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A sharp generalization on cone b-metric space over Banach algebra

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Abstract

The aim of this paper is to generalize a famous result for Banach-type contractive mapping from $\rho(k) \in [0, \frac{1}{s})$ to $\rho(k) \in [0, 1)$ in cone b-metric space over Banach algebra with coefficient $s \geqslant 1$, where $\rho(k)$ is the spectral radius of the generalized Lipschitz constant k. Moreover, some similar generalizations for the contractive constant k from $k \in [0, \frac{1}{s})$ to $k \in [0, 1)$ in cone b-metric space and in b-metric space are also obtained. In addition, two examples are given to illustrate that our generalizations are in fact real generalizations.

Keywords: Cone b-metric space over Banach algebra, fixed point, c-sequence, iterative sequence.
2010 MSC: 47H10, 54H25.

1. Introduction and preliminaries

Since Bakhtin [1] or Czerwik [4] introduced b-metric space, also called metric type space by some authors, as a large generalization of metric space, people have paid close attention to fixed point results in such spaces for many years. Recently, Jovanović et al. [10] proved Banach-type version of a fixed point result (see [2]) for contractive mapping including the contractive constant $k \in [0, \frac{1}{s})$ in b-metric space with coefficient $s \geqslant 1$. Subsequently, Huang and Xu [8] expanded the work of [10] into cone b-metric space with coefficient $s \geqslant 1$, where the contractive constant k also satisfies $k \in [0, \frac{1}{s})$. Later on, Huang and Radenović [7] gave a further generalization from cone b-metric space with coefficient $s \geqslant 1$ to cone b-metric space over Banach algebra with the same coefficient. They considered the Banach-type version of a fixed point result with the generalized Lipschitz constant k satisfying $\rho(k) \in [0, \frac{1}{s})$, where $\rho(k)$ is the spectral radius of k. So far, there have been some open questions whether the result in b-metric space or in cone b-metric space is true for $k \in [0, 1)$, and whether the result in cone b-metric space over Banach algebra is true for $\rho(k) \in [0, 1)$. In this paper, by using a new method of proof, we prove that the answers to the above questions are positive.

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Definition 1.1 ([1, 4]). Let X be a (nonempty) set and $s \geqslant 1$ be a given real number. A function $d : X \times X \to [0, \infty)$ is called a b-metric on X, if for all $x, y, z \in X$ the following conditions hold:

(b1) $d(x, y) = 0$ iff $x = y$;

(b2) $d(x, y) = d(y, x)$;

(b3) $d(x, z) \leqslant s\,[d(x, y) + d(y, z)]$.

In this case, the pair (X, d) is called a b-metric space.

For some concepts such as b-convergence, b-Cauchy sequence and b-completeness in the setting of b-metric spaces, the reader refers to [10, 13, 15].

In the following example we correct some errors from several papers (see [3, 12–16]) for b-metric space with the false coefficient $s = 2^{\frac{1}{p}}$. As a matter of fact, its correct coefficient should be $s = 2^{\frac{1}{p}-1}$.

Example 1.2. The set $l_p(\mathbb{R})$ with $0 < p < 1$, where

$$l_p(\mathbb{R}) := \left\{ \{x_n\} \subseteq \mathbb{R} \,\middle|\, \sum_{n=1}^{\infty} |x_n|^p < \infty \right\},$$

together with the mapping $d : l_p(\mathbb{R}) \times l_p(\mathbb{R}) \to [0, \infty)$ defined by

$$d(x, y) = \left(\sum_{n=1}^{\infty} |x_n - y_n|^p \right)^{\frac{1}{p}},$$

for each $x = \{x_n\}, y = \{y_n\} \in l_p(\mathbb{R})$ is a b-metric space with coefficient $s = 2^{\frac{1}{p}-1}$.

