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Some new fuzzy fixed point theorems via distance functions with applications

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Abstract

In this paper, we prove some new fuzzy fixed point theorems on a space of fuzzy sets under a G-distance function and a G′-distance function. Our results extend, generalize, and improve some existing results. Moreover, some applications are given here to illustrate the usability of the obtained results.

Keywords: Fuzzy set, fuzzy-mapping, G-distance functions, G′-distance functions, fuzzy fixed point.
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1. Introduction and preliminaries

It is well-known that functional analysis is made up of two main methods which are variational methods and fixed point methods. Variational methods are used to prove the existence of solutions for differential equations [10, 14, 15, 23, 32, 33, 35–37]. However, fixed point methods are studied by many scholars [12, 13] in different spaces. Recently, fuzzy fixed point has attracted wide attention. As far as we know, fuzzy set theory plays an important role in many scientific and engineering applications. The fuzziness appears when we need to perform, on manifold, calculations with imprecision variables. The concept of fuzzy sets was introduced initially by Zadeh [34] in 1965. Since then, Heilpern [21] defined the fuzzy mapping $T : X \to W_\alpha(X)$ and proved a fixed point theorem for fuzzy mapping T in metric linear space, which is a fuzzy extension of the Banach contraction principle. Subsequently, several other authors [1–9, 11, 16, 17, 20–22, 24–26, 29, 30, 34] have studied existence of fixed points of fuzzy mappings satisfying some different contractive type conditions.

On the one hand, in 2008, Qiu et al. [27] defined the fuzzy mapping $F : \mathcal{CB}(X) \to \mathcal{CB}(X)$ on a space of fuzzy sets and proved a fixed point theorem for fuzzy mappings F in complete metric spaces and this is different from the approach which is used by Heilpern [21]. On the other hand, in 2009, Qiu et al. [28]

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defined the fuzzy mapping $K : \mathcal{C}(X) \to \mathcal{C}(X)$ (please see [28]) on a space of fuzzy sets in another way and proved a fixed point theorem for fuzzy mappings K in compact metric spaces. Recently, following Qiu's work, Suantai et al. [31] provided some fuzzy fixed point theorems on a space of fuzzy sets equipped with supremum metric by using $\mathfrak{R}$-functions.

Motivated by [11] and [31], in this paper, we prove some new fuzzy fixed point theorems on a space of fuzzy sets under a G-distance function and G'-distance function in complete and compact metric spaces. Our results extend, generalize and improve the results of [27, 28, 31]. Moreover, some applications are given here to illustrate the usability of the obtained results.

Throughout this paper, we shall use the following notions.

Let (X, d) be a metric space, and let $CB(X)$ be the set of all nonempty bounded closed subsets of X. Recall that the Hausdorff metric is a function H on $CB(X)$ defined by

$$H(A,B) = \max\Big\{\sup_{x\in B} d(x,A), \sup_{x\in A} d(x,B)\Big\} = \max\{\rho(B,A), \rho(A,B)\} \quad \text{for all} \quad A,B \in CB(X),$$

where $\rho(A,B) = \sup_{x\in A} d(x,B)$ is the Hausdorff separation of A from B.

A fuzzy set μ in X is a function with domain X and values in $I = [0,1]$. If μ is a fuzzy set and $x \in X$, then the function value $\mu(x)$ is called the grade of membership of x in X. The α-cut set of μ, denoted by $[\mu]_\alpha$, is defined as

$$[\mu]_\alpha = \big\{x : \mu(x) \geqslant \alpha\big\},$$

where $\alpha \in (0,1]$, and we separately specify the support $[\mu]_0$ of μ to be the closure of the union of $[\mu]_\alpha$ for $0 < \alpha \leqslant 1$. We denote by $\mathcal{CB}(X)$ the totality of fuzzy sets $\mu : X \to I$ for which, for each $\alpha \in I$, the α-cut of μ is a nonempty closed bounded subset of X.

