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Optimal tracking performance of discrete-time systems with quantization

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Abstract

This paper studies optimal tracking performance issues for linear time invariant system with two-channel constraints. The specific problem under consideration is quantization for up-link and down-link communication channel which satisfies some constraints. Logarithmic quantization law is employed in the quantizers. The tracking performance is defined in an square sense, and the reference signal under consideration in this paper is a step signal. The system's reference signal is considered as a step signal. The tracking performance is measured by the minimum mean square error between the reference input and the system's output. By using dynamic programming approach, discrete-time algebraic Riccati equation (ARE) is obtained. The optimal tracking performance is obtained by output feedback control, in terms of the space equation of the given system and the unique solution of the discrete-time algebraic Riccati equation. And, the impact of quantizer for optimal tracking performance is analyzed. Finally, simulation example is given to illustrate the theoretical results. ©2017 All rights reserved.

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1. Introduction

In recent years, more and more researchers are interested in networked control systems (NCSs), for example [2, 3, 10, 15, 20, 21, 24, 25, 27], and references therein. The most problems under consideration focus on how to model the networked control system and stabilization analysis with quantization effects [9, 16, 22], time delays [9, 13, 14, 28], bandwidth constraint [5, 6, 19], and signal-to-noise ratio (SNR) constraint [4, 7, 17, 18] over the communication channels. These studies investigate mostly the problem of stability analysis and stabilization for networked control systems. However, from the angle of application, only considering the stability of the networked control system is not enough, the performance of the networked control system should also be considered. A poor performance may deteriorate stability of the system, and even make the system unstable. Therefore, the study is necessary and urgent on NCSs. At

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present, many issues of optimal control performance under such network environment remain challenging for us, such as optimal attainable tracking performance of networked control systems in terms of the key factors of the communication channel.

In recent years, most works for the performance limitation of NCS are investigated communication network with channel noise, signal-to-noise ratio (SNR) constraints, packet loss, bandwidth constraint, and delay time. For instance, in [18], SNR fundamental limitations are investigated for discrete-time single-input and single-output (SISO) NCSs. A tight condition of communication SNR is obtained for the linear time invariant output feedback stabilization of a discrete-time. In [26], the optimal tracking problem is studied for SISO discrete-time systems over communication channel with network-induced delay in the feedback path. In [23], the problem of optimal tracking performance for SISO discrete-time NCSs with packet dropouts and channel noise is studied. [11] focuses on two kinds of network parameters for bandwidth and additive colored white Gaussian noise (ACGN), the optimal tracking performance are obtained for multi-input-multi-output (MIMO) NCSs. In [12], the stabilization and tracking performance issues are investigated for MIMO control system over additive white noise channels.

In this paper, optimal tracking performance issues are investigated for linear time invariant discretetime system with quantized input and output. We are interested in the intrinsic limit on the tracking performance achievable via feedback. The tracking performance under consideration amounts to determine the minimal tracking error between the system output and the reference signals of a feedback system by output feedback stabilizing compensators. And, the tracking performance is defined in an square sense, and the reference signals under consideration are step signals. The quantization is considered in up-link and down-link communication channel. Logarithmic quantization law is employed in the quantizers. By using dynamic programming approach, discrete-time ARE is obtained. The optimal tracking performance is obtained by output feedback control, in terms of the space equation of the given system and the unique solution of the discrete-time ARE.

The notation used throughout this paper is fairly standard. For any complex number, denote its transpose by $(\cdot)^{\mathsf{T}}$, and its Moore-Penrose pseudo inverse by $(\cdot)^{\dagger}$. Denote the expectation operator and the variance operator by $\mathsf{E}\{\cdot\}$ and $\mathsf{D}\{\cdot\}$, respectively.

2. Preliminaries and problem statements

We consider control over a communication link as illustrated in Fig. 1., where the sensor and plant are connected through a network, in which quantizer maybe be necessary and channel noise may also exist, as depicted in Fig. 1.



Figure 1: Feedback control over communication channels.

