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Dynamic reliability evaluation for a multi-state component under stress-strength model

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Abstract

For many technical systems, stress-strength models are of special importance. Stress-strength models can be described as an assessment of the reliability of the component in terms of X and Y random variables where X is the random "stress" experienced by the component and Y is the random "strength" of the component available to overcome the stress. The reliability of the component is the probability that component is strong enough to overcome the stress applied on it. Traditionally, both the strength of the component and the applied stress are considered to be both time-independent random variables. But in most of real life systems, the status of a stress and strength random variables clearly change dynamically with time. Also, in many important systems, it is very necessary to estimate the reliability of the component without waiting to observe the component failure. In this paper we study multi-state component where component is subjected to two stresses. In particular, inspired by the idea of Kullback-Leibler divergence, we aim to propose a new method to compute the dynamic reliability of the component under stress-strength model. The advantage of the proposed method is that Kullback-Leibler divergence is equal to zero when the component strength is equal to applied stress. In addition, the formed function can include both stresses when two stresses exist at the same time. Also, the proposed method provides a simple way and good alternative to compute the reliability of the component in case of at least one of the stress or strengths quantities depend on time.

Keywords: Kullback-Leibler divergence, dynamic reliability, stress-strength model, multi-state component, gamma distribution.
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1. Introduction and preliminaries

The reliability of technical systems is one of the most important research subjects in the point reached by modern science. In some cases, the performance rate of the system depends on variable environmental conditions which cause degradation. The system may not fail fully but can degrade and there may exist several operation level of the system. A system that can have a finite number of operation levels is called a multi-state system. Generally, multi-state system is consisted of components that they also can be multi-state. The operation levels of the components can range from perfect functioning up to complete failure. The quality of the system is completely determined by components. Because, components failure can lead to the degradation of the entire multi-state system performance. Multi-state systems offer a flexible structure for modeling engineering systems.

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In literature, much attention has been paid to multi-state system modeling. Hudson and Kapur [17] presented some models and their applications, in terms of reliability evaluation, to situations where the system and all its components have a multiple state. Ebrahimi [8] proposed two types of multi-state system and presented various properties related to them. Brunella and Kapur [2] studied a series of reliability measures and expanded their definitions to be consisted with binary, multi-state and continuum models. Kuo and Zuo [22] focused on multi-state system reliability models and introduced several special multi-state system reliability models. Eryılmaz [9] studied mean residual and mean past lifetime concepts for multi-state systems. Gökdere and Gürcan [12] designed a system which consists of two components that can be repairable with the aging property. Firstly, the Laplace-Stieltjes transform of the system is formed. Later, the mean operating time of the system is calculated by means of Laplace-Stieltjes transform, Also, for more details about multi-state system model one can see Lisnionski and Levitin [24]. Also, Guo and Zhang [16], Zhang and Guo [27] presented a new kind of repairable system. Hwang and Yao [18] considered a linear consecutively connected system with binary system states and multistate components and provided the following equations for system reliability evaluation:

$$f(0,j) = p_{0,j}, \quad \text{for} \quad 0 < j < k_0,$$

$$f(i,j) = f(i-1,j)\sum_{h=i}^{j} p_{i,h} + p_{i,j}\sum_{h=i}^{j-1} f(i-1,h), \quad \text{for } i = 1,2,\cdots,n \text{ and } i < j \leqslant \max_{0\leqslant u\leqslant i}\{u+k_u\},$$

where k_i is the transmission capability of component i, $p_{i,i}$, is the probability that component i is completely failed, $p_{i,i}$ is the probability that component i is directly connected to components $i+1, i+2, \cdots, j$ but not to components farther than j for $i < j \leqslant i+k_i$ and $f(i,j)$ is the probability that component 0 reaches component i and component j is the farthest component that is directly connected to one of the components $0, 1, \cdots, n$ $(1 \leqslant i < j \leqslant n+1)$. Kossow and Preuss [19] provided the following equations for the reliability evaluation of a linear multistate consecutively connected systems, which is the same as the one defined by [18] except that the source is assumed to be failure prone:

