



## Applications of Mann's method to the split common fixed point problem

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### Abstract

In the present paper, we suggest a new fixed point method for solving the split common fixed point problem of directed operators. We present an iterative algorithm based on Mann's method. We prove that the presented algorithm converges weakly to a solution of the split common fixed point problem of directed operators. ©2017 All rights reserved.

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### 1. Introduction

Many problems in nonlinear analysis can be reformulated as a problem of finding a fixed point of a nonlinear operator. Iterative methods for finding fixed points of nonlinear operators have received vast investigation: Browder et al. [3], Mainge [11], Nakajo et al. [13], Shioji et al. [15], Xu [20], and Yao et al. [24, 25]. An important way for finding fixed points of nonlinear operators is Mann's method [19] which generates a sequence  $\{x^k\}$  by the form

$$\text{(Mann's method:)} \quad x^{k+1} = (1 - \alpha_k)x^k + \alpha_k \mathcal{T}x^k, k \geq 0, \quad (1.1)$$

where  $\{\alpha_k\}_{k \geq 0} \subset (0, 1)$  and  $\mathcal{T}$  is a nonlinear operator.

Mann's method and its variant form have been studied extensively. Especially, in [10, 16, 22], modified Mann's has been introduced and studied. The aim of the present article is to apply Mann's method for solving the split common fixed point problem. To begin with, let us recall some related notations and existing results in the literature.

Throughout, assume that  $\mathcal{H}_1$  and  $\mathcal{H}_2$  are two real Hilbert spaces,  $\langle \cdot, \cdot \rangle$  denotes the associated scalar product and  $\| \cdot \|$  stands for the corresponding norm. Let  $\mathcal{U} : \mathcal{H}_1 \rightarrow \mathcal{H}_1$  and  $\mathcal{V} : \mathcal{H}_2 \rightarrow \mathcal{H}_2$  be two nonlinear

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operators. Denote the fixed point sets of  $\mathcal{U}$  and  $\mathcal{V}$  by  $\text{Fix}(\mathcal{U})$  and  $\text{Fix}(\mathcal{V})$ , respectively. Let  $\mathcal{A} : \mathcal{H}_1 \rightarrow \mathcal{H}_2$  be a bounded linear operator with its adjoint  $\mathcal{A}^*$ .

Recall that the split common fixed point problem is to find an element  $u^\sharp \in \mathcal{H}_1$  satisfying

$$\text{(SCFPP:)} \quad u^\sharp \in \text{Fix}(\mathcal{U}) \quad \text{and} \quad \mathcal{A}u^\sharp \in \text{Fix}(\mathcal{V}), \quad (1.2)$$

which was firstly introduced by Censor and Segal in [6].

**An important special case:** If  $\mathcal{U}$  and  $\mathcal{V}$  are the projections, then SCFPP (1.2) reduces to the split feasibility problem, which is to find a point  $u^\sharp$  such that

$$\text{(SFP:)} \quad u^\sharp \in \mathcal{C} \quad \text{and} \quad \mathcal{A}u^\sharp \in \mathcal{Q}, \quad (1.3)$$

where  $\mathcal{C} \subset \mathcal{H}_1$  and  $\mathcal{Q} \subset \mathcal{H}_2$  are two nonempty closed convex sets.

Such problems arise in the field of intensity-modulated radiation therapy when one attempts to describe physical dose constraints and equivalent uniform dose constraints within a single model ([5]). Some efforts are being extended to solve the split feasibility problem: Byne [4], Wang and Xu [18], and Xu [21].

Note that SCFPP (1.2) can be translated to solve a fixed point problem. In fact, SCFPP (1.2) equals the following fixed point equation

$$\text{(FPE:)} \quad x^\sharp = \mathcal{U}(x^\sharp - \mu \mathcal{A}^*(\mathcal{J} - \mathcal{V})\mathcal{A}x^\sharp), \quad (1.4)$$

where  $\mu > 0$  is a constant.

