Available online at www.isr-publications.com/jnsa
J. Nonlinear Sci. Appl., 10 (2017), 1044–1050
Research Article

ISSN: 2008-1898
Journal of Nonlinear Sciences and Applications
Journal Homepage: www.tjnsa.com - www.isr-publications.com/jnsa

New dynamical behavior of two waves for $(2+1)$-dimensional Broer-Kaup equation

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Communicated by X. J. Yang

Abstract

New exact solutions including periodic breather wave, kink breather wave and doubly breather wave solutions are obtained for (2+1)D BK equation by using Painleve analysis, variable separation approach, the homoclinic test method and generalized CK method via the linearization of equation, variable separation and equivalent transformation, respectively. The dynamical behavior and interaction between different waves are investigated. These results enrich the dynamic features of higher dimensional nonlinear system.

Keywords: Broer-Kaup equation, coincidence point, equivalent transformation, variable separation, generalized CK method, dynamical behavior.

2010 MSC: 35Q51, 35Q53, 35Q99.

1. Introduction

Many nonlinear phenomena in nature and human society are usually characterized by nonlinear evolution equations. Searching for an analytical exact solution to a nonlinear system becomes one of the central themes of perpetual interest in nonlinear science. Various methods for obtaining exact solutions of a nonlinear system have been proposed, such as the Hirota bilinear method, the homogeneous balance method, inverse scattering method, Backlund transformation, variable separation approach, Painleve analysis method, Darboux transformation, symmetry reduction method, homoclinic test method, the extended homoclinic test method [2] and so on. The $(2+1)$-dimensional Broer-Kaup Equation ($(2+1)$D BK for short) comes from the constraints of the KP equation and it is of importance in mathematical physics field. Many researchers pay more and more attention to search for analytical exact solution to (2+1)D BK Equation because of its rich physical connotation [1, 3–14, 16–24]. By means of the homogeneous balance method [8, 16–19, 21, 24] solitary wave solutions, exact multi-soliton solutions and soliton-like solutions of the BK equation were obtained. Meanwhile, doubly periodic wave solutions, folded solitary wave solutions, non-Lie symmetry groups and new exact solutions were derived [6] by using variable separation

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doi:10.22436/jnsa.010.03.15

Received 2016-09-14

approach [1, 3, 11, 12, 15, 22], Painleve analysis method [5, 20] and generalized Riccati mapping method [10, 23]. In this work, we consider the following $(2+1)$D BK Equation:

$$\begin{cases} u_{ty} - u_{xxy} + 2(uu_x)_y + 2v_{xx} = 0, \\ v_t + v_{xx} + 2(uv)_x = 0. \end{cases} \tag{1.1}$$

We will study on the new exact solutions including periodic breather wave, kink breather wave and doubly breather wave solutions by Painleve analysis, variable separation approach, homoclinic test method, generalized CK method and equivalent transformation via the linearization of equation, respectively. The dynamical behavior and interaction between different waves are investigated.

2. The linearization of Equation (1.1)

Substituting $v = u_y$ into (1.1), we get

$$u_{ty} + u_{xxy} + (u^2)_{xy} = 0,$$

integrating with respect to y and set the integration constant to zero, we can obtain

$$u_t + u_{xx} + (u^2)_x = 0. \tag{2.1}$$

Equation (2.1) is a Burgers type equation. Based on the Painleve analysis, we take the following transformation

$$u = (\ln \varphi)_x. \tag{2.2}$$

Substituting (2.2) into (2.1) and integrating with respect to x, we get the following second-order LPDE.

$$\varphi_t + \varphi_{xx} = 0, \tag{2.3}$$

where φ is function of variable (x, y, t) to be determined. Based on the solutions of (2.3), we can obtain the exact solutions for the BK system.

3. Exact solutions of the Equation (1.1)

From the transformation (2.2), we assume the variable separation solutions in this form

$$\varphi = f(x, t) + g(y), \tag{3.1}$$

where $f(x, t)$ is function of (x, t) to be determined later and $g(y)$ is arbitrary function of y, respectively. Substituting (3.1) into (2.3), we have

$$f_t(x, t) + f_{xx}(x, t) = 0. \tag{3.2}$$

This is a famous heat conduction equation. Next, we will derive two types of variable separation solutions of (3.2).

