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On second-order differential subordinations for a class of analytic functions defined by convolution

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Abstract

Making use of the convolution operator we introduce a new class of analytic functions in the open unit disk and investigate some subordination results.

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1. Introduction

Let $\mathbb{C}$ be complex plane and $\mathbb{U} = \{z \in \mathbb{C} : |z| < 1\} = \mathbb{U}\setminus\{0\}$, open unit disc in $\mathbb{C}$. Let $H(\mathbb{U})$ be the class of analytic functions in $\mathbb{U}$. For $p \in N^+ = \{1, 2, 3, \cdots\}$ and $a \in \mathbb{C}$, let $H[a, k]$ be the subclass of $H(\mathbb{U})$ consisting of the functions of the form

$$f(z) = a + a_k z^k + a_{k+1} z^{k+1} + \cdots$$

with $H_0 \equiv H[0, 1]$ and $H \equiv H[1, 1]$. Let A_p be the class of all analytic functions of the form

$$f(z) = z^p + \sum_{k=p+1}^{\infty} a_k z^k \tag{1.1}$$

in the open unit disk $\mathbb{U}$ with $A_1 = A$. For functions $f \in A_p$ given by equation (1.1) and $g \in A_p$ defined by

$$g(z) = z^p + \sum_{k=p+1}^{\infty} b_k z^k,$$

their Hadamard product (or convolution) [7] of f and g is defined by

$$(f * g)(z) := z^p + \sum_{k=p+1}^{\infty} a_k b_k z^k.$$

A function $f \in H(\mathbb{U})$ is univalent if it is one to one in $\mathbb{U}$. Let S denote the subclass of A consisting of

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functions univalent in $\mathbb{U}$. If a function $f \in A$ maps $\mathbb{U}$ onto a convex domain and f is univalent, then f is called a convex function. Let

$$K = \left\{ f \in A : \Re\left\{ 1 + \frac{zf''(z)}{f'(z)} \right\} > 0,\ z \in \mathbb{U} \right\}$$

denote the class of all convex functions defined in $\mathbb{U}$ and normalized by $f(0) = 0$, $f'(0) = 1$. Let f and F be members of $H(\mathbb{U})$. The function f is said to be subordinate to φ, if there exists a Schwarz function w analytic in $\mathbb{U}$ with

$$w(0) = 0 \quad \text{and} \quad |w(z)| < 1, \quad (z \in \mathbb{U}),$$

such that

$$f(z) = \varphi(w(z)).$$

We denote this subordination by

$$f(z) \prec \varphi(z) \quad \text{or} \quad f \prec \varphi.$$

Furthermore, if the function φ is univalent in $\mathbb{U}$, then we have the following equivalence [5, 13]

$$f(z) \prec \varphi(z) \Longleftrightarrow f(0) = \varphi(0) \text{ and } f(U) \subset \varphi(U).$$

The method of differential subordinations (also known as the admissible functions method) was first introduced by Miller and Mocanu in 1978 [11] and the theory started to develop in 1981 [12]. All the details were captured in a book by Miller and Mocanu in 2000 [13]. Let $\Psi : \mathbb{C}^3 \times \mathbb{U} \longrightarrow \mathbb{C}$ and h be univalent in $\mathbb{U}$. If p is analytic in $\mathbb{U}$ and satisfies the second-order differential subordination

$$\Psi\left(p(z), zp'(z), zp''(z); z\right) \prec h(z), \tag{1.2}$$

then p is called a solution of the differential subordination. The univalent function q is called a dominant of the solution of the differential subordination or more simply dominant, if $p \prec q$ for all p satisfying (1.2). A dominant q_1 satisfying $q_1 \prec q$ for all dominants (1.2) is said to be the best dominant of (1.2).