By using FPE (1.4), some iterative algorithms have been presented for solving SCFPP (1.2), see, e.g., Chang et al. [7], He and Du [9], Moudafi [12]. Very recently, Wang [17] presented the following iterative algorithm which is generated by the following iteration

$$x^{k+1} = x^k - \mu[(x^k - \mathcal{U}x^k) + \mathcal{A}^*(\mathcal{J} - \mathcal{V})\mathcal{A}x^k], k \geq 0,$$

where the stepsize  $\mu$  is in the interval  $(0, \frac{1}{\max\{1, \|\mathcal{A}\|^2\}})$  and  $\mathcal{U}$  and  $\mathcal{V}$  are directed operators. They proved the weak convergence of the sequence  $\{x^k\}$  to the solution of SCFPP (1.2).

On the other hand, Yao et al. [23] introduced the following fixed point equation to solve SCFPP (1.2)

$$x^\sharp = \mathcal{U}x^\sharp - \mu \mathcal{A}^*(\mathcal{J} - \mathcal{V})\mathcal{A}x^\sharp, \quad (1.5)$$

where  $\mu > 0$  is a constant and  $\mathcal{U}$  and  $\mathcal{T}$  are directed operators.

It is our main purpose in this paper that we extend fixed point equation (1.5) to a general form with variant parameters. Subsequently, we apply fixed point equation (1.5) and Mann's method (1.1) for solving the split common fixed point problem (1.2). Weak convergence theorem is given under some mild assumptions.

## 2. Preliminaries

Let  $\mathcal{H}$  be a real Hilbert space, and  $\mathcal{C} \subset \mathcal{H}$  a nonempty closed convex set. Recall that an operator  $\mathcal{U} : \mathcal{C} \rightarrow \mathcal{C}$  is said to be directed if

$$\|\mathcal{U}x - x^\sharp\|^2 \leq \|x - x^\sharp\|^2 - \|\mathcal{U}x - x\|^2,$$

or equivalently,

$$\|x - \mathcal{U}x\|^2 \leq \langle x - \mathcal{U}x, x - x^\sharp \rangle, \quad (2.1)$$

for all  $x \in \mathcal{C}$  and  $x^\sharp \in \text{Fix}(\mathcal{U})$ .

The class of directed operators was introduced and investigated by Bauschke and Combettes in [2] and by Combettes in [8]. The class of directed operators is an important class since it includes the orthogonal projections and the subgradient projectors which are fundamental in the convex optimization

[1] and it allows a complete characterization of Fejér-monotonicity. Recall that a sequence  $\{x^k\}$  is called Fejér-monotone with respect to a given nonempty set  $\Omega$  if for every  $u \in \Omega$ ,

$$\|x^{k+1} - u\| \leq \|x^k - u\| \text{ for all } k \geq 0.$$

An operator  $\mathcal{U}$  is said to be demiclosed, if for any sequence  $x^k$  which weakly converges to  $\tilde{x}$ , and if the sequence  $\mathcal{U}(x^k)$  strongly converges to  $z$ , then  $\mathcal{U}(\tilde{x}) = z$ .

In any Hilbert space  $\mathcal{H}$ , the following conclusion holds:

$$\|x + y\|^2 = \|x\|^2 + 2\langle x, y \rangle + \|y\|^2, \tag{2.2}$$

for all  $x, y \in \mathcal{H}$ .

Next we adopt the following notation:

- $x_n \rightharpoonup x$  means that  $x_n$  converges weakly to  $x$ ;
- $\omega_w(x_n) := \{x : \exists x_{n_j} \rightharpoonup x\}$  is the weak  $\omega$ -limit set of the sequence  $\{x_n\}$ .

**Lemma 2.1** ([14]). *Let  $\mathcal{H}$  be a Hilbert space and  $\{x^k\}$  a sequence in  $\mathcal{H}$  such that there exists a nonempty set  $\Omega \subset \mathcal{H}$  satisfying the followings:*

- (i) *For every  $y \in \Omega$ ,  $\lim_{k \rightarrow \infty} \|x^k - y\|$  exists.*
- (ii) *Any weak-cluster point of the sequence  $\{x^k\}$  belongs to  $\Omega$ .*

*Then, there exists  $\bar{x} \in \Omega$  such that  $\{x^k\}$  weakly converges to  $\bar{x}$ .*

### 3. Main results

In this section, we will focus on SCFPP (1.2). Let  $\mathcal{H}_1$  and  $\mathcal{H}_2$  be two real Hilbert spaces. Let  $\mathcal{U} : \mathcal{H}_1 \rightarrow \mathcal{H}_1$  and  $\mathcal{V} : \mathcal{H}_2 \rightarrow \mathcal{H}_2$  be two directed operators. Let  $\mathcal{A} : \mathcal{H}_1 \rightarrow \mathcal{H}_2$  be a bounded linear operator with its adjoint operator  $\mathcal{A}^*$ . Assume that SCFPP (1.2) is consistent, i.e., its solution set, denoted by  $\Omega$ , is nonempty.

