Available online at www.isr-publications.com/jnsa
J. Nonlinear Sci. Appl., 10 (2017), 84–91
Research Article

ISSN: 2008-1898

Journal of Nonlinear Sciences and Applications

Journal Homepage: www.tjnsa.com - www.isr-publications.com/jnsa

Well-posedness for a class of strong vector equilibrium problems

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Communicated by S. S. Chang

Abstract

In this paper, we first construct a complete metric space Λ consisting of a class of strong vector equilibrium problems (for short, (SVEP)) satisfying some conditions. Under the abstract framework, we introduce a notion of well-posedness for the (SVEP), which unifies its Hadamard and Tikhonov well-posedness. Furthermore, we prove that there exists a dense G_δ set Q of Λ such that each (SVEP) in Q is well-posed, that is, the majority (in Baire category sense) of (SVEP) in Λ is well-posed. Finally, metric characterizations on the well-posedness for the (SVEP) are given.

Keywords: Strong vector equilibrium problems, well-posedness, dense set, metric characterizations.
2010 MSC: 49K40, 90C31.

1. Introduction

The concept of Hadamard well-posedness is inspired by the classical idea of Hadamard, which goes back to the beginning of the last century. It requires existence and uniqueness of the optimal solution together with continuous dependence on the problem data. For this reason, Hadamard well-posedness is often called also stability. In 1966, Tikhonov introduced another concept of well-posedness imposing convergence of each minimizing sequence to the uniqueness minimum solution. Its motivation from the approximate solution of scalar optimization problems is clear. Just after the Tikhonov's paper [26] dealing with unconstrained scalar optimization problems, Levitin and Polyak [17] extended the notion to constrained scalar optimization problems. Various extensions of Hadamard and Tikhonov well-posedness of scalar optimization problems have been developed and well studied (see [8]).

It is worth noting that in the last decades there has been an increasing interest in the study of vector optimization problems. With the development of theory of vector optimization problems, well-posedness of scalar optimization problems is gradually extended to vector optimization problems (see [7, 12–14, 19–23]). Due to the optimal solution and the optimal value of vector optimization problems usually are not unique, so the well-posedness of vector optimization problems exhibits diversity and complexity in

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doi:10.22436/jnsa.010.01.08

Received 2016-09-14

comparison with scalar optimization problems. However, we know that the uniqueness of the solution of optimization problems plays an irreplaceable role to the approximate solution of optimization problems. In many cases, the solution of optimization problems is not unique, so the study of the uniqueness of solution of optimization problems has become very difficult. For this reason, some scholars turned to seek generic uniqueness of solutions of optimization problems and get a series of research results. Kenderov [15] investigated generic uniqueness of solutions scalar optimization problems and get an important result that most of scalar optimization problems has a unique solution. Beer [3] generalized the result of [15] to a kind of constrained optimization problems with Ĉech complete metric space. Kenderov and Ribarska [16] proved that the most of two-person continues zero sum games have a unique equilibrium point and the majority of minimax problems with continuous function have a unique solution. Tan et al. [25] investigated generic uniqueness of saddle point problems with discontinuous binary payoff function. Zaslavski [31, 32] studied generic uniqueness of saddle point, scalar optimization problems and equilibrium problems in metric space. Yu et al. [28] proved that a class of equilibrium problems has a unique equilibrium point.

On the other hand, vector equilibrium problems (for short, (VEP)) has also attracted much attention in the recent years especially due to its applications within the fields of vector optimization problems and vector variational inequalities (see [5, 11]). Various types of well-posedness for (VEP) have been intensively studied in the literature, such as [4, 6, 18, 33]. By using a scalarization method, Li et al. [18] introduced two types of Levitin-Polyak well-posedness for (VEP) with variable domination structures and gave sufficient conditions and metric characterizations of Levitin-Polyak well-posedness for (VEP). Using the bounded rationality model (see, [2, 29, 30]), Deng and Xiang [6] introduced and studied well-posedness in connection with generalized vector equilibrium problems, which unifies its Hadamard and Levitin-Polyak well-posedness. Zhang et al. [33] extended the result of [6] to symmetric vector qusi-equilibrium problems.