For functions $f, g \in A_p$, the linear operator $Q^m_{\lambda,p} : A_p \longrightarrow A_p$ $(\lambda \geqslant 0, m \in N \cup \{0\})$ is defined by:

$$\begin{aligned} Q^0_{\lambda,p}(f * g)(z) &= (f * g)(z), \\ Q^1_{\lambda,p}(f * g)(z) &= Q_{\lambda,p}\left((f * g)(z)\right) \\ &= (1 - \lambda)(f * g)(z) + \frac{\lambda z}{p}\left((f * g)(z)\right)' \\ &= z^p + \sum_{k=p+1}^{\infty} \frac{p + \lambda(k - p)}{p} a_k b_k z^k, \\ Q^2_{\lambda,p}(f * g)(z) &= Q_{\lambda,p}\left[Q_\lambda, p(f * g)(z)\right]. \end{aligned}$$

Thus, we get

$$Q^m_{\lambda,p}(f * g)(z) = Q_{\lambda,p}\left(Q^{m-1}_{\lambda,p}(f * g)(z)\right) = z^p + \sum_{k=p+1}^{\infty} \left(\frac{p + \lambda(k - p)}{p}\right)^m a_k b_k z^k, \quad (\lambda \geqslant 0). \tag{1.3}$$

From (1.3) it can be easily seen that

$$\frac{\lambda z}{p}\left(Q^m_{\lambda,p}(f * g)(z)\right)' = Q^{m+1}_{\lambda,p}(f * g)(z) - (1 - \lambda)\, Q^m_{\lambda,p}(f * g)(z), \quad (\lambda \geqslant 0).$$

The operator $Q^m_{\lambda,p}(f * g)$ was introduced and studied by Selveraj and Selvakumaran [19], Aouf and Mostafa [4], and for $p = 1$ was introduced by Aouf and Mostafa [3]. Recent years, Özkan [16], Özkan and Altntaş [17], Lupaş [9], and Lupaş [10] (also see [1, 2]) investigated some applications and results of subordinations of analytic functions given by convolution. Also Bulut [6] used the same techniques by using Komatu integral operator. In some of this study, the results given by Lupaş [10] and Lupaş [9] were generalized. In order to prove our main results we need the following lemmas.

Lemma 1.1 ([8])**.** *Let* h *be convex function with* $h(0) = a$ *and let* $\gamma \in \mathbb{C}^* := \mathbb{C}\backslash\{0\}$ *be a complex number with* $\Re\{\gamma\} \geqslant 0$. *If* $p \in H[a, k]$ *and*

$$p(z) + \frac{1}{\gamma} zp'(z) \prec h(z), \tag{1.4}$$

then

$$p(z) \prec q(z) \prec h(z),$$

where

$$q(z) = \frac{\gamma}{nz^{\gamma/n}} \int_0^z t^{(\gamma/n)-1} h(t)dt.$$

The function q *is convex and is the best dominant of the subordination* (1.4)*.*

Lemma 1.2 ([15])**.** *Let* $\Re\{\mu\} > 0, n \in \mathbb{N}$*, and let*

$$w = \frac{n^2 + |\mu|^2 - |n^2 - \mu^2|}{4n\Re\{\mu\}}.$$

Let h *be an analytic function in* $\mathbb{U}$ *with* $h(0) = 1$ *and suppose that*

$$\Re\left\{1 + \frac{zh''(z)}{h'(z)}\right\} > -w.$$

If

$$p(z) = 1 + p_n z^n + p_{n+1} z^{n+1} + \cdots$$

is analytic in $\mathbb{U}$ *and*

$$p(z) + \frac{1}{\mu} zp'(z) \prec h(z), \tag{1.5}$$

then

$$p(z) \prec q(z),$$

where q *is a solution of the differential equation*

$$q(z) + \frac{n}{\mu} zq'(z) = h(z), \quad q(0) = 1,$$

given by

$$q(z) = \frac{\mu}{nz^{\sigma/n}} \int_0^z t^{(\mu/n)-1} h(t)dt \qquad (z \in \mathbb{U}).$$

Moreover q *is the best dominant of the subordination* (1.5)*.*

Lemma 1.3 ([14])**.** *Let* r *be a convex function in* $\mathbb{U}$ *and let*

$$h(z) = r(z) + n\beta zr'(z), \quad (z \in \mathbb{U}),$$

where $\beta > 0$ *and* $n \in \mathbb{N}$*. If*

$$p(z) = r(0) + p_n z^n + p_{n+1} z^{n+1} + \cdots, \quad (z \in \mathbb{U})$$

is holomorphic in $\mathbb{U}$ *and*

$$p(z) + \beta z p^{'}(z) \prec h(z),$$

then

$$p(z) \prec r(z),$$

and this result is sharp.

