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Hyers-Ulam stability of derivations in fuzzy Banach space

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Abstract

In this paper, we construct an additive functional equation, and use the fixed point alternative theorem to investigate the Hyers-Ulam stability of derivations fuzzy Banach space and fuzzy Lie Banach space associated with the following functional equation: $f(2x-y-z)+f(x-z)+f(x+y+2z)=f(4x)$.

Keywords: Fuzzy normed space, additive functional equation, Hyers-Ulam stability, fixed point alternative, fuzzy Banach space.
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1. Introduction and preliminaries

The stability problem of functional equations originated from a question of Ulam [46] concerning the stability of group homomorphisms. Hyers [22] gave a first affirmative partial answer to the question of Ulam for Banach spaces. Hyers' Theorem was generalized by Aoki [2] for additive mappings and by Rassias [40] for linear mappings by considering an unbounded Cauchy difference. Those results have been recently complemented in [8]. A generalization of the Aoki and Rassias theorem was obtained by Găvruta [21], who used a more general function controlling the possibly unbounded Cauchy difference in the spirit of Rassias' approach. The stability problems for several functional equations or inequalities have been extensively

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investigated by a number of authors and there are many interesting results concerning this problem (see [7, 10, 11, 14, 15, 23–28, 30, 31, 34, 36–39, 41–43]).

We recall a fundamental result in fixed point theory.

Let X be a set. A function $d : X \times X \to [0, \infty]$ is called a generalized metric on X, if d satisfies

(1) $d(x, y) = 0$, if and only if $x = y$;

(2) $d(x, y) = d(y, x)$ for all $x, y \in X$;

(3) $d(x, z) \le d(x, y) + d(y, z)$ for all $x, y, z \in X$.

Theorem 1.1 ([13, 18])**.** *Let (X, d) be a complete generalized metric space and let $J : X \to X$ be a strictly contractive mapping with Lipschitz constant $L < 1$. Then for each given element $x \in X$, either*

$$d(J^n x, J^{n+1} x) = \infty$$

for all nonnegative integers n, or there exists a positive integer n_0 such that

(1) *$d(J^n x, J^{n+1} x) < \infty$, for all $n \ge n_0$;*

(2) *the sequence $\{J^n x\}$ converges to a fixed point y^* of J;*

(3) *y^* is the unique fixed point of J in the set $Y = \{y \in X | d(J^{n_0} x, y) < \infty\}$;*

(4) *$d(y, y^*) \le \frac{1}{1-L} d(y, Jy)$, for all $y \in Y$.*

By using the fixed point method, the stability problems of several functional equations have been extensively investigated by a number of authors (see [1, 9, 12, 13, 17, 19, 28, 33, 35, 39]).

In 1984, Katsaras [27] defined a fuzzy norm on a linear space and at the same year Wu and Fang [47] also introduced a notion of fuzzy normed space and gave the generalization of the Kolmogoroff normalized theorem for fuzzy topological linear space. In [6], Biswas defined and studied fuzzy inner product spaces in linear space. Since then some mathematicians have defined fuzzy metrics and norms on a linear space from various points of view [5, 20, 30, 45, 48]. In 1994, Cheng and Mordeson introduced a definition of fuzzy norm on a linear space in such a manner that the corresponding induced fuzzy metric is of Kramosil and Michalek type [29]. In 2003, Bag and Samanta [3] and Saadati and Vaezpour [44] modified the definition of Cheng and Mordeson [16] by removing a regular condition. They also established a decomposition theorem of a fuzzy norm into a family of crisp norms and investigated some properties of fuzzy norms (see [4]). Following [3], we give the employing notion of a fuzzy norm.