In fact, we only need to prove that condition (b3) in Definition 1.1 is satisfied. To this end, let $x = \{x_n\}, y = \{y_n\}, z = \{z_n\} \in l_p(\mathbb{R})$, we shall show that

$$\left(\sum_{n=1}^{\infty} |x_n - z_n|^p \right)^{\frac{1}{p}} \leqslant 2^{\frac{1}{p}-1} \left[\left(\sum_{n=1}^{\infty} |x_n - y_n|^p \right)^{\frac{1}{p}} + \left(\sum_{n=1}^{\infty} |y_n - z_n|^p \right)^{\frac{1}{p}} \right]. \tag{1.1}$$

Denote $a_n = x_n - y_n$, $b_n = y_n - z_n$, then $x_n - z_n = a_n + b_n$, so (1.1) becomes

$$\left(\sum_{n=1}^{\infty} |a_n + b_n|^p \right)^{\frac{1}{p}} \leqslant 2^{\frac{1}{p}-1} \left[\left(\sum_{n=1}^{\infty} |a_n|^p \right)^{\frac{1}{p}} + \left(\sum_{n=1}^{\infty} |b_n|^p \right)^{\frac{1}{p}} \right]. \tag{1.2}$$

In order to prove (1.2), we notice that the following inequalities:

$$(a + b)^p \leqslant a^p + b^p, \qquad (a, b \geqslant 0,\ 0 < p \leqslant 1),$$
$$(a + b)^p \leqslant 2^{p-1}(a^p + b^p), \qquad (a, b \geqslant 0,\ p \geqslant 1),$$

then

$$\left(\sum_{n=1}^{\infty} |a_n + b_n|^p \right)^{\frac{1}{p}} \leqslant \left[\sum_{n=1}^{\infty} (|a_n| + |b_n|)^p \right]^{\frac{1}{p}} \leqslant \left[\sum_{n=1}^{\infty} (|a_n|^p + |b_n|^p) \right]^{\frac{1}{p}}$$
$$= \left(\sum_{n=1}^{\infty} |a_n|^p + \sum_{n=1}^{\infty} |b_n|^p \right)^{\frac{1}{p}} \leqslant 2^{\frac{1}{p}-1} \left[\left(\sum_{n=1}^{\infty} |a_n|^p \right)^{\frac{1}{p}} + \left(\sum_{n=1}^{\infty} |b_n|^p \right)^{\frac{1}{p}} \right].$$

Let $\mathbb{A}$ be a real Banach algebra, $\|\cdot\|$ be its norm and θ be its zero element. A nonempty closed subset P of $\mathbb{A}$ is called a cone, if $P^2 = P \cap P \subset P$, $P \cap (-P) = \{\theta\}$ and $\lambda P + \mu P \subset P$ for all $\lambda, \mu \geqslant 0$. We denote $\text{int}P$ as the interior of P. If $\text{int}P \neq \emptyset$, then P is said to be a solid cone. Define a partial ordering $\preceq$ with respect to P by $u \preceq v$, iff $v - u \in P$. Define $u \ll v$, iff $v - u \in \text{int}P$.

In the sequel, unless otherwise specified, we always suppose that $\mathbb{A}$ is a real Banach algebra with a unit e, P is a solid cone in $\mathbb{A}$, and " $\preceq$ " and " $\ll$ " are partial orderings with respect to P.

Definition 1.3 ([7]). Let X be a (nonempty) set, $s \geqslant 1$ be a constant and $\mathbb{A}$ be a Banach algebra. Suppose that a mapping $d : X \times X \to \mathbb{A}$ satisfies for all $x, y, z \in X$,

(d1) $\theta \preceq d(x, y)$ and $d(x, y) = \theta$, iff $x = y$;

(d2) $d(x, y) = d(y, x)$;

(d3) $d(x, z) \preceq s[d(x, y) + d(y, z)]$.

Then d is called a cone b-metric on X, and (X, d) is called a cone b-metric space over Banach algebra.

For some examples on cone b-metric space over Banach algebra, the reader refers to [6, 7].