In the present paper, making use of the subordination results of [13] and [18] we will prove our main results.

Definition 1.4. Let $\mathfrak{R}_{\lambda,m}(\beta)$ be the class of functions $f \in A$ satisfying

$$\Re\left\{\left(Q_{\lambda}^{m}(f * g)(z)\right)^{'}\right\} > \beta,$$

where $z \in \mathbb{U}$, $0 \leqslant \beta < 1$.

2. Main results

Theorem 2.1. *The set* $\mathfrak{R}_{\lambda,m}(\beta)$ *is convex.*

Proof. Let

$$\left(f_j * g_j\right)(z) = z + \sum_{k=2}^{\infty} a_{k,j} b_{k,j} z^k \qquad (z \in \mathbb{U}; j = 1, ...m)$$

be in the class of $\mathfrak{R}_{\lambda,m}(\beta)$. Then, by Definition 1.4, we get

$$\Re\left\{\left(Q_{\lambda}^{m}(f_j * g_j)(z)\right)^{'}\right\} = \Re\left\{1 + \sum_{k=2}^{\infty} a_{k,j} b_{k,j} k z^{k-1}\right\} > \beta.$$

For any positive numbers $\lambda_1, \lambda_2, ..., \lambda_\ell$ such that

$$\sum_{j=1}^{\ell} \lambda_j = 1,$$

we have to show that the function

$$h(z) = \sum_{j=1}^{\ell} \lambda \left(f_j * g_j\right)(z)$$

is member of $\mathfrak{R}_{\lambda,m}(\beta)$, that is,

$$\Re\left\{\left(Q_{\lambda}^{m} h(z)\right)^{'}\right\} > \beta. \tag{2.1}$$

Thus, we have

$$Q_{\lambda}^{m} h(z) = z + \sum_{k=2}^{\infty} \left(1 + \lambda(k-1)\right)^{m} \left(\sum_{j=1}^{\ell} \lambda_j a_{k,j} b_{k,j}\right) z^k. \tag{2.2}$$

If we differentiate (2.2) with respect to z, then we obtain

$$\left(Q_{\lambda}^{m} h(z)\right)^{'} = 1 + \sum_{k=2}^{\infty} \left(1 + \lambda(k-1)\right)^{m} \left(\sum_{j=1}^{\ell} \lambda_j a_{k,j} b_{k,j}\right) k z^{k-1}$$

$$= 1 + \sum_{j=1}^{\ell} \lambda_j \sum_{k=2}^{\infty} (1 + \lambda (k-1))^m a_{k,j} b_{k,j} k z^{k-1}.$$

Thus, we have

$$\begin{aligned} \Re \left\{ (Q_\lambda^m h(z))' \right\} &= 1 + \sum_{j=1}^{\ell} \lambda_j \Re \left\{ \sum_{k=2}^{\infty} (1 + \lambda (k-1))^m a_{k,j} b_{k,j} k z^{k-1} \right\} \\ &> 1 + \sum_{j=1}^{\ell} \lambda_j (\beta - 1) \\ &= \beta. \end{aligned}$$

Thus, the inequality (2.1) holds and we obtain desired result. □

Theorem 2.2. *Let* q *be convex function in* $\mathbb{U}$ *with* $q(0) = 1$ *and let*

$$h(z) = q(z) + \frac{1}{\gamma + 1} z q'(z) \quad (z \in \mathbb{U}),$$

where γ *is a complex number with* $\Re\{\gamma\} > -1$. *If* $f \in \mathfrak{R}_{\sigma,\theta}(\beta)$ *and* $\mathfrak{F} = \Upsilon_\gamma(f * g)$, *where*

$$\mathfrak{F}(z) = \Upsilon_\gamma(f * g)(z) = \frac{\gamma + 1}{z^\gamma} \int_0^z t^{\gamma - 1} (f * g)(t) dt, \tag{2.3}$$

then,

$$(Q_\lambda^m (f * g)(z))' \prec h(z) \tag{2.4}$$

implies

$$(Q_\lambda^m \mathfrak{F}(z))' \prec q(z),$$

and this result is sharp.