Let $\mu_1, \mu_2 \in \mathcal{CB}(X)$. Then μ_1 is said to be included in μ_2, denoted by $\mu_1 \subseteq \mu_2$, if and only if $\mu_1(x) \leqslant \mu_2(x)$ for each $x \in X$. Thus we have $\mu_1 \subseteq \mu_2$ if and only if $[\mu_1]_\alpha \subseteq [\mu_2]_\alpha$ for all $\alpha \in I$. Let X, Y be any metric space. A mapping F is said to be a fuzzy mapping if and only if F is a mapping from the space $\mathcal{CB}(X)$ into $\mathcal{CB}(X)$, i.e., $F(\mu) \in \mathcal{CB}(X)$ for each $\mu \in \mathcal{CB}(X)$. $\mu_* \in \mathcal{CB}(X)$ is said to be a fixed point of a fuzzy self-mapping F of $\mathcal{CB}(X)$ if and only if $\mu_* \subseteq F(\mu_*)$.

The d_∞-metric (called supremum or generalized Hausdorff metric) is a metric on $\mathcal{CB}(X)$ which is defined as follows:

$$d_\infty(\mu_1,\mu_2) = \sup_{0\leqslant\alpha\leqslant 1} H([\mu_1]_\alpha, [\mu_2]_\alpha) = \max\{\rho_\infty(\mu_1,\mu_2), \rho_\infty(\mu_2,\mu_1)\},$$

where $\mu_1, \mu_2 \in \mathcal{CB}(X)$, and

$$\rho_\infty(\mu_1,\mu_2) = \sup_{0\leqslant\alpha\leqslant 1} \rho([\mu_1]_\alpha, [\mu_2]_\alpha) \tag{1.1}$$

is the Hausdorff separation of μ_1 from μ_2. Notice that the supremum in (1.1) may be not attained, and so it cannot be replaced by a maximum. To clarify this, we include the following example, which can be found in [18].

Example 1.1. Let X be a set of real numbers and $\mu, \upsilon \in \mathcal{CB}(X)$ be fuzzy subsets of X such that the corresponding level sets are

$$[\mu]_\alpha = [\upsilon]_\alpha = [0,1] \ \text{ for } \ 0 \leqslant \alpha \leqslant \frac{1}{2},$$

and

$$[\mu]_\alpha = \{0\}, \ [\upsilon]_\alpha = [0, 2(1-\alpha)] \ \text{ for } \ \frac{1}{2} \leqslant \alpha \leqslant 1.$$

It follows that

$$H([\mu]_\alpha, [\upsilon]_\alpha) = \begin{cases} 0, & \text{for } \ 0 \leqslant \alpha \leqslant \frac{1}{2}, \\ 2(1-\alpha), & \text{for } \ \frac{1}{2} \leqslant \alpha \leqslant 1. \end{cases}$$

Hence, $d_\infty(\mu,\upsilon) = \sup_{0\leqslant\alpha\leqslant 1} H([\mu]_\alpha, [\upsilon]_\alpha) = 1$, but this is not attained.

Note that if $\{\mu_n\}$ is a sequence in $\mathcal{CB}(X)$, then it follows from the definition of d_∞ that $\{\mu_n\}$ converges with respect to the d_∞-metric if and only if $[\mu_n]_\alpha$ converges uniformly in $\alpha \in I$ with respect to the Hausdorff metric. Further, we know that the metric space $(\mathcal{CB}(X), d_\infty)$ and $(\mathcal{C}(X), d_\infty)$ are complete provided (X, d) is complete (see Theorem 1 in [27] and Theorem 1 in [28]). Now, we list some results of the d_∞-metric as follows.

Lemma 1.2 ([27, 28])**.** *Let $\mu_1, \mu_2, \mu_3 \in \mathcal{CB}(X)$ (or $\mathcal{C}(X)$). Then the following properties hold:*

(i) $\rho(\mu_1, \mu_2) = 0$ *if and only if* $\mu_1 \subseteq \mu_2$*,*
(ii) *if* $\mu_1 \subseteq \mu_2$*, then* $\rho_\infty(\mu_1, \mu_3) \leqslant d_\infty(\mu_2, \mu_3)$*,*
(iii) $\rho_\infty(\mu_1, \mu_3) \leqslant d_\infty(\mu_1, \mu_2) + \rho_\infty(\mu_2, \mu_3)$*.*