The main purpose of this paper is to present some generic well-posedness results for a class of (SVEP). We first introduce and study the well-posedness for a class of (SVEP) which unifies Hadamard and Tikhonov well-posedness by using the bounded rationality model and the scalarization method. Furthermore, we prove that the majority (in Baire category sense) of the kind of (SVEP) is well-posed by using the result of generic uniqueness of nonlinear problems. Finally, metric characterizations on well-posedness for the kind of (SVEP) are shown.

2. Preliminaries

Let X be a metric space, and Y be a Hausdorff topological vector space. Assume that C denotes a nonempty, closed, convex, and pointed cone in Y with apex at the origin and $\text{int}(C) \neq \emptyset$, where $\text{int}(C)$ denotes the topological interior of C.

Let $f : X \times X \to Y$ be a vector-valued mapping and consider the following strong vector equilibrium problem, which is called $P(X, f)$: there exists an element $x \in X$ such that

$$f(x, y) \in -C, \quad \forall y \in X.$$

First, we introduce the notion of the approximating solution sequence for $P(X, f)$.

Definition 2.1. A sequence $\{x_n\} \subset X$ is called the approximating solution sequence for $P(X, f)$, if there exists $\{\epsilon_n\} \subset \mathbb{R}_+$ with $\epsilon_n \to 0$ such that

$$f(x_n, y) - \epsilon_n e \in -C, \quad \forall y \in X.$$

Next, let us recall some useful definitions and lemmas.

Definition 2.2. Let $f : X \times X \to Y$ be a vector-valued mapping. f is said to be strictly pseudomonotone on $X \times X$ iff for all $x, y \in X$ with $x \neq y$,

$$f(x, y) + f(y, x) \notin -\text{int}(C).$$

Definition 2.3 ([1]). Let $F : X \rightrightarrows Y$ be a set-valued mapping.

1. F is said to be upper semicontinuous at $x \in X$ if for any open set $U \supset F(x)$, there is an open neighborhood $O(x)$ of x such that $U \supset F(x')$ for each $x' \in O(x)$;

2. F is said to be lower semicontinuous at x if for any open set $U \cap F(x) \neq \emptyset$, there is an open neighborhood $O(x)$ of x such that $U \cap F(x') \neq \emptyset$ for each $x' \in O(x)$;

3. F is said to be a usco mapping if F is upper semicontinuous and $F(x)$ is nonempty compact for each $x \in X$;

4. F is said to be closed if $\mathrm{Graph}(F)$ is closed, where $\mathrm{Graph}(F) = \{(x, y) \in X \times Y : x \in X, y \in F(x)\}$ is the graph of F.

Lemma 2.4 ([5, 10, 20]). *For any fixed $e \in \mathrm{int}(C)$, the nonlinear scalarization function is defined by*

$$\xi_e(z) := \inf\{r \in \mathbb{R} : z \in re - C\}, \quad \forall z \in Y.$$

the nonlinear scalarization function ξ_e has the following properties

(1) $\xi_e(z) < r \quad \Leftrightarrow \quad z \in re - \mathrm{int}(C)$,
(2) $\xi_e(z) \leqslant r \quad \Leftrightarrow \quad z \in re - C$,
(3) $\xi_e(z) \geqslant r \quad \Leftrightarrow \quad z \notin re - \mathrm{int}(C)$,
(4) $\xi_e(z) > r \quad \Leftrightarrow \quad z \notin re - C$.

The following Lemma 2.5 is due to Theorem 2 of Fort [9], also see Lemma 2.1 of [27].

Lemma 2.5. *Let Y be a complete metric space, X be a metric space and $F : Y \to 2^X$ be a usco mapping. Then there exists a dense G_δ set Q of Y such that F is continuous at every $y \in Q$.*

3. A unified approach to notions of well-posedness

Let X be a compact metric space supplied with a distance d, $(Y, \|\cdot\|)$ be a Banach space and C be a nonempty, closed, convex, and pointed cone in Y with apex at the origin and $\mathrm{int}(C) \neq \emptyset$. Let Λ be the collection of $P(X, f)$ such that $\Lambda = \{f : X \times X \to Y | f$ is continuous and strictly pseudomonotone on $X \times X$, for all $x \in X, f(x, x) = 0$ and there exists $x \in X$ such that $f(x, y) \in -C, \ \forall y \in X\}$.