In Fig. 1., G(z) denotes the plant that should be controlled, Q denotes the quantizer, K denotes the controller, and K₀ denotes the steady state part of the controller K. This steady state control signal and steady state system's output signal are transmitted to the plant G and the controller K through the network with a sufficient accuracy at the initial time and are held by the storage S₁ and S₂, respectively. u and y denote output signal of the controller and the plant, respectively. u_s and y_s denote the steady

state part of the control signal u and the system output y, respectively. u_t and y_t are the transient part of the control signal u and the system output y, which are quantized by the quantizer Q_1 and Q_2 . u_q and y_q are the system quantized control signal and the system quantized output signal, respectively.

The reference signal under consideration in this paper is a step signal, i.e.,

$$\mathbf{r}(\mathbf{k}) = \begin{cases} \mathbf{r}_0, & \text{for } \mathbf{k} = 0, 1, 2, \cdots \\ 0, & \text{for } \mathbf{k} < 0, \end{cases}$$

where the magnitude r_0 of the reference signal is random variable with zero mean and variance σ_r .

We consider a logarithmic quantization law of the quantizer Q_1 and Q_2 as [8], in which u_t, y_t and u_{tq}, y_{tq} are the input and output of quantizers Q_1 and Q_2 , respectively, and have

$$\begin{cases} u_{tq}(k) = Q_1(u_t(k)) = u_t(k) + u_t(k)w_1(k), \\ y_{tq(k)} = Q_2(y_t(k)) = y_t(k) + y_t(k)w_2(k), \end{cases}$$

where $w_1(k)$ and $w_2(k)$ are quantization error, and it holds that $\delta_1 = (1 - \rho_1)/(1 + \rho_1)$ and $\delta_2 = (1 - \rho_2)/(1 + \rho_2)$ where ρ_i , $i = 1, 2, (0 < \rho_1, \rho_2 < 1)$ are the quantization density. We assume that the quantization errors processes $w_1(k_1)$ and $w_1(k_2)$ are uncorrelated for any $k_1 \neq k_2$. The quantization errors processes w_2 have the same assumptions, and $w_1(k_1)$ and $w_2(k_1)$ are also uncorrelated. Furthermore, for any k_1 and k_2 , it holds that

$$E\{w_1(k_1)w_1(k_2)\} = \begin{cases} \sigma_1^2, & k_1 = k_2, \\ 0, & k_1 \neq k_2, \end{cases}$$
$$E\{w_2(k_1)w_2(k_2)\} = \begin{cases} \sigma_2^2, & k_1 = k_2, \\ 0, & k_1 \neq k_2, \end{cases}$$

where σ_1 and σ_1 are the variances of w_1 and w_2 , respectively. Additionally, the reference signal $r(k_1)$ and quantization errors $w_1(k_2)$ and $w_2(k_3)$ are uncorrelated for any k_1 , k_2 and k_3 .

Lemma 2.1 ([1]). Let matrices $F = F^T$, H, and $G = G^T$ be given with appropriate sizes. Consider the following quadratic form

$$q(x, u) = E\{x^{T}Fx + x^{T}Hu + u^{T}Hx + u^{T}Gu\}$$

where x and u are random variables defined on a probability space (Ω, \mathcal{B}, P) . Then the following conditions are equivalent:

- (i) $G \ge 0$ and $H(I GG^T) = 0$;
- (ii) there exists a symmetric matrix $S = S^T$ such that $\inf_{u} q(x, u) = E\{x^T S x\}$ for any random variable x.

Assume that the plant is strictly proper and has a state space representation:

$$\begin{cases} x(k+1) = Ax(k) + Bu_q(k), \\ y(k) = Cx(k) + Du_q(k), \end{cases}$$

where $x(\cdot)$ is the system state with initial value x(0)=0, $u(\cdot)$ is the control input, $y(\cdot)$ is the system output, and assume that (A, B) is stabilizable, (A, C) is observable. Noting Fig. 1, we have

$$\mathfrak{u}_{\mathfrak{q}}(k) = \mathfrak{u}_{\mathfrak{s}}(k) + \mathfrak{u}_{\mathfrak{t}\mathfrak{q}}(k) = \mathfrak{u}_{\mathfrak{s}}(k) + \mathfrak{u}_{\mathfrak{t}}(k) + \mathfrak{u}_{\mathfrak{t}}(k)\omega(k) = \mathfrak{u}(k) + \mathfrak{u}_{\mathfrak{t}}(k)\omega(k).$$