Let X be a real linear space. A function $N : X \times \mathbb{R} \to [0, 1]$ (the so-called fuzzy subset) is said to be a fuzzy norm on X, if for all $x, y \in X$ and all $a, b \in \mathbb{R}$:

(N_1) $N(x, a) = 0$ for $a \le 0$;

(N_2) $x = 0$, if and only if $N(x, a) = 1$ for all $a > 0$;

(N_3) $N(ax, b) = N(x, \frac{b}{|a|})$, if $a \ne 0$;

(N_4) $N(x + y, a + b) \ge \min\{N(x, a), N(y, b)\}$;

(N_5) $N(x, .)$ is a non-decreasing function on $\mathbb{R}$ and $\lim_{a \to \infty} N(x, a) = 1$;

(N_6) For $x \ne 0$, $N(x, .)$ is (upper semi) continuous on $\mathbb{R}$.

The pair (X, N) is called a fuzzy normed linear space. One may regard $N(x, a)$ as the truth value of the statement the norm of x is less than or equal to the real number a'.

Example 1.2. Let $(X, \|.\|)$ be a fuzzy normed space. Define

$$N(x,a) = \begin{cases} \frac{a}{a+\|x\|}, & a > 0, \quad x \in X, \\ 0, & a \leq 0, \quad x \in X. \end{cases}$$

Then (X, N) is called the induced fuzzy normed space.

Definition 1.3. Let (X, N) be a fuzzy normed linear space. Let x_n be a sequence in X. Then x_n is said to be convergent, if there exists $x \in X$ such that $\lim_{n\to\infty} N(x_n - x, a) = 1$ for all $a > 0$. In that case, x is called the limit of the sequence x_n and we denote it by N-$\lim_{n\to\infty}\ x_n = x$.

Definition 1.4. A sequence x_n in X is called Cauchy, if for each $\epsilon > 0$ and each $a > 0$ there exists n_0 such that for all $n \geq n_0$ and all $p > 0$, we have $N(x_{n+p} - x_n, a) > 1 - \epsilon$.

It is known that every convergent sequence in fuzzy normed space is Cauchy. If each Cauchy sequence is convergent, then the fuzzy norm is said to be complete and the fuzzy normed space is called a fuzzy Banach space.

Definition 1.5. Let (X, N) and (Y, N) be fuzzy normed algebras.

(1) An $\mathbb{R}$-linear mapping $f : X \to Y$ is called a homomorphism, if

$$f(xy) = f(x)f(y)$$

for all $x, y \in X$.

(2) An $\mathbb{R}$-linear mapping $f : X \to X$ is called a derivation, if

$$f(xy) = f(x)y + xf(y)$$

for all $x, y \in X$.

2. The stability of derivations on fuzzy C^*-algebras

Throughout this section, assume that A is a fuzzy C^*-algebra with the fuzzy norm. For any mapping $f : A \to A$, we define

$$Df(x,y,z) := f(2x-y-z) + f(x-z) + f(x+y+2z) - f(4x)$$

for all $x, y, z \in A$.

Firstly, we prove that $Df(x, y, z) = 0$ implies the additivity of f.

Lemma 2.1. *Let (Z, N) be a fuzzy normed vector space and $f : X \to Z$ be a mapping such that*

$$N\left(f\left(2x-y-z\right) + f\left(x-z\right) + f\left(x+y+2z\right), t\right) \geq N\left(f\left(4x\right), \frac{t}{2}\right) \tag{2.1}$$

for all $x, y, z \in X$ and all $t > 0$. Then f is additive.

Proof. By letting $x = y = z = 0$ in (2.1), we get

$$N(3f(0), t) = N\left(f(0), \frac{t}{3}\right) \geq N\left(f(0), \frac{t}{2}\right)$$

for all $t > 0$. By (N_5) and (N_6), $N(f(0), t) = 1$ for all $t > 0$. It follows from (N_2) that $f(0) = 0$.

By letting $x = z = 0$ in (2.1), we get

$$N(f(y) + f(-y) + f(0), t) \geq N\left(f(0), \frac{t}{2}\right) = 1$$

for all $t > 0$. It follows from (N_2) that $f(-y) + f(y) = 0$ for all $y \in X$. Thus

$$f(-y) = -f(y)$$

for all $y \in X$.