Definition 1.4 ([6]). A sequence $\{u_n\}$ in a solid cone P is said to be a c-sequence, if for each $c \gg \theta$, there exists a natural number N such that $u_n \ll c$ for all $n > N$.

Definition 1.5. Let (X, d) be a cone b-metric space over Banach algebra and $\{x_n\}$ a sequence in X. We say that

(i) $\{x_n\}$ b-converges to $x \in X$, if $\{d(x_n, x)\}$ is a c-sequence;

(ii) $\{x_n\}$ is a b-Cauchy sequence, if $\{d(x_n, x_m)\}$ is a c-sequence for n, m;

(iii) (X, d) is b-complete, if every b-Cauchy sequence in X is b-convergent.

Lemma 1.6 ([7]). *Let $\{u_n\}$ and $\{v_n\}$ be two c-sequences in a solid cone P. If $\alpha, \beta \in P$ are two arbitrarily given vectors, then $\{\alpha u_n + \beta v_n\}$ is a c-sequence.*

Lemma 1.7 ([9]). *If $u \preceq v$ and $v \ll w$, then $u \ll w$.*

Lemma 1.8 ([6]). *Let $\mathbb{A}$ be a Banach algebra with a unit e, then the spectral radius $\rho(k)$ of $k \in \mathbb{A}$ holds*

$$\rho(k) = \lim_{n\to\infty} \|k^n\|^{\frac{1}{n}} = \inf \|k^n\|^{\frac{1}{n}}.$$

If $\rho(k) < 1$, then $e - k$ is invertible in $\mathbb{A}$, moreover, $(e - k)^{-1} = \sum_{i=0}^{\infty} k^i$.

Lemma 1.9 ([6]). *Let $\mathbb{A}$ be a Banach algebra with a unit e. Let $k \in \mathbb{A}$ and $\rho(k) < 1$. Then $\{k^n\}$ is a c-sequence.*

2. Main results

Theorem 2.1. *Let (X, d) be a b-complete cone b-metric space over Banach algebra with coefficient $s \geqslant 1$. Suppose that $T : X \to X$ is a mapping such that for all $x, y \in X$ it holds:*

$$d(Tx, Ty) \preceq k d(x, y), \tag{2.1}$$

where $k \in P$ is a generalized Lipschitz constant with $\rho(k) < 1$. Then T has a unique fixed point in X. And for any $x \in X$, the iterative sequence $\{T^n x\}$ $(n \in \mathbb{N})$ b-converges to the fixed point.

Proof. Let $x_0 \in X$ and $x_{n+1} = Tx_n$ for all $n \in \mathbb{N}$. We divide the proof into three cases.

Case 1: Let $\rho(k) \in [0, \frac{1}{s})$ $(s > 1)$. By (2.1), we have

$$\begin{aligned} d(x_n, x_{n+1}) &= d(Tx_{n-1}, Tx_n) \\ &\preceq kd(x_{n-1}, x_n) \\ &= kd(Tx_{n-2}, Tx_{n-1}) \\ &\preceq k^2 d(x_{n-2}, x_{n-1}) \\ &\vdots \\ &\preceq k^n d(x_0, x_1). \end{aligned}$$

In view of $\rho(k) < \frac{1}{s}$, then $\rho(sk) = s\rho(k) < 1$, so by Lemma 1.8, we get that $e - sk$ is invertible and $(e - sk)^{-1} = \sum_{i=0}^{\infty}(sk)^i$. Thus for any $n > m$, it follows that

