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An explicit iterative algorithm for k-strictly pseudo-contractive mappings in Banach spaces

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Abstract

Let E be a real uniformly smooth Banach space. Let K be a nonempty bounded closed and convex subset of E. Let $T : K \to K$ be a strictly pseudo-contractive map and f be a contraction on K. Assume $F(T) := \{x \in K : Tx = x\} \neq \emptyset$. Consider the following iterative algorithm in K given by

$$x_{n+1} = \alpha_n f(x_n) + \beta_n x_n + \gamma_n S_n x_n,$$

where $S_n : K \to K$ is a mapping defined by $S_n x := (1 - \delta_n)x + \delta_n Tx$. It is proved that the sequence $\{x_n\}$ generated by the above iterative algorithm converges strongly to a fixed point of T. Our results mainly extend and improve the results of [C. O. Chidume, G. De Souza, Nonlinear Anal., **69** (2008), 2286–2292] and [J. Balooee, Y. J. Cho, M. Roohi, Numer. Funct. Anal. Optim., **37** (2016), 284–303].

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1. Introduction

Let E be a real normed space and E^* be its dual space, K be a nonempty subset of a real normed space E, and J denotes the normalized duality mapping from E to 2^{E^*}, which is defined by

$$J(x) := \{f \in E^* : \langle x, f\rangle = \|x\|^2 = \|f\|^2\}.$$

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Recall that $T : K \to K$ is called to be nonexpansive, if

$$\|Tx - Ty\| \leq \|x - y\|, \quad \forall x, y \in K.$$

T is called to be pseudo-contractive if there exists $j(x - y) \in J(x - y)$ such that

$$\langle Tx - Ty, j(x - y)\rangle \leq \|x - y\|^2, \quad \forall x, y \in K.$$

It is trivial to see from this, that nonexpansive mappings are pseudo-contractive mappings; numerous papers have been written on the approximation of fixed points of pseudo-contractive mappings (see, [3, 6, 8, 14, 28, 29]).

A mapping T is said to be k-strictly pseudo-contractive if there exists $j(x-y) \in J(x-y)$ and a constant $k \in (0, 1)$ such that

$$\langle Tx - Ty, j(x - y)\rangle \leq \|x - y\|^2 - k\|(I - T)x - (I - T)y\|^2, \quad \forall x, y \in K. \tag{1.1}$$

It is easy to see that such mappings are Lipschitz with Lipschitz constant $L = \frac{k+1}{k}$. In 1953, Mann [10] proposed the normal Mann's iterative algorithm defined by a fixed $x_0 \in K$ and the sequence $\{x_n\}$ is given by

$$x_{n+1} = \alpha_n x_n + (1 - \alpha_n)Tx_n, \; n \geq 0,$$

where $\{\alpha_n\}$ is a real sequence in [0,1] satisfying the following conditions:

(i) $\lim\limits_{n\to\infty} \alpha_n = 0$;

(ii) $\sum_{n=1}^{\infty} \alpha_n = \infty$,

where T is a mapping of K into itself. Since then, construction for nonexpansive mappings and k-strictly pseudo-contractive via the normal Mann's iterative algorithm has been extensively studied [2, 7, 10–12, 15]. In 2013, Yao et al. [26] presented the Ishikawa algorithms with hybrid techniques for finding the fixed points of a Lipschitzian pseudocontractive mapping. Also there are many other algorithms about the convergence analysis of fixed point theory [22, 23, 27].

In 1967, Browder and Petryshyn [2] firstly introduced the conception of strict pseudo-contraction in a real Hilbert space H. Let K be a nonempty subset of a real Hilbert space. A mapping $T : K \to K$ is called strict pseudo-contraction if

$$\|Tx - Ty\|^2 \leq \|x - y\|^2 + k\|(I - T)x - (I - T)y\|^2, \quad \forall x, y \in K, \tag{1.2}$$

holds for some $0 < k < 1$. It is easy to see that in real Hilbert spaces, (1.1) and (1.2) are equivalent. They also firstly proved the weak and strong convergence theorems for k-strict pseudo-contraction by using the following algorithm

$$x_{n+1} = (1 - \gamma)x_n + \gamma Tx_n, \; n \in N.$$

Another iteration process, so called Halpern iteration has been found to be successful for the approximation of a nonexpansive. Let K be a nonempty closed and convex subset of a Hilbert space H and $T : K \to K$ be a nonexpansive mapping. Assume $F(T) \neq \emptyset$. Halpern [5] studied the following iteration formula to approximate a fixed point of T:

For all $u \in K$, let the sequence $\{x_n\}$ in K be defined by $x_0 \in K$, and

$$x_{n+1} = \alpha_n u + (1 - \alpha_n)Tx_n, \; n \geq 0. \tag{1.3}$$

As α_n is under certain conditions, Halpern studied the special case of (1.3) in which $\alpha_n = n^{-\sigma}$, $\sigma \in (0, 1)$ and $u = 0$, and proved that $\{x_n\}$ converges strongly to a fixed point of T. Under a different restriction on $\{\alpha_n\}$, in 1977, Lions [9] improved the result of Halpern, still in Hilbert spaces. He investigated strong convergence of the sequence $\{x_n\}$, where α_n satisfies

(i) $\lim_{n\to\infty} \alpha_n = 0$;

(ii) $\sum_{n=1}^{\infty} \alpha_n = \infty$;

(iii) $\lim_{n\to\infty} \frac{|\alpha_n - \alpha_{n-1}|}{\alpha_n^2} = 0$.

Reich [16] studied the result of Halpern in the uniformly smooth Banach scheme. It was observed that both Halpern's and Lion's conditions on α_n excluded the choice $\alpha_n = \frac{1}{n+1}$. This was overcome in 1992 by Wittman [18], who proved the strong convergence of $\{x_n\}$ still in Hilbert spaces if $\{\alpha_n\}$ satisfies the conditions:

(i) $\lim_{n\to\infty} \alpha_n = 0$;

(ii) $\sum_{n=1}^{\infty} \alpha_n = \infty$;

(iii) $\sum_{n=1}^{\infty} |\alpha_n - \alpha_{n-1}| < \infty$.

In 2002, Xu [19] improved the result of Lions [9]. More precisely, he weakened the condition (iii) by removing the square in the denominator so that the choice of $\alpha_n = \frac{1}{n+1}$ is possible.

Chidume and De Souza [4] established a strong convergence theorem for strictly pseudo-contraction in Banach space scheme, the result is as follows:

Theorem CG. *Let E be a real reflexive Banach space with uniformly Gâteaux differentiable norm. Let K be a nonempty bounded closed and convex subset of E. Let $T : K \to K$ be a strictly pseudo-contractive map. Assume $F(T) \neq \emptyset$ and let $z \in F(T)$. Fix $\delta \in (0,1)$ and let δ^* be such that $\delta^* := \delta L \in (0,1)$. Define $S_n := (1-\delta_n)x + \delta_n Tx$ for all $x \in K$, where $\delta_n \in (0,1)$ and $\lim_{n\to\infty} \delta_n = 0$. Let $\{\alpha_n\}$ be a real sequence in $(0,1)$ which satisfies the conditions* (i), (ii)*. For arbitrary $x_0, u \in K$, define a sequence $\{x_n\} \in K$ by*

$$x_{n+1} = \alpha_n u + (1-\alpha_n) S_n x_n, \ n \geq 0.$$

Then, $\{x_n\}$ converges strongly to a fixed point of T.

Very recently, Yao et al. [25] studied the iterative algorithms for finding the fixed points of asymptotically pseudo-contractive mappings in Hilbert spaces. In 2016, Balooee et al. [1] presented the weak convergence of the sequence $\{x_n\}$ generated by Mann's iterative scheme to a fixed point of a uniformly Lipschitzian and pointwise asymptotically 01-strict pseudo-contractive mapping T in a Hilbert space. In 2014, [24] introduced another new iterative algorithm and got the strong convergence results in Hilbert spaces.