For any $f_1, f_2 \in \Lambda$, we define

$$\rho(f_1, f_2) := \sup_{(x,y) \in X \times X} \|f_1(x, y) - f_2(x, y)\|.$$

It is easy to check that (Λ, ρ) is a complete metric space.

Given the bounded rationality model $M = \{\Lambda, X, F, \Phi\}$ for $P(X, f)$: Λ and X are two complete metric spaces; the feasible set, the solution set and the rationality function of $P(X, f)$ are defined as

$$\begin{aligned} F(f) &:= X, \\ E(f) &:= \{x \in X : f(x, y) \in -C, \forall y \in X\}, \\ \Phi(f, x) &:= \sup_{y \in X} \xi_e \circ f(x, y). \end{aligned}$$

Lemma 3.1.

1. $\forall f \in \Lambda, E(f) \neq \emptyset$ *and* $\forall x \in X, \Phi(f, x) \geqslant 0$.
2. $\forall f \in \Lambda$, $\Phi(f, x) \leqslant \epsilon$ *if and only if* $f(x, y) - \epsilon e \in -C, \forall y \in X$.
3. $x \in E(f)$ *if and only if* $\Phi(f, x) = 0$.

Proof.

1. By the definition of Λ, for all $f \in \Lambda, E(f) \neq \emptyset$. If $x \in F(f)$, then we have
$$\Phi(f, x) \geqslant \xi_e \circ f(x, x) = 0.$$
2. If $f(x, y) - \epsilon e \in -C$, for all $y \in X$, by Lemma 2.4 (2), then we have $\xi_e \circ f(x, y) \leqslant \epsilon$, for all $y \in X$. So, $\Phi(f, x) = \sup_{y \in X} \xi_e \circ f(x, y) \leqslant \epsilon$.
Conversely, if $\Phi(f, x) = \sup_{y \in X} \xi_e \circ f(x, y) \leqslant \epsilon$, then $\xi_e \circ f(x, y) \leqslant \epsilon$, for all $y \in X$. By Lemma 2.4 (2), we get $f(x, y) - \epsilon e \in -C$, for all $y \in X$.
3. Using the above results, we can get it.

□

By Definition 2.1 and Lemma 3.1, for all $f \in \Lambda$ and $\epsilon_n > 0$ with $\epsilon_n \to 0$, solution set and ϵ-solution set for $P(f, x)$ is defined as

$$E(f) = \{x \in X : \Phi(f, x) = 0\}, \quad E(f, \epsilon) = \{x \in X : \Phi(f, x) \leqslant \epsilon\}.$$

Thus, Tikhonov and Hadamard well-posedness for $P(X, f)$ is defined as follows.

Definition 3.2.

1. $P(X, f)$ is said to be Tikhonov well-posed (in short T-wp) iff $E(f) = \{x\}$ (a singleton) and $\forall x_n \in E(f, \epsilon_n), \forall \epsilon_n > 0$ with $\epsilon_n \to 0$ implies that $\{x_n\}$ converges to x.
2. $P(X, f)$ is said to be Hadamard well-posed (in short H-wp) iff $E(f) = \{x\}$ (a singleton) and $\forall f_n \in \Lambda, f_n \to f, \forall x_n \in E(f_n)$ implies that $\{x_n\}$ converges to x.

Finally, we establish a well-posedness concept for $P(X, f)$, which unifies its Hadamard and Tikhonov well-posedness.

Definition 3.3. $P(X, f)$ is said to be well-posed (in short wp) iff $E(f) = \{x\}$ (a singleton), $\forall f_n \in \Lambda, f_n \to f, \forall x_n \in E(f_n, \epsilon_n)$, $\forall \epsilon_n > 0$ with $\epsilon_n \to 0$ implies that $\{x_n\}$ converges to x.

Lemma 3.4.

1. *If* $P(X, f)$ *is wp, then it must be T-wp.*
2. *If* $P(X, f)$ *is wp, then it must be H-wp.*

Proof.