By letting $x = 0$ and $l = y + z$ in (2.1), we get

$$N(f(-l) + f(-z) + f(l + z), t) \geq N\left(f(0), \frac{t}{2}\right) = 1$$

for all $t > 0$. It follows from (N_2) that

$$f(-l) + f(-z) + f(l + z) = 0$$

for all $l, z \in X$. Thus

$$f(l + z) = f(l) + f(z)$$

for all $l, z \in X$, as desired. □

Now, we investigate the Hyers-Ulam stability of derivations on fuzzy Banach space for the functional equation

$$Df(x, y, z) = 0$$

for all $x, y, z \in A$.

Theorem 2.2. *Let $\phi : A^3 \to [0, 1]$ be a function such that there exists an $L < 1$ with*

$$\phi\left(\frac{x}{2}, \frac{y}{2}, \frac{z}{2}\right) \leq \frac{L}{2}\phi(x, y, z) \tag{2.2}$$

for all $x, y, z \in A$. Let $f : A \to A$ be a mapping such that

$$N\left(Df(x, y, z), t\right) \geq \frac{t}{t + \phi(x, y, z)}, \tag{2.3}$$

$$N(f(xy) - f(x)y - xf(y), t) \geq \frac{t}{t + \phi(x, y, 0)} \tag{2.4}$$

for all $x, y, z \in A$, all $t > 0$. Then there exists a unique fuzzy derivation $\delta : A \to A$ such that

$$N(f(x) - \delta(x), t) \geq \frac{2(1 - L)t}{2(1 - L)t + L\phi\left(x, 0, x\right)} \tag{2.5}$$

for all $x \in A$ and all $t > 0$.

Proof. By letting $\mu = 1, y = -x$, $z = x$ in (2.4), we have

$$N\left(2f\left(x\right) - f(2x), t\right) \geq \frac{t}{t + \phi\left(\frac{x}{2}, \frac{-x}{2}, 0\right)}, \tag{2.6}$$

and so

$$N\left(2f\left(\frac{x}{2}\right) - f(x), t\right) \geq \frac{t}{t + \phi\left(\frac{x}{4}, \frac{-x}{4}, 0\right)} \geq \frac{t}{t + \frac{L}{4}\phi\left(x, -x, 0\right)}$$

for all $x \in A$. Thus

$$N\left(2f\left(\frac{x}{2}\right) - f(x), \frac{L}{4}t\right) \geq \frac{\frac{L}{4}t}{\frac{L}{4}t + \frac{L}{4}\phi\left(x, -x, 0\right)} = \frac{t}{t + \phi\left(x, -x, 0\right)} \tag{2.7}$$

for all $x \in A$.

Consider the set

$$X := \{g : A \to A\},$$

and introduce the generalized metric on X:

$$d(g, h) := \inf\{a \in \mathbb{R}^+ : N(g(x) - h(x), at) \geq \frac{t}{t + \phi\left(\frac{x}{2}, \frac{-x}{2}, 0\right)}\}$$

for all $x \in A$ and all $t > 0$, where $a \in (0, \infty)$. It is easy to show that (X, d) is complete (see the proof of [32, Lemma 2.1]).

Now, we consider the linear mapping $Q : X \to X$ such that

$$Qg(x) := 2g\left(\frac{x}{2}\right)$$

for all $x \in A$.