Thus, the system in tracking performance problem is

$$\begin{cases} x(k+1) = Ax(k) + Bu(k) + Bu_t\omega_1(k), \\ y(k) = Cx(k) + Du(k) + Du_t\omega_1(k). \end{cases}$$
(2.1)

Denote the steady state values of the input, output, and state by u_s, y_s , and x_s , respectively. The transient part of the control input and the system's state be denoted by $u_t := u - u_s, x_t := x - x_s$ and $y_t := y - y_s$, respectively. The random variables $\omega_1(k)$, $\omega_2(k)$ are defined on a given probability space (Ω, F, p) . Owning to $y(k) = y_s + y_t(k)$, when the system achieves asymptotically tracking, the steady state value of the output $y_s = y(\infty)$, and the transient parts of the system's output variable value is equal to $y_t(\infty) = 0$. Thus, the steady state value y_s of the system's output is equal to magnitude of the reference signal. The homeostatic control u_s and status x_s must satisfy equations

$$\begin{cases} x_s = Ax_s + Bu_s, \\ y_s = Cx_s + Du_s. \end{cases}$$

So, we have

$$\begin{cases} x_{s} = \frac{(I-A)^{-1}Br}{C(I-A)^{-1}B+D'}, \\ u_{s} = \frac{r}{C(I-A)^{-1}B+D}. \end{cases}$$
(2.2)

From (2.1), we can get the following equations:

$$\begin{cases} x_{t}(k+1) = A_{t}x(k) + Bu_{t}(k) + Bu_{t}(k)\omega_{1}(k), \\ y_{t}(k) = Cx_{t}(k) + Du_{t}(k) + Du_{t}(k)\omega_{1}(k). \end{cases}$$
(2.3)

3. Optimal tracking performance

The performance index of the discrete-time networked control systems is defined as

$$J = \lim_{N \to \infty} \inf_{u_i \in U_{ad}} J(u_0, u_1, \cdots, u_{N-1}) = \inf_{u_i \in U_{ad}} E \sum_{k=0}^{\infty} [r - y(k)]^2.$$
(3.1)

The admissible controller set U_{ad} is the set of all such controllers. The tracking problem under consideration is to find a controller sequence $(u_0, u_1, \dots, u_{\infty})$ that minimizes J over U_{ad} . And the control sequences $u_0, u_1, \dots, u_{\infty}, (u_i \in \mathbb{R}^n)$ are defined on a given probability space (Ω, F, p) .

From the system (2.3), performance index (3.1) can be turned to

$$J = \inf_{u_t \in U_{ad}} E\left\{\sum_{k=0}^{\infty} [Cx_t(k) + Du_t(k) + Du_t(k)\omega_1(k)]^2\right\}.$$

The optimal tracking performance is given as following theorem.

Theorem 3.1. For given discrete-time NCSs as depicted Fig. 1, considering the plant (2.3), optimal tracking controller can be designed as

$$\begin{split} \mathfrak{u}^{*} = & -\frac{1}{1+\sigma_{1}^{2}}[B^{\mathsf{T}}PB + DD^{\mathsf{T}}]^{-1}[B^{\mathsf{T}}PA + CD^{\mathsf{T}}]C^{\dagger}\mathfrak{y}_{\mathfrak{q}}\frac{1}{1+\sigma_{1}^{2}}[B^{\mathsf{T}}PB + DD^{\mathsf{T}}]^{-1}[B^{\mathsf{T}}PA + CD^{\mathsf{T}}]C^{\dagger}r\\ & +\frac{r}{C(I-A)^{-1}B + D}. \end{split}$$