3.1. Variable separation solutions with sum-form

Now, we suppose the solution of (3.2) as

$$f(x, t) = p(x) + q(t). \tag{3.3}$$

Substituting (3.3) into (2.3), we get the following results:

$$q(t) = c_0 t, \quad p(x) = \frac{-c_0 x^2}{2} + c_1 x + c_2, \quad \varphi_1 = \frac{c_0 x^2}{2} - c_1 x - c_0 t + c_2 + g(y), \tag{3.4}$$

where c_0, c_1, c_2 are arbitrary constants.

3.2. Variable separation solutions with product-form

We suppose the solution of (3.2) as $f(x, t) = p(x)q(t)$. Substituting it into (3.2), we get the following results:

$$p(x) = c_1 e^{c_0 x} + c_2 e^{-c_0 x},$$

or

$$p(x) = \sinh(c_0x), \text{ or } p(x) = \cosh(c_0x), \quad q(t) = e^{-c_0^2t},$$
$$\varphi_2 = e^{-c_0^2t}\cosh(c_0x) + g(y). \tag{3.5}$$

Based on linear superposition principle of solutions of linear equations, the following superposition solutions are available

$$\varphi_3 = c_3(\frac{c_0x^2}{2} - c_1x - c_0t + c_2) + c_4(e^{-c_0^2t}\cosh(c_0x)) + g(y), \tag{3.6}$$

where $c_i \in R(i = 0, \cdots, 4)$. Substituting (3.4), (3.5), (3.6) into (2.2) and notice $v = u_y$, we obtain the exact solutions of (1.1) as follows:

$$u_1 = \frac{c_0x - c_1}{\frac{c_0x^2}{2} - c_1x - c_0t + c_2 + g(y)}, \quad v_1 = \frac{-(c_0x - c_1)g'(y)}{[\frac{c_0x^2}{2} - c_1x - c_0t + c_2 + g(y)]^2},$$
$$u_2 = \frac{c_0e^{-c_0^2t}\sinh(c_0x)}{e^{-c_0^2t}\cosh(c_0x) + g(y)}, \quad v_2 = \frac{-c_0e^{-c_0^2t}\sinh(c_0x)g'(y)}{[e^{-c_0^2t}\cosh(c_0x) + g(y)]^2},$$
$$\begin{cases} u_3 = \frac{c_0x - c_1 - c_0e^{-c_0^2t}\sinh(c_0x)}{\frac{c_0x^2}{2} - c_1x - c_0t + c_2 + e^{-c_0^2t}\cosh(c_0x) + g(y)}, \\ v_3 = \frac{-[c_0x - c_1 - c_0e^{-c_0^2t}\sinh(c_0x)]g'(y)}{[\frac{c_0x^2}{2} - c_1x - c_0t + c_2 + e^{-c_0^2t}\cosh(c_0x) + g(y)]^2}. \end{cases}$$

3.3. *Interaction between periodic breather wave and single solitary wave*

Based on the homoclinic test method, we suppose

$$\varphi = e^{k_1x + s_1y + c_1t + r_1} + a\cos(k_2x + s_2y + c_2t + r_2) + b\cosh(k_3x + s_3y + c_3t + r_3), \tag{3.7}$$

where $a, b, k_i, s_i, c_i, r_i (i = 1, 2, 3)$ are constants to be determined later. Substituting (3.7) into (2.3), we obtain the set of algebraic equations for $a, b, k_i, s_i, c_i, r_i (i = 1, 2, 3)$

$$k_2^3 + k_1^2k_2 + c_1k_2 + c_2k_1 = 0, \cdots, c_3k_2 + c_2k_3 = 0, \quad c_3k_3 - c_2k_2 = 0. \tag{3.8}$$

The solution of (3.8) is $c_1 = -k_1^2 + k_3^2$, when $a = c_3 = 0$. Thus we have

$$\varphi_4 = e^{k_1x + s_1y - (k_1^2 - k_3^2)t + r_1} + b\cosh(k_3x + s_3y + r_3). \tag{3.9}$$