Motivated by the results of Chidume and De Souza [4] and the above other works, in this paper, we establish a new iteration process in Banach space scheme as follows:

$$x_{n+1} = \alpha_n f(x_n) + \beta_n x_n + \gamma_n S_n x_n, \tag{1.4}$$

where $S_n x := (1-\delta_n)x_n + \delta_n T x_n$, $T : K \to K$ is k-strictly pseudo-contraction and $f : K \to K$ is a contraction with the contractive coefficient α $(0 < \alpha < 1)$, and the real sequences $\{\alpha_n\}$, $\{\beta_n\}$, $\{\delta_n\}$ satisfying appropriate conditions. We will prove the sequence $\{x_n\}$ defined by (1.4) strongly converges to a fixed point of T in a real Banach space.

2. Preliminaries

In the sequel we shall make use of the following lemmas.

Lemma 2.1 ([13])**.** *Let E be a real smooth Banach space. Suppose one of the followings holds:*

(1) *j is uniformly continuous on any bounded subset of E.*

(2) $\langle x-y, j(x)-j(y)\rangle \le \|x-y\|^2$, $\forall x, y \in K$.

(3) *For any bounded subset D of E there is a c such that*

$$\langle x-y, j(x)-j(y)\rangle \le c(\|x-y\|), \ \ \forall x, y \in D,$$

where c satisfies $\lim\limits_{t\to 0^+} c(t)/t = 0$.

Then, for any $\varepsilon > 0$ and any bounded subset C there is $\delta > 0$ such that

$$\|tx + (1-t)y\|^2 \le 2t\langle x, j(y)\rangle + 2t\varepsilon + (1-2t)\|y\|^2$$

for any $x, y \in C$ and $t \in [0, \delta)$.

Lemma 2.2 ([17])**.** *Let $\{x_n\}$ and $\{z_n\}$ be bounded sequences in a Banach space E and let $\{\tau_n\}$ be a sequence in $[0,1]$ with $0 < \liminf\limits_{n\to\infty} \tau_n \le \limsup\limits_{n\to\infty} \tau_n < 1$. Suppose $x_{n+1} = \tau_n z_n + (1-\tau_n)x_n$ for all integers $n \ge 0$ and $\limsup\limits_{n\to\infty}(\|z_{n+1} - z_n\| - \|x_{n+1} - x_n\|) \le 0$. Then, $\lim\limits_{n\to\infty} \|z_n - x_n\| = 0$.*

Lemma 2.3 ([19, 20])**.** *Assume that $\{a_n\}$ is a sequence of nonnegative real numbers such that*

$$a_{n+1} \le (1-b_n)a_n + c_n,$$

where b_n is a sequence in $(0,1)$ and $\{c_n\}$ is a sequence such that

(i) $\sum_{n=1}^{\infty} b_n = \infty$;

(ii) $\limsup\limits_{n\to\infty} c_n/b_n \le 0$ *or* $\sum_{n=1}^{\infty} |c_n| < \infty$.

Then $\lim\limits_{n\to\infty} a_n = 0$.

Lemma 2.4 ([21])**.** *Let E be a uniformly smooth Banach space, K be a nonempty closed convex subset of E, $S : K \to K$ be a nonexpansive mapping with $F(S) \ne \emptyset$, and $f : K \to K$ be a contraction with the coefficient $\alpha (0 < \alpha < 1)$. If z_t is defined by*

$$z_t = tf(z_t) + (1-t)Sz_t,$$

then z_t converges strongly to a point $z \in F(S)$, which solves the variational inequality

$$\langle (I-f)z, j(z-p)\rangle \ge 0, \ \ \forall p \in F(S).$$

3. Main results

Theorem 3.1. *Let E be a real uniformly smooth Banach space and K be a nonempty bounded closed convex subset of E. Let $T : K \to K$ be a strictly pseudo-contractive map such that $F(T) \ne \emptyset$, and $f : K \to K$ be a contraction with the coefficient α $(0 < \alpha < 1)$. Consider $\{x_n\}$ as a sequence in K generated in the following manner:*

$$x_{n+1} = \alpha_n f(x_n) + \beta_n x_n + \gamma_n S_n x_n, \tag{3.1}$$

where $S_n x := (1-\delta_n)x + \delta_n Tx$, and assume that $\{z_t\}$ is defined by $z_t = tf(z_t) + (1-t)S_n z_t$. If the real sequences $\{\alpha_n\}$, $\{\beta_n\}$, $\{\gamma_n\}$, $\{\delta_n\}$ are sequences in $(0,1)$ and $\alpha_n + \beta_n + \gamma_n = 1$, which satisfy the following conditions:

(i) $\lim\limits_{n\to\infty} \alpha_n = 0$, $\sum_{n=1}^{\infty} \alpha_n = \infty$;

(ii) $0 < \liminf\limits_{n\to\infty} \beta_n \le \limsup\limits_{n\to\infty} \beta_n < 1$;

(iii) $|\delta_{n+1}-\delta_n| \to 0$ *as* $n \to \infty$,

then the sequence $\{x_n\}$ *converges strongly to a fixed point of* T.

Proof. The proof will be split into four steps.

Step 1. We show S_n is a nonexpansive mapping. Indeed, for all $x, y \in K$, taking $0 < \varepsilon < k\|Tx - Ty - (x-y)\|$, by Lemma 2.1, we have

$$\begin{aligned}
\|S_n x - S_n y\|^2 &= \|(1-\delta_n)x + \delta_n Tx - (1-\delta_n)y - \delta_n Ty\|^2 \\
&= \|\delta_n(Tx - Ty) + (1-\delta_n)(x-y)\|^2 \\
&\le 2\delta_n\langle Tx - Ty, j(x-y)\rangle + 2\varepsilon\delta_n + (1-2\delta_n)\|x-y\|^2 \\
&\le (1-2\delta_n)\|x-y\|^2 + 2\delta_n(\|x-y\|^2 - k\|Tx - Ty - (x-y)\|^2) + 2\varepsilon\delta_n \\
&\le \|x-y\|^2 - 2\delta_n k\|Tx - Ty - (x-y)\|^2 + 2\varepsilon\delta_n \\
&\le \|x-y\|^2.
\end{aligned}$$

It is observed that for each $n \in N$, $S_n x = x$ if and only if $Tx = x$, and so $F(S_n) = F(T)$. By our assumption $F(T) \neq \emptyset$, then, $F(S_n) \neq \emptyset$.

Step 2 . $\|x_{n+1} - x_n\| \to 0$ and $\|x_n - S_n x_n\| \to 0$ as $n \to \infty$.

Since K is a nonempty bounded closed convex subset of E, then $\{x_n\}$, $\{S_n x_n\}$ are bounded. Hence there exists $M = \sup\{\|x_n - Tx_n\|\}$. From Step 1, we know S_n is a nonexpansive mapping, thus by (3.1), we have

$$\begin{aligned}
\|S_n x_n - S_{n-1}x_{n-1}\| &= \|S_n x_n - S_n x_{n-1} + S_n x_{n-1} - S_{n-1}x_{n-1}\| \\
&\le \|x_n - x_{n-1}\| + M\|\delta_n - \delta_{n-1}\|.
\end{aligned} \tag{3.2}$$

Now, we define $z_n := \frac{x_{n+1} - \beta_n x_n}{1-\beta_n}$, then, $z_n = \frac{\alpha_n f(x_n) + \gamma_n S_n x_n}{1-\beta_n}$. By (3.1) and (3.2), we have