$$R_L(n) = \begin{cases} P_{0,1}, & \text{for } n = 0, \\ \sum_{\nu=1}^{n} \alpha_\nu(n) R_L(n-\nu) + \alpha_{n+1}(n), & \text{for } 1 \leqslant n < k_0, \\ \sum_{\nu=1}^{\min\{K,n\}} \alpha_\nu(n) R_L(n-\nu), & \text{for } n \geqslant k_0, \end{cases}$$

where

$$\alpha_\nu(n) = \begin{cases} P_{n,n+1}, & \text{for } \nu = 1, \\ \left(\prod_{j=1}^{\nu-1} Q_{n-j+1,n+1}\right) P_{n-\nu+1,n+1}, & \text{for } 2 \leqslant \nu < k_{n-\nu+1}, \\ 0, & \text{for } \nu \geqslant k_{n-\nu+1}, \end{cases}$$

and $P_{i,j}$ is the probability that component i can directly reach component j or beyond.

Stress-strength models are very important for reliability analysis and have a wide application area in engineering applications. In the simplest terms, stress-strength model can be described as an assessment of the reliability of the component in terms of X and Y random variables where X is the random stress experience by the component and Y is the random strength of the component available to overcome the stress.

From this simplified explanation, the reliability of the component is the probability that the component is strong enough to overcome the stress applied on it. Then the reliability of the system is defined as

$$R = P(X < Y) = \int_0^\infty F(x)\,dG(x), \tag{1.1}$$

where $F(x)$ and $G(x)$ are distribution functions of X and Y, respectively. In literature, extensive works have been done about stress-strength reliability. Chandra and Owen [3] studied the estimation of the reliability of a component where component is subject to several stresses whereas its strength is a single random variable. Eryılmaz and Işçioğlu [11], Gökdere and Gürcan [13, 14] studied multi-state systems in a stress-strength setup. Also, for comprehensive information about all methods and results on the stress-strength model one can see Kotz et al. [20].

The Kullback-Leibler divergence introduced by Kullback and Leibler [21], gives an asymmetric measure of the similarity between the distributions of two random variables. If the densities $p(x)$ and $q(x)$ of P and Q, respectively, exist with respect to Lebesque measure, the Kullback-Leibler divergence $D_{KL}(P||Q)$ of Q from P is defined as

$$D_{KL}(P||Q) = \int_s p(x)\log\frac{p(x)}{q(x)}dx, \tag{1.2}$$

where S is the support set of $p(x)$. Note that $D_{KL}(P||Q)$ is finite only, if P is absolutely continuous with respect to Q, and $+\infty$ otherwise. Also, the Kullback-Leibler divergence remains non-negative and is zero, if and only if $P = Q$. It is important to point that the Kullback-Leibler divergence is not a symmetrical quantity.

In information theory and machine learning, the Kullback-Leibler divergence plays an important role. In literature, much attention has been paid to it. Dahlhaus [6] calculated the asymptotic Kulback-Leibler information divergence of two locally stationary sequences and the limit of the Fisher information matrix. Do [7] proposed a fast algorithm to approximate the Kullback-Leibler distance between two hidden Markov models. Rached et al. [25] provided an explicit computable expression for the Kullback-Leibler divergence rate between two arbitrary time-invariant finite-alphabet Markov sources. Lee and Park [23] considered estimation of the Kullback-Leibler divergence between the true density and a selected parametric model. Yari et al. [26] used the new divergence measure, called Kullback-Leibler divergence of Survival functions, to estimate the parameters of a Weibull distribution. The following definitions are available in [26].

Definition 1.1. Let $X_1, X_2, \cdots$ be a sequence of positive, independent and identically distributed random variables from a non-increasing survival function $F(x, \theta) = P_\theta(X > x)$ with support S_x and vector of parameters θ. Define the empirical survival function of random sample of size n by

$$G_n(x) = \sum_{i=0}^{n-1}(1-\frac{i}{n})I_{(X_{i:n},X_{i+1:n})}(x),$$

where I is the indicator function and $X_{0:n} \leqslant X_{1:n} \leqslant ... \leqslant X_{n:n}$ are ordered sample $(X_{0:n} = 0)$.