The optimal performance under uplink and downlink channels with quantization are given by

$$J^{*} = \frac{B^{\mathsf{T}}(I - A^{\mathsf{T}})^{-1}}{B^{\mathsf{T}}(I - A^{\mathsf{T}})^{-1}C^{\mathsf{T}}} P \frac{(I - A)^{-1}B}{C(I - A)^{-1}B} \sigma_{\mathsf{r}}^{2},$$

where, P is the unique solution of discrete-time ARE

$$\mathbf{P} = \mathbf{C}^{\mathsf{T}}\mathbf{C} + \mathbf{A}^{\mathsf{T}}\mathbf{P}\mathbf{A} - \frac{(1+\sigma_2^2)}{(1+\sigma_1^2)} \Big[\mathbf{A}^{\mathsf{T}}\mathbf{P}\mathbf{B} + \mathbf{D}\mathbf{C}^{\mathsf{T}}\Big] \Big(\mathbf{B}^{\mathsf{T}}\mathbf{P}\mathbf{B} + \mathbf{D}\mathbf{D}^{\mathsf{T}}\Big)^{-1} \Big[\mathbf{B}^{\mathsf{T}}\mathbf{P}\mathbf{A} + \mathbf{C}\mathbf{D}^{\mathsf{T}}\Big].$$

Proof. Consider the following performance index:

$$J_{N} = E \sum_{k=0}^{N} [Cx_{t}(k) + Du_{t}(k) + Du_{t}(k)\omega_{1}(k)]^{2},$$

and let $V_j = J_N - J_{j-1}, (j \in 1, \cdots, N).$ Then

$$\begin{split} V_{N} &= J_{N} - J_{N-1} \\ &= E[Cx_{t}(N) + Du_{t}(N) + Du_{t}(N)\omega_{1}(N)]^{2} \\ &= E\left\{x_{t}^{\mathsf{T}}(N)C^{\mathsf{T}}Cx_{t}(N) + x_{t}(N)^{\mathsf{T}}C^{\mathsf{T}}Du_{t}(N) + u_{t}^{\mathsf{T}}(N)D^{\mathsf{T}}Cx_{t}(N) + (1 + \sigma_{1}^{2})u_{t}(N)^{\mathsf{T}}D^{\mathsf{T}}Du_{t}(N)\right\}. \end{split}$$

Following Lemma 2.1, we have

$$\inf_{u_t \in U_{ad}} V_N = E[x_t^T(N)P_N(N)x_t(N)],$$
(3.2)

where $P_N(N)$ is a symmetric matrix. From equations (2.1) and (3.2), we have

$$\begin{split} \inf_{u_t(N)\in U_{ad}} V_N = & E[x_t^T(N)P_N(N)x_t(N)] \\ = & x_t^T(N-1)A^TP_N(N-1)Ax(N-1) + u_t^T(N-1)B^TP_N(N-1)Ax(N-1) \\ & + x_t^T(N-1)A^TP_N(N-1)Bu_t(N-1) + x_t^T(N-1)A^TP_N(N-1)Bu_t(N-1) \\ & + (1+\sigma_1^2)u_t^T(N-1)B^TP_N(N-1)Bu_t(N-1). \end{split}$$

Then, we have

$$\begin{split} V_{N-1} &= J_N - J_{N-2} \\ &= (J_N - J_{N-1}) + (J_{N-1} - J_{N-2}) \\ &= V_N + (J_{N-1} - J_{N-2}) \\ &= E\Big\{\Big[Cx_t(N-1) + Du_t(N-1) + Du_t(N-1)\omega_1(N-1)\Big]^2\Big\} + V_N \\ &= E\Big\{x_t^T(N-1)[C^TC + A^TP_N(N-1)A]x_t(N-1) \\ &\quad + x_t^T(N-1)[A^TP_N(N-1)B + C^TD]u_t(N-1) + u_t^T(N-1)[B^TP_N(N-1)A + D^TC]x_t(N-1) \\ &\quad + (1 + \sigma_1^2)u_t^T(N-1)[B^TP_N(N-1)B + D^TD]u_t(N-1)\Big\} + V_N - \inf_{u_i \in U_{ad}} V_N \\ &= E\Big\{x_t^T(N-1)P_N(N-1)x_t(N-1) \\ &\quad + (1 + \sigma_1^2)[u_t(N-1) + K_t(N-1)y_{tq}(N-1)]^T[B^TP_N(N)B + DD^T][u_t(N-1) \\ &\quad + K_t(N-1)y_{tq}(N-1)]\Big\} + V_N - \inf_{u_i \in U_{ad}} V_N, \end{split}$$

where

$$P_{N}(N-1) = C^{T}C + A^{T}P_{N}(N)A - \frac{1+\sigma_{2}^{2}}{1+\sigma_{1}^{2}}C^{T}K_{t}^{T}(N-1)[B^{T}P_{N}(N)B + DD^{T}]K_{t}(N-1)C, \quad (3.3)$$
$$K_{t}(N-1) = \frac{1}{1+\sigma_{1}^{2}}[B^{T}P_{N}(N)B + DD^{T}]^{-1}[B^{T}P_{N}(N)A + CD^{T}]C^{\dagger}. \quad (3.4)$$