First, we prove that SCFPP (1.2) is equivalent to a new fixed point equation which plays an important role for constructing our algorithm.

**Lemma 3.1.**  *$u^\sharp \in \Omega$  if and only if  $u^\sharp \in \text{Fix}(\mathcal{U} - \mu_k \mathcal{A}^*(\mathcal{J} - \mathcal{V})\mathcal{A})$  where  $\mu_k > 0$  for all  $k \geq 0$ .*

*Proof.* If  $u^\sharp$  solves SCFPP (1.2), then  $u^\sharp = \mathcal{U}u^\sharp$  and  $(\mathcal{J} - \mathcal{V})\mathcal{A}u^\sharp = 0$ . It is obvious that  $u^\sharp \in \text{Fix}(\mathcal{U} - \mu_k \mathcal{A}^*(\mathcal{J} - \mathcal{V})\mathcal{A})$  for any  $\mu_k > 0$ .

Conversely, setting  $u^\sharp \in \text{Fix}(\mathcal{U} - \mu_k \mathcal{A}^*(\mathcal{J} - \mathcal{V})\mathcal{A})$ ,  $\forall \mu_k > 0$ , we have

$$u^\sharp = \mathcal{U}u^\sharp - \mu_k \mathcal{A}^*(\mathcal{J} - \mathcal{V})\mathcal{A}u^\sharp.$$

Picking up any  $z \in \Omega$ , we get

$$\begin{aligned} 0 &= \langle u^\sharp - \mathcal{U}u^\sharp + \mu_k \mathcal{A}^*(\mathcal{J} - \mathcal{V})\mathcal{A}u^\sharp, u^\sharp - z \rangle \\ &= \langle u^\sharp - \mathcal{U}u^\sharp, u^\sharp - z \rangle + \mu_k \langle \mathcal{A}^*(\mathcal{J} - \mathcal{V})\mathcal{A}u^\sharp, u^\sharp - z \rangle \\ &= \langle u^\sharp - \mathcal{U}u^\sharp, u^\sharp - z \rangle + \mu_k \langle (\mathcal{J} - \mathcal{V})\mathcal{A}u^\sharp, \mathcal{A}u^\sharp - \mathcal{A}z \rangle. \end{aligned}$$

Since  $\mathcal{U}$  and  $\mathcal{V}$  are directed operator, from (2.1), we deduce

$$\langle u^\sharp - \mathcal{U}u^\sharp, u^\sharp - z \rangle \geq \|u^\sharp - \mathcal{U}u^\sharp\|^2,$$

and

$$\langle (\mathcal{J} - \mathcal{V})\mathcal{A}u^\sharp, \mathcal{A}u^\sharp - \mathcal{A}z \rangle \geq \|(\mathcal{J} - \mathcal{V})\mathcal{A}u^\sharp\|^2.$$

Hence,

$$0 = \langle u^\sharp - \mathcal{U}u^\sharp + \mu_k \mathcal{A}^*(\mathcal{J} - \mathcal{V})\mathcal{A}u^\sharp, u^\sharp - z \rangle \geq \|u^\sharp - \mathcal{U}u^\sharp\|^2 + \mu_k \|(\mathcal{J} - \mathcal{V})\mathcal{A}u^\sharp\|^2.$$

Thus,  $u^\sharp \in \text{Fix}(\mathcal{U})$  and  $\mathcal{A}u^\sharp \in \text{Fix}(\mathcal{V})$ . That is,  $u^\sharp$  solves SCFPP (1.2). The proof is completed. □

Applying Lemma 3.1, we present the following algorithm.