Substituting (3.9) into (2.2) and $v = u_y$ in turn, we can derive the following exact solutions of (1.1):

$$\begin{cases} u_4 = \frac{k_1e^{k_1x + s_1y - (k_1^2 - k_3^2)t + r_1} + bk_3\sinh(k_3x + s_3y + r_3)}{e^{k_1x + s_1y - (k_1^2 - k_3^2)t + r_1} + b\cosh(k_3x + s_3y + r_3)}, \\ v_4 = \frac{s_1e^{k_1x + s_1y - (k_1^2 - k_3^2)t + r_1} + bs_3\sinh(k_3x + s_3y + r_3)}{[e^{k_1x + s_1y - (k_1^2 - k_3^2)t + r_1} + b\cosh(k_3x + s_3y + r_3)]^2}. \end{cases}$$

3.4. *Equivalence transformation of Equation* (2.3)

Based on generalized CK method, we suppose φ as

$$\varphi = \alpha + \beta f(\xi, \eta), \tag{3.10}$$

where $\alpha = \alpha(x, y, t), \beta = \beta(x, y, t), \xi = \xi(x, y, t), \eta = \eta(x, y, t)$ are functions of (x, y, t) to be determined later and $f(\xi, \eta)$ is a solution of linear equations

$$f_\eta + f_{\xi\xi} = 0. \tag{3.11}$$

Substituting (3.10) and (3.11) into (2.3), we obtain

$$\alpha = g(y), \quad \beta = 1, \quad \xi = h(y)x, \quad \eta = h^2(y)t, \tag{3.12}$$

where $h(y), g(y)$ are arbitrary functions of y. Substituting (3.12) into (3.10), we obtain equivalence transformation of (2.3)

$$\varphi = f(h(y)x, h^2(y)t) + g(y).$$

Base on transformation (3.10) of (2.3), we obtain invariant form solutions,

$$\varphi_5 = \frac{c_0h^2(y)x^2}{2} - c_1h(y)x - c_0h^2(y)t + c_2 + g(y), \tag{3.13}$$

$$\varphi_6 = h(y)\cosh(c_0h(y)x)e^{-c_0h^2(y)t} + g(y), \tag{3.14}$$

$$\varphi_7 = c_3[\frac{c_0h^2(y)x^2}{2} - c_1h(y)x - c_0h^2(y)t + c_2] + c_4h(y)e^{k^2h^2(y)t}\cosh(c_0h(y)x) + g(y). \tag{3.15}$$

We can derive the following exact solutions of (1.1) from (3.13), (3.14), (3.15):

$$u_5 = \frac{c_0h^2(y)x - c_1h(y)}{\frac{c_0h^2(y)x^2}{2} - c_1h(y)x - c_0h^2(y)t + c_2 + g(y)}, \quad v_5 = [\frac{c_0h^2(y)x - c_1h(y)}{\frac{c_0h^2(y)x^2}{2} - c_1h(y)x - c_0h^2(y)t + c_2 + g(y)}]_y,$$

$$u_6 = \frac{c_0h^2(y)x - c_1h(y)}{\frac{c_0h^2(y)x^2}{2} - c_1h(y)x - c_0h^2(y)t + c_2 + g(y)}, \quad v_6 = [\frac{c_0h^2(y)x - c_1h(y)}{\frac{c_0h^2(y)x^2}{2} - c_1h(y)x - c_0h^2(y)t + c_2 + g(y)}]_y,$$

$$\begin{cases} u_7 = \frac{c_3[c_0h^2(y)x - c_1h(y)] - c_4kh^2(y)e^{k^2h^2(y)t}\sin(kh(y)x}{c_3[c_0h^2(y)(\frac{x^2}{2} - t) - c_1h(y)x + c_2] + c_4h(y)e^{k^2h^2(y)t}\cos(kh(y)x + g(y)}, \\ v_7 = [\frac{c_3[c_0h^2(y)x - c_1h(y)] - c_4kh^2(y)e^{k^2h^2(y)t}\sin(kh(y)x}{c_3[c_0h^2(y)(\frac{x^2}{2} - t) - c_1h(y)x + c_2] + c_4h(y)e^{k^2h^2(y)t}\cos(kh(y)x + g(y)}]_y. \end{cases}$$