Thus, the following result can be obtained

$$\begin{split} \inf_{u_t \in U_{\alpha d}} V_{N-1} = & E \Big\{ x_t^\mathsf{T}(N-1) \mathsf{P}_N(N-1) x_t(N-1) + (1+\sigma^2) [u_t(N-1) + K_t(N-1) y_{tq}(N-1)]^\mathsf{T} \\ & \times [B^\mathsf{T} \mathsf{P}_N(N) B + DD^\mathsf{T}] [u_t(N-1) + K_t(N-1) y_{tq}(N-1)] \Big\}. \end{split}$$

Therefore, when equations (3.3) and (3.4) are satisfied, we have

$$\inf_{u_i \in U_{ad}} V_{N-1} = E\{x^T(N-1)P_N(N-1)x(N-1)\}.$$

We use the same treatment process, we can obtain

$$\begin{split} \mathsf{P}_{\mathsf{N}}(j-1) = & \mathsf{C}^{\mathsf{T}}\mathsf{C} + \mathsf{A}^{\mathsf{T}}\mathsf{P}_{\mathsf{N}}(j)\mathsf{A} - (1+\sigma_{1}^{2})(1+\sigma_{2}^{2})\mathsf{C}^{\mathsf{T}}\mathsf{K}_{t}^{\mathsf{T}}(j-1)[\mathsf{B}^{\mathsf{T}}\mathsf{P}_{\mathsf{N}}(\mathsf{N})\mathsf{B} + \mathsf{D}\mathsf{D}^{\mathsf{T}}]\mathsf{K}_{t}(j-1)\mathsf{C}_{t}^{\mathsf{T}}\\ \mathsf{K}_{t}(j-1) = & \frac{1}{1+\sigma_{1}^{2}}[\mathsf{B}^{\mathsf{T}}\mathsf{P}_{\mathsf{N}}(j)\mathsf{B} + \mathsf{D}\mathsf{D}^{\mathsf{T}}]^{-1}[\mathsf{B}^{\mathsf{T}}\mathsf{P}_{\mathsf{N}}(j)\mathsf{A} + \mathsf{C}\mathsf{D}^{\mathsf{T}}]\mathsf{C}^{\dagger}. \end{split}$$

Accordingly, we have

$$\inf_{u_i \in U_{ad}} V_{j-1} = E[x^{\mathsf{T}}(j-1)P_{\mathsf{N}}(j-1)x(j-1)],$$

where $j = 2, \dots, N-1$. It is implied that when

$$u_{t}(j) = -K_{t}(j)y_{tq}(j),$$

the cost function obtains the minimum. Therefore, the optimal tracking performance for system (2.3) is given by

$$J_{N}^{*} = \inf_{u_{i} \in U_{ad}} V_{1} = E\{x_{t}^{\mathsf{T}}(0)P_{N}(0)x_{t}(0)\},$$
(3.5)

where $P_N(0) > 0$ is solution of the following discrete-time infinite ARE

$$P_{N}(j-1) = C^{T}C + A^{T}P_{N}(j)A - \frac{(1+\sigma_{2}^{2})}{(1+\sigma_{1}^{2})} \Big[A^{T}P_{N}(j)B + DC^{T}\Big] (B^{T}P_{N}(j)B + DD^{T})^{-1} \Big[B^{T}P_{N}(j)A + CD^{T}\Big].$$