**Algorithm 3.2.** Initialization: let  $x^0$  be arbitrary.

Iterative step: for  $k \geq 0$ , given the current iterate  $x^k$  calculates the next iterate  $x^{k+1}$  by the following Mann’s form

$$x^{k+1} = (1 - \alpha_k)x^k + \alpha_k[\mathcal{U}x^k - \mu_k\mathcal{A}^*(\mathcal{J} - \mathcal{V})\mathcal{A}x^k], \quad k \geq 0, \tag{3.1}$$

where  $\{\alpha_k\}$  and  $\{\mu_k\}$  are two sequences in  $(0, 1)$ .

**Theorem 3.3.** Suppose  $\mathcal{J} - \mathcal{U}$  and  $\mathcal{J} - \mathcal{V}$  are demiclosed at zero. Assume the following conditions are satisfied:

- (i)  $0 < \liminf_{k \rightarrow \infty} \alpha_k \leq \limsup_{k \rightarrow \infty} \alpha_k < 1$ ;
- (ii)  $\mu_k \in (0, \frac{1}{\alpha_k \|\mathcal{A}\|^2})$ .

Then the sequence  $\{x^k\}$  generated by (3.1) converges weakly to a solution  $u^\#$ .

*Proof.* We first show that the sequence  $\{x^k\}$  is Fejér-monotone with respect to  $\Omega$ . Picking up any  $z \in \Omega$ , from (2.1), we have

$$\begin{aligned} \langle x^k - \mathcal{U}x^k + \mu_k\mathcal{A}^*(\mathcal{J} - \mathcal{V})\mathcal{A}x^k, x^k - z \rangle &= \langle x^k - \mathcal{U}x^k, x^k - z \rangle + \mu_k \langle \mathcal{A}^*(\mathcal{J} - \mathcal{V})\mathcal{A}x^k, x^k - z \rangle \\ &= \langle x^k - \mathcal{U}x^k, x^k - z \rangle + \mu_k \langle (\mathcal{J} - \mathcal{V})\mathcal{A}x^k, \mathcal{A}x^k - \mathcal{A}z \rangle \\ &\geq \|x^k - \mathcal{U}x^k\|^2 + \mu_k \|(\mathcal{J} - \mathcal{V})\mathcal{A}x^k\|^2. \end{aligned} \tag{3.2}$$

By (3.1) and (2.2), we get

$$\begin{aligned} \|x^{k+1} - z\|^2 &= \|(1 - \alpha_k)x^k + \alpha_k[\mathcal{U}x^k - \mu_k\mathcal{A}^*(\mathcal{J} - \mathcal{V})\mathcal{A}x^k] - z\|^2 \\ &= \|x^k - z - \alpha_k[x^k - \mathcal{U}x^k + \mu_k\mathcal{A}^*(\mathcal{J} - \mathcal{V})\mathcal{A}x^k]\|^2 \\ &= \|x^k - z\|^2 - 2\alpha_k \langle x^k - \mathcal{U}x^k + \mu_k\mathcal{A}^*(\mathcal{J} - \mathcal{V})\mathcal{A}x^k, x^k - z \rangle \\ &\quad + \alpha_k^2 \|x^k - \mathcal{U}x^k + \mu_k\mathcal{A}^*(\mathcal{J} - \mathcal{V})\mathcal{A}x^k\|^2 \\ &\leq \|x^k - z\|^2 - 2\alpha_k \langle x^k - \mathcal{U}x^k + \mu_k\mathcal{A}^*(\mathcal{J} - \mathcal{V})\mathcal{A}x^k, x^k - z \rangle \\ &\quad + \alpha_k^2 [\|x^k - \mathcal{U}x^k\| + \|\mu_k\mathcal{A}^*(\mathcal{J} - \mathcal{V})\mathcal{A}x^k\|]^2 \\ &\leq \|x^k - z\|^2 - 2\alpha_k \langle x^k - \mathcal{U}x^k + \mu_k\mathcal{A}^*(\mathcal{J} - \mathcal{V})\mathcal{A}x^k, x^k - z \rangle \\ &\quad + \alpha_k^2 [2\|x^k - \mathcal{U}x^k\|^2 + 2\mu_k^2 \|\mathcal{A}\|^2 \|(\mathcal{J} - \mathcal{V})\mathcal{A}x^k\|^2]. \end{aligned}$$