$$\begin{aligned} d(x_m, x_n) &\preceq s[d(x_m, x_{m+1}) + d(x_{m+1}, x_n)] \\ &\preceq sd(x_m, x_{m+1}) + s^2[d(x_{m+1}, x_{m+2}) + d(x_{m+2}, x_n)] \\ &\preceq sd(x_m, x_{m+1}) + s^2 d(x_{m+1}, x_{m+2}) + s^3[d(x_{m+2}, x_{m+3}) + d(x_{m+3}, x_n)] \\ &\preceq sd(x_m, x_{m+1}) + s^2 d(x_{m+1}, x_{m+2}) + s^3 d(x_{m+2}, x_{m+3}) \\ &\quad + \cdots + s^{n-m-1} d(x_{n-2}, x_{n-1}) + s^{n-m-1} d(x_{n-1}, x_n) \\ &\preceq sk^m d(x_0, x_1) + s^2 k^{m+1} d(x_0, x_1) + s^3 k^{m+2} d(x_0, x_1) \\ &\quad + \cdots + s^{n-m-1} k^{n-2} d(x_0, x_1) + s^{n-m-1} k^{n-1} d(x_0, x_1) \\ &\preceq sk^m (e + sk + s^2 k^2 + \cdots + s^{n-m-2} k^{n-m-2} + s^{n-m-1} k^{n-m-1}) d(x_0, x_1) \\ &\preceq sk^m \left[\sum_{i=0}^{\infty} (sk)^i\right] d(x_0, x_1) \\ &= sk^m (e - sk)^{-1} d(x_0, x_1). \end{aligned}$$

Note that $\rho(k) < \frac{1}{s} < 1$ and Lemma 1.9, it is easy to see that $\{k^m\}$ is a c-sequence. Therefore, by using Lemma 1.6 and Lemma 1.7, we claim that $\{x_n\}$ is a b-Cauchy sequence. Since (X, d) is b-complete, then there exists $x^* \in X$ such that $\{x_n\}$ b-converges to x^*.

Next, let us show that x^* is a fixed point of T. Indeed, by (2.1), we have

$$d(x_{n+1}, Tx^*) \preceq kd(x_n, x^*). \tag{2.2}$$

Since $\{d(x_n, x^*)\}$ is a c-sequence, then by Lemma 1.6, it is not hard to verify that $\{kd(x_n, x^*)\}$ is a c-sequence. Hence, by Lemma 1.7, (2.2) implies that $\{d(x_{n+1}, Tx^*)\}$ is also a c-sequence, which means that $\{x_n\}$ b-converges to Tx^*. By the uniqueness of limit of a b-convergent sequence, we get $Tx^* = x^*$. That is to say, x^* is a fixed point of T.

Further, x^* is the unique fixed point of T. Actually, assume that there is another fixed point y^*, then by (2.1), it is obvious that

$$d(x^*, y^*) = d(Tx^*, Ty^*) \preceq kd(x^*, y^*) \preceq k^2 d(x^*, y^*) \preceq \cdots \preceq k^n d(x^*, y^*).$$

Now that $\{k^n\}$ is a c-sequence, then by Lemma 1.6 and Lemma 1.7, we obtain $d(x^*, y^*) = \theta$. This leads to $x^* = y^*$.

Case 2: Let $\rho(k) \in [\frac{1}{s}, 1)$ $(s > 1)$. In this case, we have $[\rho(k)]^n \to 0$ as $n \to \infty$, then there is $n_0 \in \mathbb{N}$ such that $[\rho(k)]^{n_0} < \frac{1}{s}$. Notice that

$$\rho(k^{n_0}) = \lim_{n\to\infty} \|(k^{n_0})^n\|^{\frac{1}{n}}$$

$$
\begin{aligned}
&= \lim_{n\to\infty} \| \underbrace{k^n \cdot \cdots \cdot k^n}_{n_0 \text{ terms}} \|^{\frac{1}{n}} \\
&\leqslant \lim_{n\to\infty} \big(\underbrace{\|k^n\| \cdot \cdots \cdot \|k^n\|}_{n_0 \text{ terms}} \big)^{\frac{1}{n}} \\
&= \underbrace{\Big(\lim_{n\to\infty} \|k^n\|^{\frac{1}{n}} \Big) \cdot \cdots \cdot \Big(\lim_{n\to\infty} \|k^n\|^{\frac{1}{n}} \Big)}_{n_0 \text{ terms}} \\
&= [\rho(k)]^{n_0} \\
&< \frac{1}{s},
\end{aligned}
$$