Lemma 1.3 ([27])**.** *Let (X, d) be a metric space and $\mu_1, \mu_2 \in \mathcal{CB}(X)$. Then for any $\beta > 1$ and any $\mu_3 \in \mathcal{CB}(X)$ satisfying $\mu_3 \subseteq \mu_1$, there exists a $\mu_4 \in \mathcal{CB}(X)$ such that $\mu_4 \subseteq \mu_2$ and $d_\infty(\mu_3, \mu_4) \leqslant \beta d_\infty(\mu_1, \mu_2)$.*

Lemma 1.4 ([28])**.** *Let (X, d) be a metric space and $\mu_1, \mu_2 \in \mathcal{C}(X)$. Then for any $\mu_3 \in \mathcal{C}(X)$ satisfying $\mu_3 \subseteq \mu_1$, there exists a $\mu_4 \in \mathcal{C}(X)$ such that $\mu_4 \subseteq \mu_2$ and $d_\infty(\mu_3, \mu_4) \leqslant d_\infty(\mu_1, \mu_2)$.*

In [31], the authors gave an important tool related to our considered class of mappings. A function $\varphi : [0, \infty) \to [0, 1)$ is said to be an $\mathfrak{R}$-function if

$$\sup_{s \to t^+} \varphi(s) < 1 \quad \text{for all } \ t \in [0, \infty).$$

Note that if $\varphi : [0, \infty) \to [0, 1)$ is a non-decreasing function or a non-increasing function, then φ is an $\mathfrak{R}$-function. This means the set of $\mathfrak{R}$-functions is a rich class. In [19], Du proved some of the following properties for the class of $\mathfrak{R}$-functions.

Theorem 1.5. *Let $\varphi : [0, \infty) \to [0, 1)$ be a function. Then the following statements are equivalent*

(a) *φ is an $\mathfrak{R}$-function.*
(b) *For any nonincreasing sequence $\{x_n\}_{n \in \mathbf{N}}$ in $[0, \infty)$, we have $0 \leqslant \sup_{n \in \mathbf{N}} \varphi(x_n) < 1$.*

By Lemma 1.2, Lemma 1.3, and Theorem 1.5, Qiu et al. [27] and Suantai et al. [31] proved the following common fixed point theorems under the assumption of a closed bounded cut set of $\mathcal{CB}(X)$.

Theorem 1.6. *Let (X, d) be a complete metric space and let $\{F_i\}_{i=1}^\infty$ be a sequence of self-mappings of $\mathcal{CB}(X)$. If there exists a constant $q \in (0, 1)$ such that for each $\mu_1, \mu_2 \in \mathcal{CB}(X)$, and for arbitrary positive integers i and j, $i \neq j$,*

$$d_\infty(F_i(\mu_1), F_j(\mu_2)) \leqslant q M_{i,j}(\mu_1, \mu_2),$$

where

$$M_{i,j}(\mu_1, \mu_2) = \max\left\{ d_\infty(\mu_1, \mu_2), \rho_\infty(\mu_1, F_i(\mu_1)), \rho_\infty(\mu_2, F_j(\mu_2)), \frac{\rho_\infty(\mu_2, F_i(\mu_1)) + \rho_\infty(\mu_1, F_j(\mu_2))}{2} \right\},$$

then there exists a $\mu_* \in \mathcal{CB}(X)$ such that $\mu_* \subseteq F_i(\mu_*)$ for all $i \in \mathbf{N}$.

Theorem 1.7. *Let (X, d) be a complete metric space and let $\{F_i\}_{i=1}^\infty$ be a sequence of self-mappings of $\mathcal{CB}(X)$. Assume that there exists an $\mathfrak{R}$-function $\varphi : [0, \infty) \to [0, 1)$ such that for each $\mu_1, \mu_2 \in \mathcal{CB}(X)$, and for arbitrary positive integers i and j, $i \neq j$,*

$$d_\infty(F_i(\mu_1), F_j(\mu_2)) \leqslant \varphi(d_\infty(\mu_1, \mu_2)) M_{i,j}(\mu_1, \mu_2),$$

where

$$M_{i,j}(\mu_1, \mu_2) = \max\left\{ d_\infty(\mu_1, \mu_2), \rho_\infty(\mu_1, F_i(\mu_1)), \rho_\infty(\mu_2, F_j(\mu_2)), \frac{\rho_\infty(\mu_2, F_i(\mu_1)) + \rho_\infty(\mu_1, F_j(\mu_2))}{2} \right\}.$$

Then there exists a $\mu_* \in \mathcal{CB}(X)$ such that $\mu_* \subseteq F_i(\mu_*)$ for all $i \in \mathbf{N}$.