Definition 1.2. Let $F(x, \theta)$ be the true survival function with unknown parameter θ and $G_n(x)$ be the empirical survival function of a random sample of size n from $F(x, \theta)$. Define the Kullback-Leibler divergence of Survival function G_n and F by

$$KLS(G_n||F) = \int_0^\infty G_n(x)\ln\frac{G_n(x)}{F(x)} - [G_n(x) - F(x)]dx.$$

The following theorems show that the Kullback-Leibler divergence of survival function is a divergence measure which converges to zero with increasing sample size. Also the following theorems are available in [26] and hence the proof of theorems are not presented here.

Theorem 1.3. $KLS(G_n||F) \geqslant 0$ *for all* $n \in \mathbb{N}$, *and the equality holds if and only if* $G_n = F$.

Theorem 1.4. *If* $\int_0^\infty F(x)\mathit{ln}F(x)dx < \infty$, *the introduced measure converges to zero as* n *tends to infinity.*

In this paper, we suppose that the component is subject to X_1 and X_2 stresses, which remain fixed over time, whereas its strength, $Y(t)$, is a single random variable, which is stochastically decreasing in time. Let, X_2 is stochastically larger than X_1, i.e., for real α, $P(X_2 > \alpha) \geqslant P(X_1 > \alpha)$. The failure of the component occurs when the firstly time-independent X_1 stress process and then X_2 stress process exceed the time-dependent strength. The use of multiple stresses rather than single stress is more realistic approach in the reliability of technical systems.

The rest of the paper is organized as three sections. In Section 2, we explain the proposed approach for evaluation of the components dynamic operational level under stress-strength setup. Section 3 gives a gamma distributed example to illustrate the theoretical results for the proposed approach. In Section 4, we summarize what we have done in the paper.

2. Operation performance of the component

In this section, we use Kullback-Leibler divergence for computing the component operation level where component is subject to X_1 and X_2 stresses with continuous cumulative distribution function $F_l(x) = P\{X_l \leqslant x\}(l = 1, 2)$, whereas its strength, $Y(t)$, is a single random variable with marginal distribution function $G_t(x) = P\{Y(t) \leqslant x\}$.

In our method, we first set Kullback-Leibler divergence $D^{(t)}_{KL(l)} = D_{KL}(Y(t)||X_l)$ of X_l from $Y(t)$ by using (1.2) for $l = 1, 2$. After setting Kullback-Leibler divergence, we calculate the $D^{(t)}_{KL(l)}$ for selected values of the parameters of marginal lifetime distributions of the stress and strength random variables. Then using these values, the operation level of the component depending on the both stresses can be defined as follows.

$$\text{The operation level of the component} = \begin{cases} 2, & t < t_2, \\ 1, & t_2 \leqslant t < t_1, \\ 0, & t_1 \leqslant t, \end{cases}$$

where $t_l(l = 1, 2)$ denotes the time when $D^{(t)}_{KL(l)}$ is equal to zero. Also using $D^{(t)}_{KL(l)}$ and t_l values we can define the following equations

$$\zeta^1_t = \begin{cases} D^{(t)}_{KL(1)}, & t_2 \leqslant t < t_1, \\ 0, & t \leqslant t_2, \end{cases}$$

and

$$\zeta^2_t = \begin{cases} D^{(t)}_{KL(2)}, & t \leqslant t_2, \\ 0, & t_2 \leqslant t < t_1. \end{cases}$$

Now with the help of the above equalities, the reliability degree $Rd(t)$ of the component depending on its operation level can be expressed as follows:

$$Rd(t) = (1 - \alpha_t)\frac{\zeta^1_t}{u_1} + \alpha_t(1 + \frac{\zeta^2_t}{u_2}), \tag{2.1}$$

where

$$\alpha_t = \begin{cases} 1, & \zeta^2_t > 0, \\ 0, & \zeta^2_t = 0, \end{cases}$$

and $u_l = \sup D^{(t)}_{KL(l)}$. The superiority of the $Rd(t)$ which is presented in (2.1) is that when two stresses exist at the same time the formed function can include both stresses at the same time. However, when R which is presented in (1.1) is used, this is not possible because of stresses are evaluated separately. In probabilistic design it is common to use parametric statistical models to compute the reliability obtained from stress-strength interference theory. In the following section we apply our method to a gamma distributional example.