and by (2.1) for all $x, y \in X$, it follows that

$$
\begin{aligned}
d(T^{n_0}x, T^{n_0}y) &= d\left(T(T^{n_0-1}x), T(T^{n_0-1}y)\right) \\
&\preceq kd(T^{n_0-1}x, T^{n_0-1}y) \\
&= kd\left(T(T^{n_0-2}x), T(T^{n_0-2}y)\right) \\
&\preceq k^2 d(T^{n_0-2}x, T^{n_0-2}y) \\
&\ \ \vdots \\
&\preceq k^{n_0} d(x, y).
\end{aligned}
$$

Then by Case 1, we claim that the mapping T^{n_0} has a unique fixed point $x^{**} \in X$.

Now we prove that x^{**} is also a fixed point of T. As a matter of fact, on account of $T^{n_0}x^{**} = x^{**}$, we have

$$T^{n_0}(Tx^{**}) = T^{n_0+1}x^{**} = T(T^{n_0}x^{**}) = Tx^{**},$$

then Tx^{**} is also a fixed point of T^{n_0}. Thus, by the uniqueness of fixed point of T^{n_0}, it ensures us that $Tx^{**} = x^{**}$. In other words, x^{**} is also a fixed point of T.

Finally, we show that the fixed point of T is unique. Virtually, we suppose for absurd that there exists another fixed point x^{***} of T, that is, $Tx^{**} = x^{**}$, $Tx^{***} = x^{***}$, then

$$
\begin{aligned}
T^{n_0}x^{**} &= T^{n_0-1}(Tx^{**}) = T^{n_0-1}x^{**} = \cdots = Tx^{**} = x^{**}, \\
T^{n_0}x^{***} &= T^{n_0-1}(Tx^{***}) = T^{n_0-1}x^{***} = \cdots = Tx^{***} = x^{***},
\end{aligned}
$$

which imply that x^{**} and x^{***} are two fixed points of T^{n_0}. Because the fixed point of T^{n_0} is unique, we claim that $x^{**} = x^{***}$.

Case 3: $s = 1$. Since $\rho(k) < 1$, repeat the process of Case 1, then the claim holds. □

Corollary 2.2. *Let (X, d) be a b-complete cone b-metric space with coefficient $s \geqslant 1$. Suppose that $T : X \to X$ is a mapping such that for all $x, y \in X$, it holds:*

$$d(Tx, Ty) \preceq kd(x, y),$$

where $k \in [0, 1)$ is a real constant. Then T has a unique fixed point in X. And for any $x \in X$, the iterative sequence $\{T^n x\}$ $(n \in \mathbb{N})$ b-converges to the fixed point.

Proof. Choose $k \in \mathbb{R}$ in Theorem 2.1, then the proof is completed. □

Corollary 2.3. *Let (X, d) be a b-complete b-metric space with coefficient $s \geqslant 1$. Suppose that $T : X \to X$ is a mapping such that for all $x, y \in X$, it holds:*

$$d(Tx, Ty) \leqslant kd(x, y),$$

where $k \in [0, 1)$ is a real constant. Then T has a unique fixed point in X. And for any $x \in X$, the iterative sequence $\{T^n x\}$ $(n \in \mathbb{N})$ b-converges to the fixed point.

Remark 2.4. Theorem 2.1 greatly generalizes [7, Theorem 2.1] from $\rho(k) \in [0, \frac{1}{s})$ to $\rho(k) \in [0, 1)$. Corollary 2.2 greatly generalizes [8, Theorem 2.1] from $k \in [0, \frac{1}{s})$ to $k \in [0, 1)$. Corollary 2.3 greatly generalizes [10, Theorem 3.3] from $k \in [0, \frac{1}{s})$ to $k \in [0, 1)$.