Thus, we know that the discrete-time finite ARE

$$P_{M}(j-1) = C^{T}C + A^{T}P_{M}(j)A - \frac{(1+\sigma_{2}^{2})}{(1+\sigma_{1}^{2})} \Big[A^{T}P_{M}(j)B + DC^{T} \Big] (B^{T}P_{M}(j)B + DD^{T})^{-1} \Big[B^{T}P_{M}(j)A + CD^{T} \Big],$$
(3.6)

has a unique solution $P_M(k) > 0, k \in \{0, 1, \dots, M\}$. It is obvious that $P_M(k) = P_{M-k}(0)$. If the output of system (2.1) asymptotically tracks the reference signal, the corresponding tracking performance limitation must exist, namely, $P_M(0)$ in equation (3.6) must exist, and

$$\lim_{M \to \infty} \mathsf{P}_{\mathsf{M}}(0) = \lim_{M \to \infty} \mathsf{P}_{\mathsf{M}-k}(0) = \lim_{M \to \infty} \mathsf{P}_{\mathsf{M}}(k) = \mathsf{P}_{\mathsf{M}}(k)$$

exists, and

$$\lim_{j \to \infty} \mathsf{K}_t(j-1) = \frac{1}{1 + \sigma_1^2} [\mathsf{B}^\mathsf{T} \mathsf{P} \mathsf{B} + \mathsf{D} \mathsf{D}^\mathsf{T}]^{-1} [\mathsf{B}^\mathsf{T} \mathsf{P} \mathsf{A} + \mathsf{C} \mathsf{D}^\mathsf{T}] \mathsf{C}^\dagger \triangleq \mathsf{K}_t^*.$$

Therefore, the discrete-time ARE can be converted to the following ARE

$$\mathbf{P} = \mathbf{C}^{\mathsf{T}}\mathbf{C} + \mathbf{A}^{\mathsf{T}}\mathbf{P}\mathbf{A} - \frac{(1+\sigma_2^2)}{(1+\sigma_1^2)} \Big[\mathbf{A}^{\mathsf{T}}\mathbf{P}\mathbf{B} + \mathbf{D}\mathbf{C}^{\mathsf{T}}\Big] \Big(\mathbf{B}^{\mathsf{T}}\mathbf{P}\mathbf{B} + \mathbf{D}\mathbf{D}^{\mathsf{T}}\Big)^{-1} \Big[\mathbf{B}^{\mathsf{T}}\mathbf{P}\mathbf{A} + \mathbf{C}\mathbf{D}^{\mathsf{T}}\Big].$$

Noting equation (3.5) and the fact that x(0) = 0 and

$$\mathbf{x}_{\mathbf{t}}(0) = \mathbf{x}(0) - \mathbf{x}_{\mathbf{s}} = -\mathbf{x}_{\mathbf{s}},$$

therefore

$$J^* = \lim_{N \to \infty} J^*_N = E\{x_s^T P x_s\}.$$

Noting the equation (2.2), the minimum tracking performance for system (2.3) can be given by

$$J^* = \frac{B^{\mathsf{T}}(I - A^{\mathsf{T}})^{-1}}{B^{\mathsf{T}}(I - A^{\mathsf{T}})^{-1}C^{\mathsf{T}}} P \frac{(I - A)^{-1}B}{C(I - A)^{-1}B} \sigma_r^2.$$

Additionally, we have

$$u^{*} = u_{t}^{*} + u_{s} = -\frac{1}{1 + \sigma_{1}^{2}} [B^{\mathsf{T}}PB + DD^{\mathsf{T}}]^{-1} [B^{\mathsf{T}}PA + CD^{\mathsf{T}}]C^{\dagger}y_{\mathfrak{q}} + \frac{1}{1 + \sigma_{1}^{2}} [B^{\mathsf{T}}PB + DD^{\mathsf{T}}]^{-1} [B^{\mathsf{T}}PA + CD^{\mathsf{T}}]C^{\dagger}r + \frac{r}{C(I - A)^{-1}B + D}$$

Thus, the proof is completed.

4. Conclusion

In this paper, the best attainable tracking performance of networked control systems in tracking step signal has been discussed for linear time-invariant unstable plants using output feedback control. By using dynamic programming approach, discrete-time algebraic Riccati equation (ARE) is obtained. Then the best attainable tracking performance is obtained, in terms of the space equation of given system and the unique solution of the discrete-time ARE.

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