$$\begin{aligned}
\|z_{n+1} - z_n\| - \|x_{n+1} - x_n\| &= \|\frac{\alpha_{n+1} f(x_{n+1}) + \gamma_{n+1} S_{n+1} x_{n+1}}{1-\beta_{n+1}} - \frac{\alpha_n f(x_n) + \gamma_n S_n x_n}{1-\beta_n}\| - \|x_{n+1} - x_n\| \\
&= \|\frac{\alpha_{n+1}(f(x_{n+1}) - S_n x_n) + \alpha_{n+1} S_n x_n + \gamma_{n+1} S_{n+1} x_{n+1}}{1-\beta_{n+1}} \\
&\quad - \frac{\alpha_n(f(x_n) - S_n x_n) + \alpha_n S_n x_n + \gamma_n S_n x_n}{1-\beta_n}\| - \|x_{n+1} - x_n\| \\
&= \|\frac{\alpha_{n+1}(f(x_{n+1}) - S_n x_n)}{1-\beta_{n+1}} - \frac{\alpha_n(f(x_n) - S_n x_n)}{1-\beta_n} \\
&\quad + \frac{\alpha_{n+1} S_n x_n + \gamma_{n+1} S_{n+1} x_{n+1}}{1-\beta_{n+1}} - s_n x_n\| - \|x_{n+1} - x_n\| \\
&\le \frac{\alpha_{n+1}}{1-\beta_{n+1}}\|f(x_{n+1}) - S_n x_n\| + \frac{\alpha_n}{1-\beta_n}\|f(x_n) - S_n x_n\| \\
&\quad + \frac{\gamma_{n+1}}{1-\beta_{n+1}}\|S_{n+1}x_{n+1} - S_n x_n\| - \|x_{n+1} - x_n\| \\
&\le \frac{\alpha_{n+1}}{1-\beta_{n+1}}\|f(x_{n+1}) - S_n x_n\| + \frac{\alpha_n}{1-\beta_n}\|f(x_n) - S_n x_n\| \\
&\quad + \|S_{n+1}x_{n+1} - S_n x_n\| - \|x_{n+1} - x_n\| \\
&\le \frac{\alpha_{n+1}}{1-\beta_{n+1}}\|f(x_{n+1}) - S_n x_n\| + \frac{\alpha_n}{1-\beta_n}\|f(x_n) - S_n x_n\| + M\|\delta_{n+1} - \delta_n\|.
\end{aligned}$$

By the assumptions on $\{\alpha_n\}$, $\{\beta_n\}$, $\{\delta_n\}$, we have

$$\limsup_{n\to\infty}(\|z_{n+1} - z_n\| - \|x_{n+1} - x_n\|) \le 0.$$

By using Lemma 2.2, we have

$$\|z_n - x_n\| \to 0 \text{ as } n \to \infty.$$

Applying

$$\lim_{n\to\infty} \|x_{n+1} - x_n\| = \lim_{n\to\infty} (1-\beta_n)\|z_n - x_n\| = 0,$$

together with

$$x_n - S_n x_n = x_n - x_{n+1} + x_{n+1} - S_n x_n = x_n - x_{n+1} + \alpha_n(f(x_n) - S_n x_n) + \beta_n(x_n - S_n x_n),$$

we have

$$\|x_n - S_n x_n\| \le \frac{1}{1-\beta_n}\|x_n - x_{n+1}\| + \frac{\alpha_n}{1-\beta_n}\|f(x_n) - S_n x_n\|.$$

Hence

$$\lim_{n\to\infty} \|S_n x_n - x_n\| = 0.$$

Step 3. Claim: $\limsup_{n\to\infty}\langle f(z) - z, j(x_n - z)\rangle \le 0$.

It is observed that from Lemma 2.4, there exist z_t satisfying $z_t = tf(z_t) + (1-t)S_n z_t$ and z_t converges to a fixed point of $S_n(F(T) = F(S_n))$. Let $z_t \to z \in F(T) = F(S_n)$, using equality

$$z_t - x_n = (1-t)(S_n z_t - x_n) + t(f(z_t) - x_n),$$

and inequality

$$\langle S_n x - S_n y, j(x-y)\rangle \le \|x-y\|^2,$$

we get that

$$\begin{aligned}
\|z_t - x_n\|^2 &= (1-t)\langle S_n z_t - x_n, j(z_t - x_n)\rangle + t(\langle f(z_t) - x_n, j(z_t - x_n)\rangle) \\
&\le (1-t)(\langle S_n z_t - S_n x_n, j(z_t - x_n)\rangle + \langle S_n x_n - x_n, j(z_t - x_n)\rangle) \\
&\quad + t(\langle f(z_t) - z_t, j(z_t - x_n)\rangle) + t\|z_t - x_n\|^2 \\
&\le \|z_t - x_n\|^2 + \|S_n x_n - x_n\|\|j(z_t - x_n)\| + t(\langle f(z_t) - z_t, j(z_t - x_n)\rangle),
\end{aligned}$$

and hence

$$\langle f(z_t) - z_t, j(x_n - z_t)\rangle \le \frac{\|S_n x_n - x_n\|}{t}\|z_t - x_n\|. \tag{3.3}$$