Let $g, h \in X$ be given such that $d(g, h) = \epsilon$. Then

$$N(g(x) - h(x), \epsilon t) \geq \frac{t}{t + \phi\left(x, -x, 0\right)}$$

for all $x \in A$ and all $t > 0$. Hence

$$\begin{aligned} N(Qg(x) - Qh(x), L\epsilon t) &= N\left(2g\left(\frac{x}{2}\right) - 2h\left(\frac{x}{2}\right), L\epsilon t\right) = N\left(g\left(\frac{x}{2}\right) - h\left(\frac{x}{2}\right), \frac{L}{2}\epsilon t\right) \\ &\geq \frac{\frac{Lt}{2}}{\frac{Lt}{2} + \phi\left(\frac{x}{2}, \frac{-x}{2}, 0\right)} \geq \frac{\frac{Lt}{2}}{\frac{Lt}{2} + \frac{L}{2}\phi\left(x, -x, 0\right)} \\ &= \frac{t}{t + \phi\left(x, -x, 0\right)} \end{aligned}$$

for all $x \in A$ and all $t > 0$. Thus $d(g, h) = \epsilon$ implies that $d(Qg, Qh) \leq L\epsilon$. This means that

$$d(Qg, Qh) \leq Ld(g, h)$$

for all $g, h \in X$.

It follows from (2.7) that $d(f, Qf) \leq \frac{L}{4}$.

By Theorem 1.1, there exists a mapping $\delta : A \to A$ satisfying the following:

(1) δ is a fixed point of Q, i.e.,

$$\delta\left(\frac{x}{2}\right) = \frac{1}{2}\delta(x), \tag{2.8}$$

for all $x \in A$. The mapping δ is a unique fixed point of Q in the set

$$M = \{g \in G : d(f, g) < \infty\}.$$

This implies that δ is a unique mapping satisfying (2.8) such that there exists an $a \in (0, \infty)$ satisfying

$$N(f(x) - \delta(x), at) \geq \frac{t}{t + \phi\left(x, -x, 0\right)}$$

for all $x \in A$ and $t > 0$.

(2) $d(Q^k f, \delta) \to 0$ as $k \to \infty$. This implies the equality

$$N - \lim_{k\to\infty} 2^k f\left(\frac{x}{2^k}\right) = \delta(x)$$

for all $x \in A$.

(3) $d(f, \delta) \leq \frac{1}{1-L} d(f, Qf)$, which implies the inequality

$$d(f, A) \leq \frac{L}{4(1-L)}.$$

This implies that the inequality (2.5) holds.

Next we show that δ is additive. It follows from (2.2) that

$$\begin{aligned}\sum_{k=0}^{\infty} 2^k \phi\left(\frac{x}{2^k}, \frac{y}{2^k}, \frac{z}{2^k}\right) &= \phi(x,y,z) + 2\phi\left(\frac{x}{2}, \frac{y}{2}, \frac{z}{2}\right) + 2^2\phi\left(\frac{x}{2^2}, \frac{y}{2^2}, \frac{z}{2^2}\right) + \cdots \\ &\leq \phi(x,y,z) + L\phi(x,y,z) + L^2\phi(x,y,z) + \cdots \\ &= \frac{1}{1-L}\phi(x,y,z) < \infty\end{aligned}$$

for all $x, y, z \in A$.

By (2.3),

$$N\left(2^k f\left(\frac{2x-y-z}{2^k}\right) + 2^k f\left(\frac{x-z}{2^k}\right) + f\left(\frac{x+y+2z}{2^k}\right) - 2^k f\left(\frac{4}{2^k}x\right), 2^k t\right) \geq \frac{t}{t + \phi\left(\frac{x}{2^k}, \frac{y}{2^k}, \frac{z}{2^k}\right)},$$

and so

$$\begin{aligned}&N\left(2^k f\left(\frac{2x-y-z}{2^k}\right) + 2^k f\left(\frac{x-z}{2^k}\right) + 2^k f\left(\frac{x+y-2z}{2^k}\right) - 2^k f\left(\frac{4}{2^k}x\right), t\right) \\ &\geq \frac{\frac{t}{2^k}}{\frac{t}{2^k} + \phi\left(\frac{x}{2^k}, \frac{y}{2^k}, \frac{z}{2^k}\right)} = \frac{t}{t + 2^k\phi\left(\frac{x}{2^k} \cdot \frac{y}{2^k}, \frac{z}{2^k}\right)}\end{aligned}$$