3. Gamma distributional example

A gamma process is frequently used for lifetime analysis and reliability testing. Let us suppose that $Y(t)$ is gamma random variables with cumulative distribution function

$$G_t(x; k, \theta_t) = \gamma(k, \frac{x}{\theta(t)})\frac{1}{\Gamma(k)}, \tag{3.1}$$

where $\gamma(k, \frac{x}{\theta(t)})$ is the lower incomplete gamma function, $k > 0$ is the shape parameter and $\theta(t)$ is the scale parameter which decreases over time. Similarly, assume that X_1 and X_2 stresses are gamma random variables with cumulative distribution functions

$$F_l(x; k_l, \theta_l) = \gamma(k_l, \frac{x}{\theta_l})\frac{1}{\Gamma(k_l)}, \tag{3.2}$$

where $\gamma(k_l, \frac{x}{\theta_l})$ is the lower incomplete gamma function, $\Gamma(k_l)$ is the gamma function, $k_l > 0$ is the shape parameter, θ_l is the scale parameter and $l = 1, 2$. Also both k_l and θ_l are constant with aging time. For obtaining the reliability degree $Rd(t)$ of the component, at first we derive a closed form solution for the Kullback-Leibler divergence $D^{(t)}_{KL(l)}$ for $l = 1, 2$. Using probability density functions of (3.1) and (3.2) in (1.2) and referring to Bauckhage [1], the $D^{(t)}_{KL(l)}$ can be obtained as

$$\begin{aligned} D^{(t)}_{KL(l)} &= \int_0^\infty \frac{x^{k-1}}{(\theta(t))^k\Gamma(k)}\exp(\frac{-x}{\theta(t)}) \ln \frac{\dfrac{x^{k-1}}{(\theta(t))^k\Gamma(k)}\exp(\dfrac{-x}{\theta(t)})}{\dfrac{x^{k_l-1}}{(\theta_l)^{k_l}\Gamma(k_l)}\exp(\dfrac{-x}{\theta_l})} \\ &= \ln\frac{(\theta_l)^{k_l}\Gamma(k_l)}{(\theta(t))^k\Gamma(k)} + \frac{k-k_l}{(\theta(t))^k\Gamma(k)}\int_0^\infty x^{k-1}\exp(\frac{-x}{\theta(t)})\ln x dx + \frac{\frac{1}{\theta_l}-\frac{1}{\theta(t)}}{(\theta(t))^k\Gamma(k)}\int_0^\infty x^{k-1}\exp(\frac{-x}{\theta(t)})dx. \end{aligned}$$

By making the substitution $t = x/\theta(t)$ in above equation and then using following integrals,

$$\int_0^\infty x^{\nu-1}\exp(-\mu x)\ln x dx = \frac{1}{\mu^\nu}\Gamma(\nu)[\psi(\nu) - \ln\mu],$$

where $\psi(\nu) = \frac{d}{d\nu}\ln\Gamma(\nu)$ is the psi function (Gradshteyn and Ryzhik, [15, Eq. 4.352.1]) and

$$\int_0^\infty x^{\nu-1}\exp(-\mu x)dx = \frac{1}{\mu^\nu}\Gamma(\nu),$$

where $\mu > 0$ and $\nu > 0$ (Gradshteyn and Ryzhik, [15, Eq. 3.381.4]) we have

$$D^{(t)}_{KL(l)} = \ln\frac{(\theta_l)^{k_l}\Gamma(k_l)}{(\theta(t))^k\Gamma(k)} + (k-k_l)(\ln\theta(t) + \psi(k))) + k(\frac{\theta(t)}{\theta_l} - 1). \tag{3.3}$$

Because of $\theta(t)$ decreases over time, in (3.3), let $\theta(t) = 1/t$, then we finally have

$$D^{(t)}_{KL(l)} = \ln\frac{t^k(\theta_l)^{k_l}\Gamma k_l}{\Gamma k} + (k-k_l)(\ln\frac{1}{t} + \psi(k)) + k(\frac{1}{t\theta_l} - 1), \tag{3.4}$$

where $l = 1, 2$. In order to compare $Rd(t)$ and R expediently, let $k = k_1 = k_2 = 1$ in (3.4), then we can get