Since $\{z_t\}$, $\{x_n\}$ and $\{S_n x_n\}$ are bounded and $\|x_n - S_n x_n\| \to 0$, taking $n \to \infty$ in Eq. (3.3), we get

$$\limsup_{n\to\infty}\langle f(z_t) - z_t, j(x_n - z_t)\rangle \le 0. \tag{3.4}$$

Since z_t converges strongly to z, as $t \to 0$, and $\{z_t - x_n\}$ is bounded, and in view of the fact that the duality map j is norm-to-weak* uniformly continuous on bounded subsets of E, we get that

$$\begin{aligned}
|\langle f(z) - z, j(x_n - z)\rangle - \langle f(z_t) - z_t, j(x_n - z_t)\rangle| &= |\langle f(z) - z, j(x_n - z) - j(x_n - z_t)\rangle \\
&\quad + \langle (f(z) - z) - (f(z_t) - z_t), j(x_n - z_t)\rangle| \\
&\le |\langle f(z) - z, j(x_n - z) - j(x_n - z_t)\rangle| \\
&\quad + \|(f(z) - z) - (f(z_t) - z_t)\|\|x_n - z_t\| \to 0, \text{ as } t \to 0.
\end{aligned}$$

Hence, for all $\varepsilon > 0$, there exists $\sigma > 0$ such that for all $t \in (0, \sigma)$, and $n \ge 0$, we have that

$$\langle f(z) - z, j(x_n - z)\rangle < \langle f(z_t) - z_t, j(x_n - z_t)\rangle + \varepsilon.$$

By Eq. (3.4), we have that

$$\begin{aligned}\limsup_{n\to\infty}\langle f(z)-z, j(x_n-z)\rangle &\le \limsup_{n\to\infty}\langle f(z_t)-z_t, j(x_n-z_t)\rangle+\varepsilon\\ &\le \varepsilon.\end{aligned}$$

Since ε is arbitrary, we get that

$$\limsup_{n\to\infty}\langle f(z)-z, j(x_n-z)\rangle \le 0.$$

Step 4. Show that $x_n \to z$. As a matter of fact, from (3.1), we have

$$\begin{aligned}\|x_{n+1}-z\|^2 &= \alpha_n\langle f(x_n)-z, j(x_{n+1}-z)\rangle+\beta_n\langle x_n-z, j(x_{n+1}-z)\rangle+\gamma_n\langle S_nx_n-z, j(x_{n+1}-z)\rangle\\ &\le \alpha_n\langle f(x_n)-f(z), j(x_{n+1}-z)\rangle+\alpha_n\langle f(z)-z, j(x_{n+1}-z)\rangle\\ &\quad+\beta_n\|x_n-z\|\|x_{n+1}-z\|+\gamma_n\|x_n-z\|\|x_{n+1}-z\|\\ &\le (\alpha_n\alpha+\beta_n+\gamma_n)\|x_n-z\|\|x_{n+1}-z\|+\alpha_n\langle f(z)-z, j(x_{n+1}-z)\rangle\\ &\le [1-(1-\alpha)\alpha_n][\frac{1}{2}\|x_n-z\|^2+\frac{1}{2}\|x_{n+1}-z\|^2]+\alpha_n\langle f(z)-z, j(x_{n+1}-z)\rangle\\ &\le \frac{1}{2}\|x_{n+1}-z\|^2+\frac{1-(1-\alpha)\alpha_n}{2}\|x_n-z\|^2+\alpha_n\langle f(z)-z, j(x_{n+1}-z)\rangle.\end{aligned}$$

It follows that

$$\|x_{n+1}-z\|^2 \le [1-(1-\alpha)\alpha_n]\|x_n-z\|^2+2\alpha_n\langle f(z)-z, j(x_{n+1}-z)\rangle. \tag{3.5}$$

Using Lemma 2.3 onto (3.5) we conclude that $x_n \to z$. The proof is completed. □

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