1. For all $x_n \in E(f, \epsilon_n)$, for any $\epsilon_n > 0$ with $\epsilon_n \to 0$, we define $f_n = f, \forall n \in N$, then $f_n \to f$ and $x_n \in E(f_n, \epsilon_n)$. Since $P(X, f)$ is wp, then there must have $x_n \to x \in E(f)$. Thus, $P(X, f)$ must be T-wp.
2. For all $f_n \in \Lambda, f_n \to f$, for any $x_n \in E(f_n)$, we define $\epsilon_n = 0, \forall n \in N$, then $x_n \in E(f_n, \epsilon_n)$. Since $P(X, f)$ is wp, then there must have $x_n \to x \in E(f)$. Thus, $P(X, f)$ must be H-wp.

□

4. Generic well-posedness of (SVEP)

In order to show sufficient conditions of generic well-posedness for $P(X, f)$, we first give the following lemmas.

Lemma 4.1. Φ *is lower semicontinuous on* $\Lambda \times X$.

Proof. It is only needed to show that $\forall\epsilon > 0, \forall f_n \in \Lambda, f_n \to f \in \Lambda, \forall x_n \in X, x_n \to x \in X$, there exists $n_0 \in N$ such that for all $n \geqslant n_0$,

$$\Phi(f_n, x_n) > \Phi(f, x) - \epsilon. \tag{4.1}$$

By definition of the least upper bound, there exists $y_0 \in X$ such that

$$\xi_e \circ f(x, y_0) > \Phi(f, x) - \frac{\epsilon}{2}. \tag{4.2}$$

Since X is compact, there exists $y_n \in X$ such that $y_n \to y_0$. Since $f_n \to f$ and f is continuous on $X \times X$, then we have

$$\|f_n(x_n, y_n) - f(x, y_0)\| \leqslant \|f_n(x_n, y_n) - f(x_n, , y_n)\| + \|f(x_n, y_n) - f(x, y_0)\| \to 0. \tag{4.3}$$

By continuity of ξ_e and (4.3), we have

$$\xi_e \circ f_n(x_n, y_n) \to \xi_e \circ f(x, y_0). \tag{4.4}$$

By (4.4), there exists $n_1 \in N$ such that for any $n \geqslant n_1 \geqslant n_0$,

$$\xi_e \circ f_n(x_n, y_n) > \xi_e \circ f(x, y_0) - \frac{\epsilon}{2}. \tag{4.5}$$

From (4.2) and (4.5), for any $n \geqslant n_1 \geqslant n_0$, we get (4.1), that is,

$$\Phi(f_n, x_n) \geqslant \xi_e \circ f_n(x_n, y_n) > \xi_e \circ f(x, y_0) - \frac{\epsilon}{2} > \Phi(f, x) - \epsilon.$$

□

Lemma 4.2. $E : \Lambda \to 2^X$ *is a usco mapping.*

Proof. Since X is compact, by Lemma 4.1, we have $\{x \in X : \Phi(f, x) \leqslant 0\}$ is closed and hence $E(f)$ is compact.

Next, we show that E is upper semicontinuous at f. Suppose to the contrary that there is an open set O of X with $O \supset E(f)$ such that there are a sequence $\{f_n\}$ with $f_n \to f$ and a sequence $\{x_n\}$ with $x_n \in E(f_n)$, but $x_n \notin O$. Note that $x_n \in E(f_n)$, we have $x_n \in X$. Since X is compact, there is a subsequence $\{x_{n_k}\} \subset \{x_n\}$ such that $x_{n_k} \to x \in X$. Note that $x_{n_k} \in E(x_{n_k})$ and thus $\Phi(f_{n_k}, x_{n_k}) = 0$. By Lemma 4.1, we have

$$0 \leqslant \Phi(f, x) \leqslant \liminf \Phi(f_{n_k}, x_{n_k}) = 0.$$

Hence $x \in E(f) \subset O$, which is a contradiction with $x_{n_k} \to x$ and O is open but $x_{n_k} \notin O$ for all $n_k \in N$. This shows that $E : \Lambda \to 2^X$ is upper semicontinuous. □

Referring to [24], it is easy to check the following.

Lemma 4.3. *There exists a dense* G_δ *set* Q *of* Λ *such that* $E(f)$ *is a singleton for each* $f \in Q$.