for all $x, y, z \in A$, all $t > 0$ and all $\mu \in \mathbb{T}^1$. Since $\lim_{k\to\infty} \frac{t}{t+2^k\phi\left(\frac{x}{2^k}, \frac{y}{2^k}, \frac{z}{2^k}\right)} = 1$, for all $x, y, z \in A$ and all $t > 0$,

$$N\left(\delta\left(2x-y-z\right) + \delta\left(x-z\right) + \delta\left(x+y+2z\right) - \delta\left(4x\right), t\right) = 1$$

for all $x, y, z \in X$, all $t > 0$ and all $\mu \in \mathbb{T}^1$. So

$$\delta\left(2x-y-z\right) + \delta\left(x-z\right) + \delta\left(x+y+2z\right) = \delta\left(4x\right)$$

for all $x, y, z \in A$, all $t > 0$. By Lemma 2.1, we get $\delta(x)$ is an additive mapping.

It follows from (2.4) that

$$\begin{aligned}N\left(f\left(\frac{xy}{2^{2k}}\right) - \frac{y}{2^k} f\left(\frac{x}{2^k}\right) - \frac{x}{2^k} f\left(\frac{y}{2^k}\right), t\right) &= N\left(2^{2k} f\left(\frac{xy}{2^{2k}}\right) - y2^k f\left(\frac{x}{2^k}\right) - x2^k f\left(\frac{y}{2^k}\right), \frac{t}{2^{2k}}\right) \\ &\geq \frac{\frac{t}{2^{2k}}}{\frac{t}{2^{2k}} + \phi\left(\frac{x}{2^k}, \frac{y}{2^k}, 0\right)} \\ &= \frac{t}{t + 2^{2k}\phi\left(\frac{x}{2^k}, \frac{y}{2k}, 0\right)} \\ &\geq \frac{t}{t + (2L)^k \phi\left(x, y, 0\right)}\end{aligned}$$

for all $x, y \in A$ and all $t > 0$. Since $\lim_{k\to\infty} \frac{t}{t+(2^{n-1}L)^k\phi(x,y,0)} = 1$, for all $x, y \in A$ and all $t > 0$, we get

$$\delta(xy) = y\delta(x) + x\delta(y)$$

for all $x, y \in A$.

□

Corollary 2.3. *Let p be a real number with $p > 1$, $\theta \geq 0$, and X be a normed vector space with norm $\|\cdot\|$. Let $f : X \to X$ be a mapping satisfying*

$$N\left(Df(x,y,z),t\right) \geq \frac{t}{t+\theta(\|x\|^p+\|y\|^p+\|z\|^p)},$$

$$N(f(xy)-f(x)y-xf(y),t) \geq \frac{t}{t+\theta(\|x\|^p+\|y\|^p)}$$

for all $x, y, z \in A$, all $t > 0$. Then there exists a unique derivation $\delta : A \to A$ satisfying

$$N(f(x)-\delta(x),t) \geq \frac{(2^p-2)t}{(2^p-2)t+\theta\|x\|^p}$$

for all $x \in X$ and all $t > 0$.

Proof. The proof follows from Theorem 2.2 by taking

$$\phi(x,y,z) = \theta(\|x\|^p+\|y\|^p+\|z\|^p),$$

and $L = 3^{1-p}$. □

3. Stability of derivations on fuzzy Lie Banach space

A fuzzy Banach space, endowed with the Lie product

$$[x,y] := \frac{xy-yx}{2},$$

on $\mathbb{R}$, is called a fuzzy Lie Banach space.

Definition 3.1. Let A be a fuzzy Lie Banach space. An $\mathbb{R}$-linear mapping $\delta : A \to A$ is called a fuzzy Lie derivation, if

$$\delta([x,y]) = [\delta(x),y] + [x,\delta(y)]$$

for all $x, y \in A$.