$$D^{(t)}_{KL(l)} = \ln(t\theta_l) + \frac{1}{t\theta_l} - 1.$$

Also, taking into account that $k = k_1 = k_2 = 1$ and $\theta(t) = 1/t$, using (3.1) and (3.2) in (1.1) the reliability function can be easily derived as

$$R_l(t) = P(X_1 < Y(t)) = \frac{1}{t\theta_l}. \tag{3.5}$$

For more details, see Chiodo et al. [4], Chiodo and Mazzanti [5] and Eryılmaz [10]. Clearly, using (3.4) and (3.5) for $t = 0.1, 0.2, \cdots, 2$ and selected values of the parameters θ_1 and θ_2, we can obtain $D^{(t)}_{KL(l)}$ and $R_l(t)$ values presented in Table 1.

Table 1: Numerical values obtained from Equation (3.4) and (3.5) for $\theta_1 = 0.6$ and $\theta_2 = 0.8$.

t	$D^{(t)}_{KL(1)}$	$D^{(t)}_{KL(2)}$	$R_1(t)$	$R_2(t)$
0.1	12.8533	8.9742	0.9433	0.9259
0.2	5.2130	3.4174	0.8928	0.8620
0.3	2.8407	1.7395	0.8474	0.8064
0.4	1.7395	0.9855	0.8064	0.7575
0.5	1.1293	0.5837	0.7692	0.7142
0.6	0.7561	0.3493	0.7352	0.6756
0.7	0.5134	0.2058	0.7042	0.6410
0.8	0.3493	0.1162	0.6756	0.6097
0.9	0.2356	0.0603	0.6493	0.5813
1.0	0.1558	0.0268	0.6250	0.5555
1.1	0.0996	0.0085	0.6024	0.5319
1.2	0.0603	0.0008	0.5813	0.5102
t_2	0.0456	0.	0.5714	0.5
1.3	0.0335	0.0007	0.5617	0.4901
1.4	0.0161	0.0061	0.5434	0.4716
1.5	0.0057	0.0156	0.5263	0.4545
1.6	0.0008	0.0281	0.5102	0.4385
t_1	0.	0.0366	0.5	0.4295
1.7	0.0001	0.0427	0.4950	0.4237
1.8	0.0028	0.0590	0.4807	0.4098
1.9	0.0082	0.0766	0.4672	0.3968
2.0	0.0156	0.0950	0.4545	0.3846

In Table 1, $t_1 = 1.66$ and $t_2 = 1.25$. Also, it can be observed from numerical values in $D^{(t)}_{KL(1)}$ and $D^{(t)}_{KL(2)}$ columns how stresses affect the performance of the component that operates under different parameters. When the component starts working, its strength is greater than either stresses. However, because the components strength is decreasing depending on the selected time, as the uptime increases at first the Kullback-Leibler divergence $D^{(t)}_{KL(2)}$ decreases to near zero. In this period, the strength of the component will begin to move to the average position declined from a good position. From the moment that $D^{(t)}_{KL(2)} = 0$, the component will pass to the average working period from a good working period, the Kullback-Leibler divergence $D^{(t)}_{KL(2)}$ is not considered and instead of the Kullback-Leibler divergence $D^{(t)}_{KL(1)}$ is taken into account. The Kullback-Leibler divergence $D^{(t)}_{KL(1)}$ will be reduced again depending on the time. From the moment it is equal to zero, the operation of the machine will end and because the machine's durability remains weak in both stresses the machine will be impaired. In the table, there are also separately calculated $R_1(t)$ and $R_2(t)$ reliabilities of the machine according to either stresses. When attention is paid to their numerical values, for the stress and strength have the same parameters the reliability values becomes 0.5. This numeric value does not provide clear information about the component's operating performance. Considering two stresses, a joint reliability is not calculated but instead the reliability is calculated separately according to the stresses.

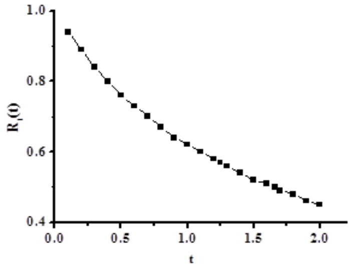

Figure 1: Reliability for $R_1(t)$ at time t.