Theorem 4.4. *There exists a dense* G_δ *set* Q *of* Λ *such that, for any* $f \in Q$, $P(X, f)$ *is wp.*

Proof. For all $f_n \in \Lambda, f_n \to f$, and any $x_n \in E(f_n, \epsilon_n)$, $\forall\epsilon_n > 0$ with $\epsilon_n \to 0$, then we have $x_n \in X$ and $\Phi(f_n, x_n) \leqslant \epsilon_n$. Since X is compact, then there exists a subsequence $\{x_{n_k}\} \subset \{x_n\}$ such that $x_{n_k} \to x \in X$. Secondly, by $\Phi(f_n, x_n) \leqslant \epsilon_n$ and Lemma 4.1, we have

$$0 \leqslant \Phi(f, x) \leqslant \liminf_{n_k \to \infty} \Phi(f_{n_k}, x_{n_k}) \leqslant \liminf_{n_k \to \infty} \epsilon_{n_k} = 0,$$

which implies that $\Phi(f, x) = 0$. By Lemma 4.3, there exists a dense G_δ set Q of Λ such that, for any $f \in Q$, $E(f)$ is a singleton. Thus, there exists a dense G_δ set Q of Λ such that, for any $f \in Q$, $P(X, f)$ is wp. □

Finally, by Lemma 3.4, and Theorem 4.4, it is easy to check the following.

Corollary 4.5. *There exists a dense* G_δ *set* Q *of* Λ *such that, for any* $f \in Q$, $P(X, f)$ *is H-wp.*

Corollary 4.6. *There exists a dense* G_δ *set* Q *of* Λ *such that, for any* $f \in Q$, $P(X, f)$ *is T-wp.*

5. Metric characterizations of well-posedness for (SVEP)

In order to give metric characterization of well-posedness of $P(X,f)$, we introduce the following notation.

Let $f \in \Lambda$ be given sequence. The approximating solution set of $P(X,f)$ is defined for all $\delta > 0$ and $\epsilon > 0$, by

$$S(\delta,\epsilon) := \bigcup_{f' \in B(f,\delta)\cap\Lambda} E(f',\epsilon) = \bigcup_{f' \in B(f,\delta)\cap\Lambda} \{x \in X : \Phi(f',x) \leqslant \epsilon\}. \tag{5.1}$$

where, $B(f,\delta)$ denotes the ball centered at f with radius δ.

Clearly, we have, for all $f \in \Lambda$,

(1) $S(0,0) = E(f)$;

(2) $\forall\delta > 0$ and $\epsilon > 0$, $E(f) \subseteq S(\delta,\epsilon)$;

(3) if $0 \leqslant \delta_2 \leqslant \delta_1$ and $0 \leqslant \epsilon_2 \leqslant \epsilon_1$, then $S(\delta_2,\epsilon_2) \subseteq S(\delta_1,\epsilon_1)$.

Lemma 5.1. *For all* $f \in \Lambda, E(f) = \bigcap_{\delta,\epsilon>0} cl(S(\delta,\epsilon))$, *where* $cl(S(\delta,\epsilon))$ *denotes closure of* $S(\delta,\epsilon)$.

Proof. Clearly, $E(f) \subseteq \bigcap_{\delta,\epsilon>0} cl(S(\delta,\epsilon))$. Thus, we only need to show $\bigcap_{\delta,\epsilon>0} cl(S(\delta,\epsilon)) \subseteq E(f)$. Indeed, if $x \in \bigcap_{\delta,\epsilon>0} cl(S(\delta,\epsilon))$, then $\forall\delta > 0$ and $\epsilon > 0, x \in cl(S(\delta,\epsilon))$. Thus, there exists a sequence $\{x_n\}$ such that $x_n \in S(\delta,\epsilon)$ and $x_n \to x$. So, for each $n \in N$, we have $x_n \in S(\frac{1}{n},\frac{1}{n})$ and, so there exists $f_n \in B(f,\frac{1}{n})$ such that $x_n \in X$ and $\Phi(f_n,x_n) \leqslant \frac{1}{n}$. Since X is compact, there exists $\{x_{n_k}\} \subset \{x_n\}$ such that $x_{n_k} \to x \in X$. Since $\Phi(f,x) \geqslant 0$ and Φ is lower semicontinuous at (f,x), then

$$0 \leqslant \Phi(f,x) \leqslant \liminf_{n_k\to\infty} \Phi(f_{n_k},x_{n_k}) \leqslant \lim_{n_k\to\infty} \frac{1}{n_k} = 0.$$