In this section, suppose that A is a fuzzy Lie Banach space with norm N. We prove the Hyers-Ulam stability of fuzzy Lie derivations on fuzzy Lie Banach space for the functional equation

$$Df(x,y,z) = 0.$$

Theorem 3.2. *Let $\phi : A^3 \to [0,\infty)$ be a function such that there exists an $L < 1$ with*

$$3L\phi\left(\frac{x}{2},\frac{y}{2},\frac{z}{2}\right) \leq \phi(x,y,z)$$

for all $x, y, z \in A$. Let $f : A \to A$ be a mapping satisfying

$$N(Df(x,y,z),t) \geq \frac{t}{t+\phi(x,y,z)},$$

and

$$N(f([x,y]) - [f(x),y] - [x,f(y)],t) \geq \frac{t}{t+\phi(x,y,0)}.$$

Then there exists a unique fuzzy Lie derivation $\delta : A \to A$ satisfying

$$N(f(x)-\delta(x),t) \geq \frac{2(1-L)t}{2(1-L)t+\phi(x,0,x)} \tag{3.1}$$

for all $x \in A$ and all $t > 0$.

Proof. Let (X,d) be the generalized metric space defined in the proof of Theorem 2.2. Consider the linear mapping $Q : X \to X$ such that

$$Qg(x) := \frac{1}{2}g(2x)$$

for all $x \in A$.

It follow from (2.6) that

$$N\left(f(x) - \frac{1}{2}f(2x), \frac{1}{2}t\right) \geq \frac{t}{t+\phi(x,0,x)}$$

for all $x \in A$ and all $t > 0$. Thus $d(f,Qf) \leq \frac{1}{2}$. Hence

$$d(f,A) \leq \frac{1}{2(1-L)},$$

which implies that the inequality (3.1) holds.

The rest of the proof is similar to the proof of Theorem 2.2. □

Corollary 3.3. *Let $\theta \geq 0$ and let p be a positive real number with $p < 1$. Let A be a fuzzy Lie Banach space with norm $\|\cdot\|$. Let $f : A \to A$ be a mapping satisfying*

$$N(Df(x,y,z),t) \geq \frac{t}{t+\theta(\|x\|^p+\|y\|^p+\|z\|^p)},$$

and

$$N(f([x,y]) - [f(x),y] - [x,f(y)],t) \geq \frac{t}{t+\theta(\|x\|^p+\|y\|^p)}$$

for all $x,y,z \in A$ and $t > 0$. Then there exists a unique fuzzy Lie derivation $\delta : A \to A$ such that

$$N(f(x)-\delta(x),t) \geq \frac{(2-2^p)t}{(2-2^p)t+\theta\|x\|^p}$$

for every $x \in A$ and all $t > 0$.

Proof. The proof follows from Theorem 3.2 by taking

$$\phi(x,y,z) = \theta(\|x\|^p+\|y\|^p+\|z\|^p),$$

and $L = 3^{p-1}$. □

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References

[1] S. Alizadeh, F. Moradlou, *Approximate a quadratic mapping in multi-Banach spaces, a fixed point approach*, Int. J. Nonlinear Anal. Appl., **7** (2016), 63–75. 1

[2] T. Aoki, *On the stability of the linear transformation in Banach spaces*, J. Math. Soc. Japan, **2** (1950), 64–66. 1

[3] T. Bag, S. K. Samanta, *Finite dimensional fuzzy normed linear spaces*, J. Fuzzy Math., **11** (2003), 687–705. 1

[4] T. Bag, S. K. Samanta, *Fuzzy bounded linear operators*, Fuzzy Sets and Systems, **151** (2005), 513–547. 1

[5] V. Balopoulos, A. G. Hatzimichailidis, B. K. Papadopoulos, *Distance and similarity measures for fuzzy operators*, Inform. Sci., **177** (2007), 2336–2348. 1

[6] R. Biswas, *Fuzzy inner product spaces and fuzzy norm functions*, Inform. Sci., **53** (1991), 185–190. 1