Proof. From the equality (2.3) we can write

$$z^\gamma \mathfrak{F}(z) = (\gamma + 1) \int_0^z t^{\gamma - 1} (f * g)(t) dt, \tag{2.5}$$

by differentiating the equality (2.5) with respect to z, we obtain

$$(\gamma) \mathfrak{F}(z) + z \mathfrak{F}'(z) = (\gamma + 1) (f * g)(z).$$

If we apply the operator Q_λ^m to the last equation, then we get

$$(\gamma) Q_\lambda^m \mathfrak{F}(z) + z (Q_\lambda^m \mathfrak{F}(z))' = (\gamma + 1) Q_\lambda^m (f * g)(z). \tag{2.6}$$

If we differentiate (2.6) with respect to z, we can obtain

$$(Q_\lambda^m \mathfrak{F}(z))' + \frac{1}{\gamma + 1} z (Q_\lambda^m \mathfrak{F}(z))'' = (Q_\lambda^m f(z))'. \tag{2.7}$$

By using the differential subordination given by (2.4) in the equality (2.7), we have

$$(Q_\lambda^m \mathfrak{F}(z))' + \frac{1}{\gamma + 1} z (Q_\lambda^m \mathfrak{F}(z))'' \prec h(z). \tag{2.8}$$

Now, we define

$$p(z)=\left(Q_{\lambda}^{m}\mathfrak{F}(z)\right)'. \tag{2.9}$$

Then by a simple computation we get

$$p(z)=\left[z+\sum_{k=2}^{\infty}(1+\lambda(k-1))^{m}\frac{\gamma+1}{\gamma+k}a_k b_k z^k\right]'=1+p_1z+p_2z+\cdots,\qquad (p\in H[1,1]).$$

Using the equation (2.9) in the subordination (2.8), we obtain

$$p(z)+\frac{1}{\gamma+1}zp'(z)\prec h(z)=q(z)+\frac{1}{\gamma+1}zq'(z).$$

If we use Lemma 1.2, then we get

$$p(z)\prec q(z).$$

So we obtain the desired result and q is the best dominant. □

Example 2.3. *If we choose in Theorem 2.2*

$$\gamma=i+1,\quad q(z)=\frac{1}{1-z},$$

thus we get

$$h(z)=\frac{(i+2)-(i+1)z}{(i+2)(1-z)^2}.$$

If $(f*g)\in\mathfrak{R}_{\lambda,m}(\beta)$ *and* $\mathfrak{F}$ *is given by*

$$\mathfrak{F}(z)=\Upsilon_i(f*g)(z)=\frac{i+2}{z^{i+1}}\int_0^z t^i(f*g)(t)dt,$$

then by Theorem 2.2, we obtain

$$\left(Q_{\lambda}^{m}f(z)\right)'\prec h(z)=\frac{(i+2)-(i+1)z}{(i+2)(1-z)^2}\Longrightarrow\left(Q_{\lambda}^{m}\mathfrak{F}(z)\right)'\prec q(z))=\frac{1}{1-z}.$$

Theorem 2.4. *Let* $\Re\{\gamma\}>-1$ *and let*

$$w=\frac{1+|\gamma+1|^2-\left|\gamma^2+2\gamma\right|}{4\Re\{\gamma+1\}}.$$

Let h *be an analytic function in* $\mathbb{U}$ *with* $h(0)=1$ *and assume that*

$$\Re\left\{1+\frac{zh''(z)}{h'(z)}\right\}>-w.$$

If $f*g\in\mathfrak{R}_{\lambda,m}(\beta)$ *and* $\mathfrak{F}=\Upsilon_{\gamma}^{\delta}(f*g)$*, where* $\mathfrak{F}$ *is defined by equation (2.3), then*