This together with (3.2) implies that

$$\begin{aligned} \|x^{k+1} - z\|^2 &\leq \|x^k - z\|^2 - 2\alpha_k [\|x^k - \mathcal{U}x^k\|^2 + \mu_k \|(\mathcal{J} - \mathcal{V})\mathcal{A}x^k\|^2] \\ &\quad + \alpha_k^2 [2\|x^k - \mathcal{U}x^k\|^2 + 2\mu_k^2 \|\mathcal{A}\|^2 \|(\mathcal{J} - \mathcal{V})\mathcal{A}x^k\|^2] \\ &= \|x^k - z\|^2 - 2\alpha_k(1 - \alpha_k) \|x^k - \mathcal{U}x^k\|^2 - 2\alpha_k\mu_k(1 - \alpha_k\mu_k \|\mathcal{A}\|^2) \|(\mathcal{J} - \mathcal{V})\mathcal{A}x^k\|^2. \end{aligned} \tag{3.3}$$

By virtue of (3.3), we deduce that the sequence  $\{x^k\}$  is Fejér-monotone due to  $\alpha_k \in (0, 1)$  and  $\mu_k \in (0, \frac{1}{\alpha_k \|\mathcal{A}\|^2})$ . Hence, the sequence  $\|x^k - z\|$  is convergent.

Next, we show that every weak cluster point of the sequence  $\{x^k\}$  belongs to the solution set of problem (1.2), i.e.,  $\omega_w(x^k) \subset \Omega$ .

From (3.3) and the conditions (i) and (ii), we obtain that the sequence  $\{\|x^k - z\|\}$  is monotonically decreasing and thus converges to some positive real  $l(z)$ . Therefore,

$$\lim_{k \rightarrow \infty} \|(\mathcal{J} - \mathcal{V})\mathcal{A}x^k\| = 0. \tag{3.4}$$

From the Fejér-monotonicity of  $\{x^k\}$  it follows that the sequence is bounded. Denoting by  $\tilde{x}$  a weak-cluster point of  $\{x^k\}$ , let  $i = 0, 1, 2, \dots$  be the sequence of indices, such that

$$w - \lim_{i \rightarrow \infty} x^{k_i} = \tilde{x}.$$

Then from (3.4) and closedness of  $\mathcal{J} - \mathcal{V}$  at 0 we obtain

$$\mathcal{V}(A\tilde{x}) = A\tilde{x},$$

from which it follows that  $A\tilde{x} \in \text{Fix}(\mathcal{V})$ .

Again from (3.3) and the convergence of the sequence  $\{\|x^k - z\|\}$  we also have

$$\lim_{k \rightarrow \infty} \|x^k - \mathcal{U}x^k\| = 0,$$

which combined with the demiclosedness of  $\mathcal{J} - \mathcal{U}$  at 0 and the weak convergence of  $\{x^{k_i}\}$  to  $\tilde{x}$  yields

$$\mathcal{U}(\tilde{x}) = \tilde{x}.$$

Hence,  $\tilde{x} \in \text{Fix}(\mathcal{U})$ . Therefore,  $\tilde{x} \in \Omega$ .

To this end, all conditions of the well-known Opial's lemma 2.1 are all fulfilled. Consequently,  $x^k \rightharpoonup \tilde{x}$ . The proof is completed.  $\square$

We can apply our results to SFP (1.3).

**Algorithm 3.4.** Initialization: let  $x^0$  be arbitrary.

Iterative step: for  $k \geq 0$ , given the current iterate  $x^k$  calculates the next iterate  $x^{k+1}$  by the following Mann's form

$$x^{k+1} = (1 - \alpha_k)x^k + \alpha_k[\text{proj}_{\mathcal{C}}x^k - \mu_k \mathcal{A}^*(\mathcal{J} - \text{proj}_{\mathcal{Q}})\mathcal{A}x^k], \quad k \geq 0, \quad (3.5)$$

where  $\alpha_k \in (0, 1)$  is a constant.

**Theorem 3.5.** *If  $0 < \liminf_{k \rightarrow \infty} \alpha_k \leq \limsup_{k \rightarrow \infty} \alpha_k < 1$  and  $\mu_k \in (0, \frac{1}{\alpha_k \|\mathcal{A}\|^2})$ , then the sequence  $\{x^k\}$  generated by (3.5) converges weakly to a solution of SFP (1.3).*

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