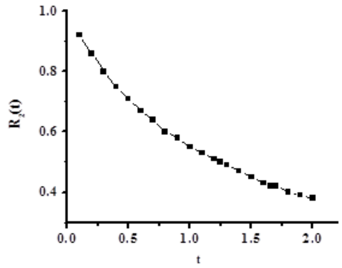

Figure 2: Reliability for $R_2(t)$ at time t.

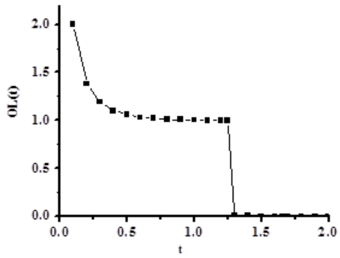

Figure 3: Reliability degree $Rd(t)$ at time t.

In Figures 1 and 2, we plot the reliability scores $R_1(t)$ and $R_2(t)$ at time t for $\theta_1 = 0.6$ and $\theta_2 = 0.8$, respectively. In Figure 3, we plot the reliability degree $Rd(t)$ of the component at time t for $\theta_1 = 0.6$ and $\theta_2 = 0.8$. Finally, using (2.1) for ζ_t^1 and ζ_t^2 values in Table 1, $u_1 = 12.8533$ and $u_2 = 8.9742$, we can obtain dynamic reliability degree presented Table 2 for the component under stress-strength setup.

Table 2: Dynamic reliability degree for the component when $\theta_1 = 0.6$ and $\theta_2 = 0.8$.

t	$Rd(t)$	t	$Rd(t)$
0.1	2.	1.2	1.00008
0.2	1.3808	1.25	1.
0.3	1.1938	1.3	0.0026
0.4	1.1098	1.4	0.0012
0.5	1.0650	1.5	0.0004
0.6	1.0389	1.6	0.000006
0.7	1.0229	1.66	0.
0.8	1.0129	1.7	0.
0.9	1.0067	1.8	0.
1.0	1.0029	1.9	0.
1.1	1.0009	2.0	0.

4. Conclusion

In this study, we aimed to measure the damage caused by stresses which is applied to the component. In the method that we have offered, it is preferred to use Kullback-Leibler divergence to obtain this measurement. Because Kullback-Leibler divergence plays an important role in information theory and statistics. The Kullback-Leibler divergence is not a metric. But this feature does not adversely affect on the use of our method. Also it is assumed that the Kullback-Leibler divergence is known. In our method, it is theoretically assumed that a component operates under two different stresses and when the components strength remains weak in both stresses the component fails. Let us consider two stresses by using (1.1) a joint reliability is not calculated. However, instead the reliability can be calculated separately according to the stresses. Here, for reliability evaluation we provide a new approach for obtaining the component operation performance. The proposed method can clearly show the change of component operation performance depending on time while under both stresses. The reliability degree $\mathrm{Rd}(t)$ can denote the change of component operation performance under two stresses depending on time. When the related parameters of the component in Table 2 are selected as θ_1=0.6 and θ_2=0.8, the values of reliability score are calculated in the interval $0< t \leqslant 1.6$. The method used in the study does not depend on probability distribution. When different effect functions are used instead of probability functions of stress and strength, the recommended method can be easily used. In terms of theoretical applications, the Kullback-Leibler divergence used in this study is a useful method for leveling the stress-strength models. The results of the study are widely used in technical applications. In this respect, the study brings a different perspective to the subject.