4. Dynamic behavior analysis of solution of Equation (1.1)

In this section, we mainly discuss the dynamical behavior and localized structure of solution u_7 and v_7 for (1.1). u_7 and v_7 describe the behavior of inelastic collision between line soliton $g(y)$ and different kink-type breather solitons. At the same time, they show different structures of solitons in different regions.

4.1 Interaction between breather wave and kink wave.

For example, if we choose $g(y) = y^6 + 1, h(y) = \tanh(y) + 1, k = 0.03, c_i = 1 (i = 0, \cdots, 4)$, we can obtain the localized structure of u_7 and v_7 at time $t = -0.8, t = 0, t = 0.8$, respectively. They are expressed by Figure 1 in turn.

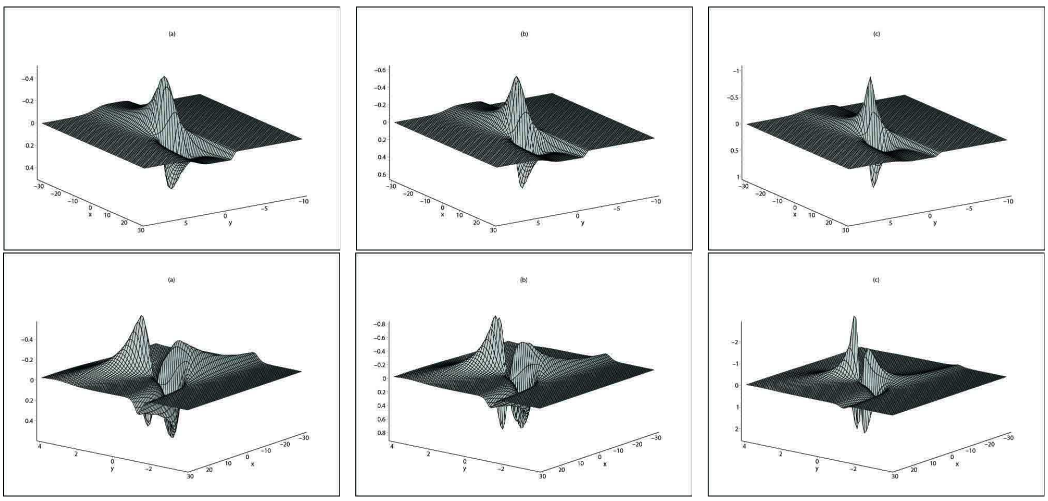

Figure 1: Row 1 Shows that u_7 is kink-type breather wave and it describes the interaction between kink wave and breather wave with different directions. Row 2 Shows that v_7 is doubly breather-type wave and it describes the interaction between kink wave and doubly breather-type wave with different directions.

4.2 Interaction between line solition and dromion solition.

If we choose $g(y) = y^4 + 1, h(y) = y\sin(\ln(y^2)), k = 0.11, c_i = 1(i = 0, \cdots, 4)$, we can obtain the localized structure of u_7 and v_7 at time $t = -3.6, t = 0, t = 3.6$, respectively. They are expressed by Figure 2 in turn.

