begin{aligned}
&\|(A_1 u)(t, x) - (A_1 v)(t, x)\|^2 \\
&\quad= \left\|\int_{t_1}^t (F(s, x, u(s, x), \nabla u(s, x), \Delta u(s, x)) - F(s, x, v(s, x), \nabla v(s, x), \Delta v(s, x)))ds\right\|^2 \\
&\quad\le T\int_{t_1}^t \|F(s, x, u, \nabla u, \Delta u) - F(s, x, v, \nabla v, \Delta v))\|^2 ds \\
&\quad\le T\int_{t_1}^t q^2(s)\|u(s, x) - v(s, x)\|^2 ds \\
&\quad\le T\left(\int_0^t |q^2(s) - \psi(s)|ds + \int_0^t |\psi(s)|ds\right)\|u - v\|_{SPC}^2 \\
&\quad\le T(\varepsilon + Mt)\|u - v\|_{SPC}^2.
\end{aligned}$$

Similar to the proof of Step 1, A_1^n is a contraction operator on $H_2(J_1, \Omega)$. By the Banach Contraction Theorem, we conclude that A_1 has only one fixed point $\eta_1 \in H_2(J_1, \Omega)$, that is (3.8) has a unique solution $\eta_1 \in SPC(J_1, \Omega) \cap H_1(J_1', \Omega)$ such that

$$\begin{cases} (\eta_1)_t(t, x) = F(t, x, \eta_1(t, x), \nabla\eta_1(t, x), \Delta\eta_1(t, x)), & t \in J_1,\ x \in \Omega, \\ \eta_1(t_1^+, x) = I_1(\eta_0(t_1, x)) + \eta_0(t_1, x). \end{cases} \tag{3.10}$$

Step 3. For $i = 2, 3, \cdots, p$, we repeat the above procedure, then the following problem:

$$\begin{cases} u_t(t,x) = F(t,x,u(t,x),\nabla u(t,x),\Delta u(t,x)), & t \in J_i,\ x \in \Omega, \\ u(t_i^+,x) = I_i(\eta_{i-1}(t_i,x)) + \eta_{i-1}(t_i,x), \end{cases}$$

has a unique solution $\eta_i(t,x) \in SPC(J_i,\Omega) \cap H_1(J_i')$ such that

$$\begin{cases} (\eta_i)_t(t,x) = F(t,x,\eta_i(t,x),\nabla \eta_i(t,x),\Delta \eta_i(t,x)), & t \in J_i,\ x \in \Omega, \\ \eta_i(t_i^+,x) = I_i(\eta_{i-1}(t_i,x)) + \eta_{i-1}(t_i,x). \end{cases} \tag{3.11}$$

Let

$$\eta(t,x) = \begin{cases} \eta_0(t,x), & t \in J_0, \\ \eta_1(t,x), & t \in J_1, \\ \cdots, & \cdots, \\ \eta_p(t,x), & t \in J_p,\ x \in \Omega. \end{cases} \tag{3.12}$$

Then, from (3.7), (3.10)-(3.12), $\eta(t,x) \in SPC(J,\Omega) \cap H_1(J',\Omega)$ is unique solution of impulsive problem (1.1)-(1.3).

Step 4. For any $y_0 \in SPC(J,\Omega)$, let $y_n = Ay_{n-1}$, where

$$(Ay_{n-1})(t,x) = \begin{cases} (A_0 y_{(n-1)0})(t,x), & t \in J_0, \\ (A_1 y_{(n-1)1})(t,x), & t \in J_1, \\ \cdots, & \cdots, \\ (A_p y_{(n-1)p})(t,x), & t \in J_p,\ x \in \Omega, \end{cases}$$

$$y_{n-1}(t,x) = \begin{cases} y_{(n-1)0}(t,x), & t \in J_0, \\ y_{(n-1)1}(t,x), & t \in J_1, \\ \cdots, & \cdots, \\ y_{(n-1)p}(t,x), & t \in J_p,\ x \in \Omega. \end{cases}$$

From the proof of Step 1-Step 3, it is easy to prove that the iterative convergence theorem holds. Thus, we complete the proof of Theorem 3.1. □

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