$$\left(Q_{\lambda}^{m}(f*g)(z)\right)'\prec h(z) \tag{2.10}$$

implies

$$\left(Q_{\lambda}^{m}\mathfrak{F}(z)\right)'\prec q(z),$$

where q *is the solution of the differential equation*

$$h(z) = q(z) + \frac{1}{\gamma+1} zq^{'}(z), \quad q(0) = 1,$$

given by

$$q(z) = \frac{\gamma+1}{z^{\gamma+1}} \int\limits_0^z t^{\gamma}(f*g)(t)dt.$$

Moreover q *is the best dominant of the subordination* (2.10)*.*

Proof. If we choose $n = 1$ and $\mu = \gamma + 1$ in Lemma 1.2, then the proof is obtained by means of the proof of Theorem 2.4. □

Letting

$$h(z) = \frac{1 + (2\beta - 1)z}{1+z}, \quad 0 \leqslant \beta < 1$$

in Theorem 2.4, we obtain the following result.

Corollary 2.5. *If* $0 \leqslant \beta < 1$, $0 \leqslant \xi < 1$, $\lambda \geqslant 0$, $\mathfrak{R}\{\gamma\} > -1$, *and* $\mathfrak{F} = \Upsilon_\gamma(f*g)$ *is defined by the equation* (2.3)*, then*

$$\Upsilon_\gamma\left(\mathfrak{R}_{\lambda,m}(\beta)\right) \subset \mathfrak{R}_{\lambda,m}(\xi),$$

where

$$\xi = \min_{|z|=1} \mathfrak{R}\{q(z)\} = \xi(\gamma, \beta)$$

and this result is sharp. Also,

$$\xi = \xi(\gamma, \beta) = (2\beta - 1) + 2(\gamma+1)(1-\beta)\tau(\gamma), \tag{2.11}$$

where

$$\tau(\gamma) = \int_0^1 \frac{t^\gamma}{1+t} dt. \tag{2.12}$$

Proof. Let $f \in \mathfrak{R}_{\lambda,m}(\beta)$. Then from Definition 1.4 it is known that

$$\mathfrak{R}\left\{ \left(Q_\lambda^m (f*g)(z)\right)^{'} \right\} > \beta,$$

which is equivalent to

$$\left(Q_\lambda^m (f*g)(z)\right)^{'} \prec h(z).$$

By using Theorem 2.2, we have

$$\left(Q_\lambda^m \mathfrak{F}(z)\right)^{'} \prec q(z).$$

If we take

$$h(z) = \frac{1 + (2\beta - 1)z}{1+z}, \quad 0 \leqslant \beta < 1,$$

then h is convex and by Theorem 2.4, we obtain

$$\left(Q_\lambda^m \mathfrak{F}(z)\right)^{'} \prec q(z) = \frac{\gamma+1}{z^{\gamma+1}} \int_0^z t^\gamma \frac{1+(2\beta-1)t}{1+t} dt = (2\beta - 1) + 2\frac{(1-\beta)(\gamma+1)}{z^{\gamma+1}} \int_0^z \frac{t^\gamma}{1+t} dt.$$

On the other hand if $\mathfrak{R}\{\gamma\} > -1$, then from the convexity of q and the fact that $q(\mathbb{U})$ is symmetric with respect to the real axis, we get

$$\mathfrak{R}\left\{ \left(Q_\lambda^m \mathfrak{F}(z)\right)^{'} \right\} \geqslant \min_{|z|=1} \mathfrak{R}\{q(z)\} = \mathfrak{R}\{q(1)\} = \xi(\gamma, \beta) = 2\beta - 1 + 2(1-\beta)(\gamma+1)\tau(\gamma), \tag{2.13}$$

where $\tau(\gamma)$ is given by equation (2.12). From inequality (2.13), we get

$$\Upsilon_{\gamma}\left(\mathfrak{R}_{\lambda,m}(\beta)\right) \subset \mathfrak{R}_{\lambda,m}(\xi),$$

where ξ is given by (2.11). □

Theorem 2.6. *Let* q *be a convex function with* $q(0) = 1$ *and* h *a function such that*

$$h(z) = q(z) + zq^{'}(z), \quad (z \in \mathbb{U}).$$

If $f \in A$*, then the following subordination*

$$\left(Q_{\lambda}^{m}(f * g)(z)\right)^{'} \prec h(z) = q(z) + zq^{'}(z) \tag{2.14}$$

implies that

$$\frac{\left(Q_{\lambda}^{m}(f * g)(z)\right)}{z} \prec q(z),$$

and the result is sharp.