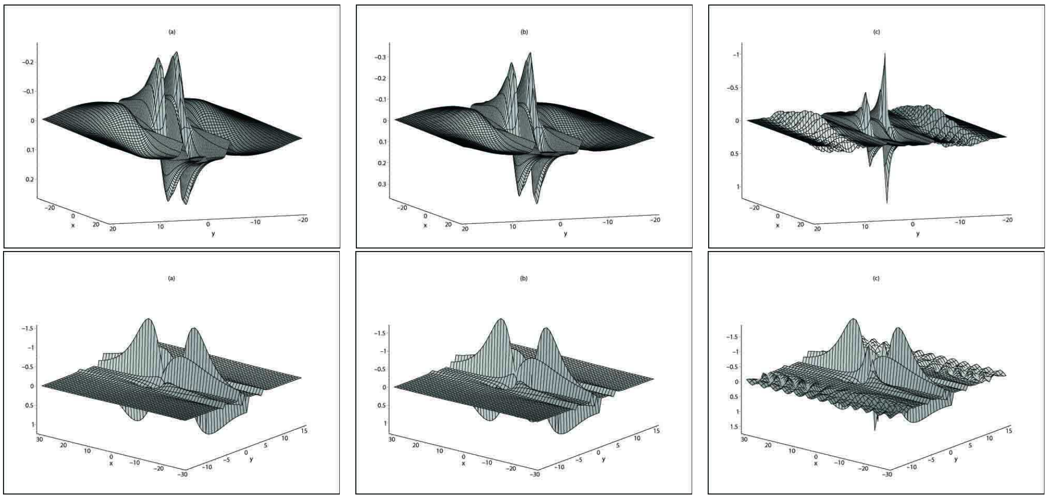

Figure 2: Row 1 and Row 2 show interaction between line solition and dromion solition, which is inelastic collision, and exhibit the evolution of the two solitons from the chaotic line solition to the chaotic structure.

4.3 Interaction between line solition and compaction.

If we choose $g(y) = \text{sech}(y) + 1, h(y) = \sin(y), k = 0.3, c_i = 1(i = 0, \cdots, 4)$, we can obtain the localized structure of u_7 and v_7 at time $t = -0.5, t = 0, t = 0.5$, respectively. They are expressed by Figure 3 in turn.

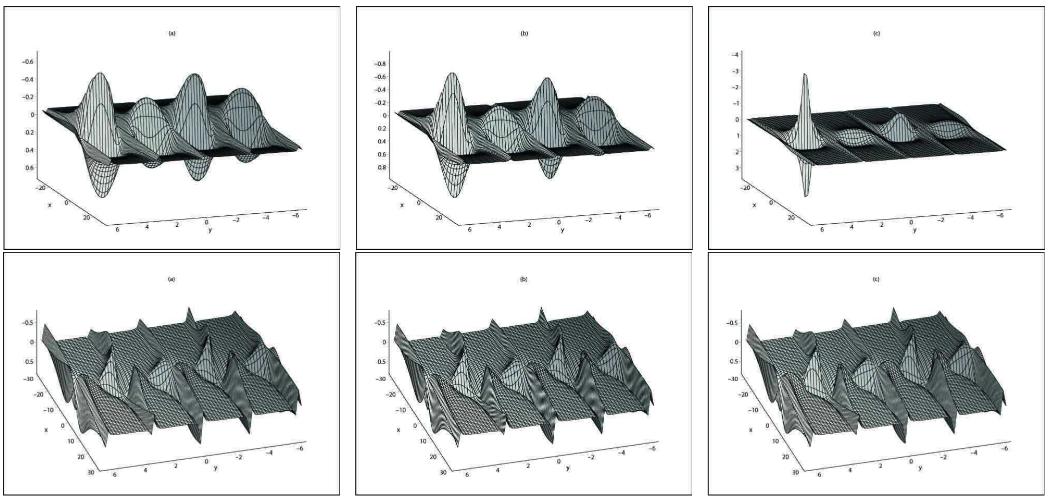

Figure 3: Row 1 and Row 2 show interaction between line solition and compaction, which are inelastic collision, and exhibit degradation of chaotic line soliton, but they have not chaotic structure.

5. Conclusions

In this paper, we study the linearization of $(2+1)$D BK Equation and obtain some new exact solutions including periodic breather-type wave, kink breather wave and doubly breather wave using Painleve analysis, variable separation approach, the homoclinic test method and generalized CK method. Furthermore, we discuss its dynamical behavior and localized structure of interaction between breather wave and kink wave, line solition and dromion solution, line solition and compaction, etc. These results enrich the dynamic significance of the higher dimensional nonlinear system.

Acknowledgment

This work was supported by Chinese Natural Science Foundation Grant No. 11204250 , No. 11202175 and No. 11361048.

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