It shows that $x \in E(f)$ and this implies that $\bigcap_{\delta,\epsilon>0} cl(S(\delta,\epsilon)) \subseteq E(f)$. Therefore, $E(f) = \bigcap_{\delta,\epsilon>0} cl(S(\delta,\epsilon))$. □

Theorem 5.2. $P(X,f)$ *is wp iff*

$$diam(cl(S(\delta,\epsilon))) \to 0 \quad as \quad (\delta,\epsilon) \to (0,0), \tag{5.2}$$

where, $diam(cl(S(\delta,\epsilon)))$ *denotes the diameter of* $cl(S(\delta,\epsilon))$ *defined by*

$$diam(cl(S(\delta,\epsilon))) := \sup\{d(x_1,x_2) : x_1,x_2 \in cl(S(\delta,\epsilon))\}.$$

Proof. By way of contradiction, if (5.2) does not hold, then there exists a real number $a > 0$ and $\delta_n,\epsilon_n > 0$ with $(\delta_n,\epsilon_n) \to (0,0)$ such that $diam(cl(S(\delta_n,\epsilon_n))) \geqslant a$. Hence, there exist two sequences $\{u_n\},\{v_n\} \subset S(\delta_n,\epsilon_n)$ such that $d(u_n,v_n) > \frac{a}{2}$. Since $P(X,f)$ is wp, then we have $E(f) = \{x\}$ (a singleton), and $u_n \to x, v_n \to x$. This contradicts $d(u_n,v_n) > \frac{a}{2}$.

Conversely, $\forall x_n \in S(\delta_n,\epsilon_n)$, $\delta_n,\epsilon_n > 0$ with $(\delta_n,\epsilon_n) \to (0,0)$. Without loss of generality, suppose that $0 \leqslant \delta_{n+1} \leqslant \delta_n$ and $0 \leqslant \epsilon_{n+1} \leqslant \epsilon_n$. Since X is complete, $cl(S(\delta_n,\epsilon_n)) \supseteq cl(S(\delta_{n+1},\epsilon_{n+1}))$, and $diam(cl(S(\delta_n,\epsilon_n))) \to 0$ $(n \to \infty)$, there exists a unique $x \in X$ such that

$$\bigcap_{n=1}^{+\infty} cl(S(\delta_n,\epsilon_n)) = \{x\}.$$

By Lemma 5.1, we have

$$E(f) = \bigcap_{n=1}^{+\infty} cl(S(\delta_n,\epsilon_n)) = \{x\}.$$

It shows that $P(X,f)$ is wp. □

In (5.1), let $\delta = 0$, then the approximating solution set of $P(X, f)$ is defined for any $\epsilon > 0$, by

$$E(f, \epsilon) := \{x \in X : \Phi(f, x) \leqslant \epsilon\}.$$

Corollary 5.3. $P(X, f)$ *is T-wp if and only if* $\mathrm{diam}(E(f, \epsilon)) \to 0$ *as* $\epsilon \to 0$.

Proof. For all $\epsilon \geqslant 0$, for any $w_n \in E(f, \epsilon)$ and $w_n \to w$, then we have $w_n \in X$ and $\Phi(f, w_n) \leqslant \epsilon$. Since X is compact, then we have $w \in X$. By $\Phi(f, w_n) \leqslant \epsilon$ and Φ is lower semicontinuous on $\Lambda \times X$, we have

$$\Phi(f, w) \leqslant \liminf_{n\to\infty} \Phi(f, w_n) \leqslant \epsilon.$$

It implies that $E(f, \epsilon)$ is closed. Thus, by Theorem 5.2, the above result holds. □

Acknowledgment

The authors would like to thank the editor and the reviewers for their helpful comments and suggestions, which have improved the presentation of the paper. This work is supported by NSFC, (Grant Number:11561013) and Natural Science Foundation Guizhou Province, PR China (Grant Number: 20132235).

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