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References

[1] C. Bauckhage, *Computing the Kullback-Leibler divergence between two Generalized Gamma distributions*, ArXiv, **2014** (2014), 7 pages. 3
[2] R. D. Brunelle, K. C. Kapur, *Review and classification of reliability measures for multistate and continuum models*, IIE Trans., **31** (1999), 1171–1180. 1
[3] S. Chandra, D. B. Owen, *On estimating the reliability of a component subject to several different stresses (strengths)*, Naval Res. Logist. Quart., **22** (1975), 31–39. 1
[4] E. Chiodo, D. Fabiani, G. Mazzanti, *Bayes inference for reliability of HV insulation systems in the presence of switching voltage surges using a Weibull stress-strength model*, IEEE Power Tech. Conf. Proc., Bologna, **3** (2003). 3
[5] E. Chiodo, G. Mazzanti, *Bayesian reliability estimation based on a Weibull stress-strength model for aged power system components subjected to voltage surges*, IEEE Trans. Dielectr. Electr. Insul., **13** (2006), 146–159. 3
[6] R. Dahlhaus, *On the Kullback-Leibler information divergence of locally stationary processes*, Stochastic Process. Appl., **62** (1996), 139–168. 1
[7] M. N. Do, *Fast approximation of Kullback-Leibler distance for dependence trees and hidden Markov models*, IEEE Signal Process. Lett., **10** (2003), 115–118. 1
[8] N. Ebrahimi, *Multistate reliability models*, Naval Res. Logist. Quart., **31** (1984), 671–680. 1
[9] S. Eryılmaz, *Mean residual and mean past lifetime of multi-state systems with identical components*, IEEE Trans. Rel., **59** (2010), 644–649. 1
[10] S. Eryılmaz, *On stress-strength reliability with a time-dependent strength*, J. Qual. Reliab. Eng., **2013** (2013), 6 pages. 3
[11] S. Eryılmaz, F. İşçioğlu, *Reliability evaluation for a multi-state system under stress-strength setup*, Comm. Statist. Theory Methods, **40** (2011), 547–558. 1
[12] G. Gökdere, M. Gürcan, *Erlang Strength Model for Exponential Effects*, Open Phys., **13** (2015), 395–399. 1
[13] G. Gökdere, M. Gürcan, *Laplace-Stieltjes transform of the system mean lifetime via geometric process model*, Open Math., **14** (2016), 384–392. 1
[14] G. Gökdere, M. Gürcan, *New Reliability Score for Component Strength Using Kullback-Leibler Divergence*, Eksploatacja i Niezawodność.-Maintenance and Reliability, **18** (2016), 367–372. 1
[15] I. S. Gradshteyn, I. M. Ryzhik, *Table of integrals, series, and products*, Translated from the Russian, Sixth edition, Translation edited and with a preface by Alan Jeffrey and Daniel Zwillinger, Academic Press, Inc., San Diego, CA, (2000). 3
[16] L. Guo, M.-M. Zhang, *A Time-varying repairable system with repairman vacation and warning device*, J. Nonlinear Sci. Appl., **9** (2016), 316–331. 1
[17] J. C. Hudson, K. C. Kapur, *Reliability analysis for multistate systems with multistate components*, IIE Trans., **15** (1983), 127–135. 1

[18] F. K. Hwang, Y.-C. Yao, *Multistate consecutively-connected systems*, IEEE Trans. Rel., **38** (1989), 472–474. 1

[19] A. Kossow, W. Preuss, *Reliability of linear consecutively-connected systems with multistate components*, IEEE Trans. Rel., **44** (1995), 518–522. 1

[20] S. Kotz, Y. Lumelskii, M. Pensky, *The stress-strength model and its generalizations: theory and applications*, World Scientific Publishing Co. Inc., Singapore, (2003). 1

[21] S. Kullback, R. A. Leibler, *On information and sufficiency*, Ann. Math. Statistics, **22** (1951), 79–86. 1

[22] W. Kuo, M. J. Zuo, *Optimal reliability modeling: principles and applications*, John Wiley & Sons, New York, (2003). 1

[23] Y. K. Lee, B. U. Park, *Estimation of Kullback-Leibler divergence by local likelihood*, Ann. Inst. Statist. Math., **58** (2006), 327–340. 1

[24] A. Lisnionski, G. Levitin, *Multi-state system reliability: assessment, optimization and applications*, Series on Guality, Reliability and Engineering Statistics, World Scientific Publishing Co. Inc., Singapore, (2003). 1

[25] Z. Rached, F. Alajaji, L. L. Campbell, *The Kullback-Leibler divergence rate between Markov sources*, IEEE Trans. Inf. Theory, **50** (2004), 917–921. 1

[26] G. Yari, A. Mirhabibi, A. Saghafi, *Estimation of the Weibull parameters by Kullback-Leibler divergence of survival functions*, Appl. Math. Inf. Sci., **7** (2013), 187–192. 1, 1

[27] X. Zhang, L. Guo, *A new kind of repairable system with repairman vacations*, J. Nonlinear Sci. Appl., **8** (2015), 324–333. 1