[7] N. Brillouët-Belluot, J. Brzdęk, K. Ciepliński, *On some recent developments in Ulam's type stability*, Abstr. Appl. Anal., **2012** (2012), 41 pages. 1

[8] J. Brzdęk, *Hyperstability of the Cauchy equation on restricted domains*, Acta Math. Hungar., **141** (2013), 58–67. 1

[9] J. Brzdęk, J. Chudziak, Z. Páles, *A fixed point approach to stability of functional equations*, Nonlinear Anal., **74** (2011), 6728–6732. 1

[10] J. Brzdęk, K. Ciepliński, *Hyperstability and superstability*, Abstr. Appl. Anal., **2013** (2013), 13 pages. 1

[11] J. Brzdęk, A. Fošner, *Remarks on the stability of Lie homomorphisms*, J. Math. Anal. Appl., **400** (2013), 585–596. 1

[12] L. Cădariu, L. Găvruţa, P. Găvruţa, *Fixed points and generalized Hyers-Ulam stability*, Abstr. Appl. Anal., **2012** (2012), 10 pages. 1

[13] L. Cădariu, V. Radu, *Fixed points and the stability of Jensen's functional equation*, J. Inequal. Pure Appl. Math., **4** (2003), 7 pages. 1.1, 1

[14] L. S. Chadli, S. Melliani, A. Moujahid, M. Elomari, *Generalized solution of Sine-Gordon equation*, Int. J. Nonlinear Anal. Appl., **7** (2016), 87–92. 1

[15] I. S. Chang, M. Eshaghi Gordji, H. Khodaei, H. M. Kim, *Nearly quartic mappings in β-homogeneous F-spaces*, Results Math., **63** (2013), 529–541. 1

[16] S. C. Cheng, J. N. Mordeson, *Fuzzy linear operators and fuzzy normed linear spaces*, Bull. Calcutta Math. Soc., **86** (1994), 429–436. 1

[17] K. Ciepliński, *Applications of fixed point theorems to the Hyers-Ulam stability of functional equations–a survey*, Ann. Funct. Anal., **3** (2012), 151–164. 1

[18] J. B. Diaz, B. Margolis, *A fixed point theorem of the alternative, for contractions on a generalized complete metric space*, Bull. Amer. Math. Soc., **74** (1968), 305–309. 1.1

[19] M. Eshaghi Gordji, H. Khodaei, T. M. Rassias, R. Khodabakhsh, *J^*-homomorphisms and J^*-derivations on J^*-algebras for a generalized Jensen type functional equation*, Fixed Point Theory, **13** (2012), 481–494. 1

[20] C. Felbin, *Finite-dimensional fuzzy normed linear space*, Fuzzy Sets and Systems, **48** (1992), 239–248. 1

[21] P. Găvruţa, *A generalization of the Hyers-Ulam-Rassias stability of approximately additive mappings*, J. Math. Anal. Appl., **184** (1994), 431–436. 1

[22] D. H. Hyers, *On the stability of the linear functional equation*, Proc. Nat. Acad. Sci. U. S. A., **27** (1941), 222–224. 1

[23] D. H. Hyers, G. Isac, T. M. Rassias, *Stability of functional equations in several variables*, Progress in Nonlinear Differential Equations and their Applications, Birkhäuser Boston, Inc., Boston, MA, (1998). 1

[24] G. Isac, T. M. Rassias, *On the Hyers-Ulam stability of ψ-additive mappings*, J. Approx. Theory, **72** (1993), 131–137.

[25] W. Jabłoński, *Sum of graphs of continuous functions and boundedness of additive operators*, J. Math. Anal. Appl., **312** (2005), 527–534.

[26] S.-M. Jung, *Hyers-Ulam-Rassias stability of functional equations in nonlinear analysis*, Springer Optimization and Its Applications, Springer, New York, (2011).