Proof. Let

$$p(z) = \frac{\left(Q_{\lambda}^{m}(f * g)(z)\right)}{z}. \tag{2.15}$$

Differentiating (2.15), we have

$$\left(Q_{\lambda}^{m}(f * g)(z)\right)^{'} = p(z) + zp^{'}(z), \quad (z \in \mathbb{U})$$

and thus (2.14) becomes

$$p(z) + zp^{'}(z) \prec h(z) = q(z) + zq^{'}(z).$$

Hence by applying Lemma1.3, we conclude that

$$p(z) \prec q(z),$$

that is,

$$\frac{\left(Q_{\lambda}^{m}(f * g)(z)\right)}{z} \prec q(z),$$

and this result is sharp. □

Theorem 2.7. *Let* q *be a convex function with* $q(0) = 1$ *and* h *the function*

$$h(z) = q(z) + zq^{'}(z) \quad (z \in \mathbb{U}).$$

If $m \in \mathbb{N}, f \in A$ *and verifies the differential subordination*

$$\left(\frac{Q_{\lambda}^{m+1}(f * g)(z)}{Q_{\lambda}^{m}(f * g)(z)}\right)^{'} \prec h(z), \tag{2.16}$$

then

$$\frac{Q_{\lambda}^{m+1}(f * g)(z)}{Q_{\lambda}^{m}(f * g)(z)} \prec q(z),$$

and this result is sharp.

Proof. For the function $f \in A$, given by the equation (1.1), we have

$$Q_{\lambda}^{m}(f * g)(z) = z + \sum_{k=2}^{\infty} (1 + \lambda(k-1))^{m} \frac{\gamma + 1}{k + \gamma} a_k b_k z^k, \quad (z \in \mathbb{U}).$$

Let us consider

$$p(z)=\frac{Q_{\lambda}^{m+1}(f*g)(z)}{Q_{\lambda}^{m}(f*g)(z)}=\frac{z+\sum\limits_{k=2}^{\infty}(1+\lambda(k-1))^{m+1}\frac{\gamma+1}{k+\gamma}a_kb_kz^k}{z+\sum\limits_{k=2}^{\infty}(1+\lambda(k-1))^{m}\frac{\gamma+1}{k+\gamma}a_kb_kz^k}$$
$$=\frac{1+\sum\limits_{k=2}^{\infty}(1+\lambda(k-1))^{m+1}\frac{\gamma+1}{k+\gamma}a_kb_kz^{k-1}}{1+\sum\limits_{k=2}^{\infty}(1+\lambda(k-1))^{m}\frac{\gamma+1}{k+\gamma}a_kb_kz^{k-1}}.$$

We get

$$(p(z))^{'}=\frac{(Q_{\lambda}^{m+1}(f*g)(z))^{'}}{Q_{\lambda}^{m}(f*g)(z)}-p(z)\frac{(Q_{\lambda}^{m}(f*g)(z))^{'}}{Q_{\lambda}^{m}(f*g)(z))}$$

and

$$p(z)+zp^{'}(z)=\left(\frac{zQ_{\lambda}^{m+1}(f*g)(z)}{Q_{\lambda}^{m}(f*g)(z)}\right)^{'}\quad(z\in\mathbb{U}).$$

Thus, the relation (2.16) becomes

$$p(z)+zp^{'}(z)\prec h(z)=q(z)+zq^{'}(z),\quad(z\in\mathbb{U}),$$

and by using Lemma 1.3, we obtain

$$p(z)\prec q(z),$$

that is,

$$\frac{\left(Q_{\lambda}^{m}(f*g)(z)\right)}{z}\prec q(z).$$

□

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