Remark 2.5. Regarding the improvement of contractive coefficients, there have been some articles dealing with them. For instance, compared with [11], [5] generalizes the range of the coefficient λ from $\lambda \in (0, \frac{1}{2})$ to $\lambda \in (0, 1)$ for quasi-contraction, which is an interesting generalization. Whereas, our results generalize some famous results on Banach-type contractions for the coefficient k from $\rho(k) \in [0, \frac{1}{s})$ to $\rho(k) \in [0, 1)$, as well as from $k \in [0, \frac{1}{s})$ to $k \in [0, 1)$. Consequently, our generalizations are indeed sharp generalizations. The following examples illustrate our conclusions.

Example 2.6. Let $X = [0, 1]$, $\mathbb{A} = C^1_{\mathbb{R}}(X)$ and define a norm on $\mathbb{A}$ by $\|u\| = \|u\|_\infty + \|u'\|_\infty$. Define multiplication in $\mathbb{A}$ as just pointwise multiplication. Then $\mathbb{A}$ is a real Banach algebra with a unit $e = 1$ ($e(t) = 1$ for all $t \in X$). The set $P = \{u \in \mathbb{A} : u(t) \geqslant 0, t \in X\}$ is a non-normal solid cone (see [9]). Define a mapping $d : X \times X \to \mathbb{A}$ by $d(x, y)(t) = |x - y|^2 e^t$. We have that (X, d) is a b-complete cone b-metric space over Banach algebra $\mathbb{A}$ with coefficient $s = 2$. Define a self-mapping T on X by $Tx = \frac{\sqrt{2}}{2}x$. Put $k = \frac{1}{2} + \frac{1}{4}t$. Then $d(Tx, Ty) \preceq kd(x, y)$ for all $x, y \in X$. Simple calculations show that $\frac{1}{s} = \frac{1}{2} < \rho(k) = \frac{3}{4} < 1$. Clearly, $\rho(k) \notin [0, \frac{1}{s})$, but $\rho(k) \in [\frac{1}{s}, 1)$. Hence, [7, (i) of Theorem 2.1] is not satisfied. That is to say, [7, Theorem 2.1] cannot be used in this example. However, our Theorem 2.1 is satisfied. Accordingly, T has a unique fixed point $x = 0$.

Example 2.7. Let $X = [0, \frac{3}{5}]$, $E = \mathbb{R}^2$ and $p \geqslant 5$ be a constant. Put $P = \{(x, y) \in E : x, y \geqslant 0\}$. We define $d : X \times X \to E$ as $d(x, y) = |x - y|^p$, for all $x, y \in X$. Then (X, d) is a b-complete b-metric space with coefficient $s = 2^{p-1}$. Define a self-mapping T on X by $Tx = \frac{1}{2}\big(\cos\frac{x}{2} - |x - \frac{1}{2}|\big)$, for all $x \in X$. Hence, for all $x, y \in X$, we speculate that

$$\begin{aligned} d(Tx, Ty) &= |Tx - Ty|^p \\ &= \frac{1}{2^p}\left|\left(\cos\frac{x}{2} - \cos\frac{y}{2}\right) - \left(\left|x - \frac{1}{2}\right| - \left|y - \frac{1}{2}\right|\right)\right|^p \\ &\leqslant \frac{1}{2^p}\left(\left|\cos\frac{x}{2} - \cos\frac{y}{2}\right| + |x - y|\right)^p \\ &\leqslant \frac{1}{2^p}\left(\frac{|x + y|}{8}|x - y| + |x - y|\right)^p \\ &\leqslant 0.575^p|x - y|^p. \end{aligned}$$

In view of $p \geqslant 5$, then $k = 0.575^p \notin [0, \frac{1}{s})$, but $k = 0.575^p \in [\frac{1}{s}, 1)$. Thus, [10, Theorem 3.3] does not hold in this case. In other words, [10, Theorem 3.3] is not applicable in this example. However, our Corollary 2.3 can be utilized in this case. To sum up, $x_0 \in X$ satisfied with $0.472251591454 < x_0 < 0.472251591479$ is the unique fixed point of T.

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