[27] A. K. Katsaras, *Fuzzy topological vector spaces*, II, Fuzzy Sets and Systems, **12** (1984), 143–154. 1

[28] H. Khodaei, R. Khodabakhsh, M. Eshaghi Gordji, *Fixed points, Lie $*$-homomorphisms and Lie $*$-derivations on Lie C^*-algebras*, Fixed Point Theory, **14** (2013), 387–400. 1, 1

[29] I. Kramosil, J. Michálek, *Fuzzy metrics and statistical metric spaces*, Kybernetika (Prague), **11** (1975), 336–344. 1

[30] S. V. Krishna, K. K. M. Sarma, *Separation of fuzzy normed linear spaces*, Fuzzy Sets and Systems, **63** (1994), 207–217. 1, 1

[31] G. Lu, C. K. Park, *Hyers-Ulam stability of additive set-valued functional equations*, Appl. Math. Lett., **24** (2011), 1312–1316. 1

[32] D. Mihet, V. Radu, *On the stability of the additive Cauchy functional equation in random normed spaces*, J. Math. Anal. Appl., **343** (2008), 567–572. 2

[33] F. Moradlou, M. Eshaghi Gordji, *Approximate Jordan derivations on Hilbert C^*-modules*, Fixed Point Theory, **14** (2013), 413–425. 1

[34] C.-G. Park, *Homomorphisms between Poisson JC^*-algebras*, Bull. Braz. Math. Soc. (N.S.), **36** (2005), 79–97. 1

[35] C. K. Park, *Fixed points and Hyers-Ulam-Rassias stability of Cauchy-Jensen functional equations in Banach algebras*, Fixed Point Theory Appl., **2007** (2007), 15 pages. 1

[36] C. K. Park, K. Ghasemi, S. Ghaffary Ghaleh, *Fuzzy n-Jordan $*$-derivations on induced fuzzy C^*-algebras*, J. Comput. Anal. Appl., **16** (2014), 494–502. 1

[37] C. K. Park, S. O. Kim, J. R. Lee, D. Y. Shin, *Quadratic ρ-functional inequalities in β-homogeneous normed spaces*, Int. J. Nonlinear Anal. Appl., **6** (2015), 21–26.

[38] C. Park, A. Najati, *Generalized additive functional inequalities in Banach algebras*, Int. J. Nonlinear Anal. Appl., **1** (2010), 54–62.

[39] C. Park, J. M. Rassias, *Stability of the Jensen-type functional equation in C^*-algebras: a fixed point approach*, Abstr. Appl. Anal., **2009** (2009), 17 pages. 1, 1

[40] T. M. Rassias, *On the stability of the linear mapping in Banach spaces*, Proc. Amer. Math. Soc., **72** (1978), 297–300. 1

[41] T. M. Rassias, *Functional equations and inequalities*, Mathematics and its Applications, Kluwer Academic Publishers, Dordrecht, (2000). 1

[42] T. M. Rassias, *On the stability of functional equations and a problem of Ulam*, Acta Math. Appl., **62** (2000), 23–130.

[43] T. M. Rassias, *On the stability of functional equations in Banach spaces*, J. Math. Anal. Appl., **251** (2000), 264–284. 1

[44] R. Saadati, S. M. Vaezpour, *Some results on fuzzy Banach spaces*, J. Appl. Math. Comput., **17** (2005), 475–484. 1

[45] B.-S. Shieh, *Infinite fuzzy relation equations with continuous t-norms*, Inform. Sci., **178** (2008), 1961–1967. 1

[46] S. M. Ulam, *Problems in modern mathematics*, Wiley, New York, Chapter VI, (1960). 1

[47] C. X. Wu, J. X. Fang, *Fuzzy generalization of Klomogoroffs theorem*, J. Harbin Inst. Technol., **1** (1984), 1–7. 1

[48] J.-Z. Xiao, X.-H. Zhu, *Fuzzy normed space of operators and its completeness*, Fuzzy Sets and Systems, **133** (2003), 389–399. 1