By Lemmas 1.2 and 1.3, Qiu et al. [28] proved the following common fixed point theorems under the assumption of a compact cut set of $\mathcal{C}(X)$.

Theorem 1.8. *Let* (X, d) *be a compact metric space and let* $\{F_i\}_{i=1}^{\infty}$ *be a sequence of self-mappings of* $\mathcal{C}(X)$. *Let* $\Phi : [0, \infty) \to [0, \infty)$ *be a non-decreasing function satisfying the following condition:* Φ *is continuous from the right and*

$$\Sigma_{n=1}^{\infty} \Phi^n(t) < \infty \quad \textit{for all} \;\; t > 0,$$

where Φ^n *denotes the nth iterative function of* Φ. *Suppose that for arbitrary positive integers* i *and* j, $i \neq j$,

$$d_\infty(F_i(\mu_1), F_j(\mu_2)) \leqslant \Phi(M_{i,j}(\mu_1, \mu_2)),$$

where

$$M_{i,j}(\mu_1, \mu_2) = \max \left\{ d_\infty(\mu_1, \mu_2), \rho_\infty(\mu_1, F_i(\mu_1)), \rho_\infty(\mu_2, F_j(\mu_2)), \frac{\rho_\infty(\mu_2, F_i(\mu_1)) + \rho_\infty(\mu_1, F_j(\mu_2))}{2} \right\}.$$

Then there exists a $\mu_* \in \mathcal{CB}(X)$ *such that* $\mu_* \subseteq F_i(\mu_*)$ *for all* $i \in \mathbf{N}$.

Next, we introduce some classes of functions.

Let Ψ be the set of all functions ϕ such that $\phi : [0, +\infty) \to [0, +\infty)$ be a continuous and nondecreasing function with $\phi(t) = 0$ if and only if $t = 0$.

Let Υ be the set of all functions η such that $\eta : [0, +\infty) \to [0, +\infty)$ be lower semi continuous with $\eta(t) = 0$ if and only if $t = 0$.

Let Ω be the set of all functions ψ such that $\psi : [0, +\infty) \to [0, +\infty)$ be nondecreasing and continuous from the right with $\lim_{n\to\infty} \psi^n(t) = 0$ for all $t \in (0, +\infty)$. If $\psi \in \Omega$, then ψ is called Ω-map, then it is an easy matter to show that

(1) $\psi(t) < t$ for all $t \in (0, +\infty)$,
(2) $\psi(0) = 0$.

Remark 1.9. By Theorem 1.8 and the above function classes, it is an easy matter to show that $\Phi \in \Omega$.

2. Fuzzy fixed point theorems under a G-distance function

In this section, inspired by Constantin [17] and Chen et al. [11], we will show some fuzzy fixed point theorems on a space of fuzzy sets via a G-distance function. In what follows, we slightly modified the definition of G-distance functions which was introduced by Chen et al. [11].

Definition 2.1. A function g is said to be a G-distance function if $g : [0, \infty)^5 \to [0, \infty)$ is continuous function with the following properties hold:

(i) g is nondecreasing in the 2nd, 3rd, 4th, and 5th variables;
(ii) if $u, v \in [0, \infty)$ are such that $u \leqslant g(v, v, u, 0, u + v)$ or $u \leqslant g(v, u, v, 0, u + v)$ then $u \leqslant hv$ where $0 < h < 1$ is a given constant;
(iii) if $u \in [0, \infty)$ is such that $u \leqslant g(u, 0, 0, u, u)$, then $u = 0$.

Next, we introduce and prove the following result which generalizes some existing results.

Theorem 2.2. *Let* (X, d) *be a complete metric space and* g *be a* G*-distance function and* $\{F_i\}_{i=1}^{\infty}$ *a sequence of self-mappings of* $\mathcal{CB}(X)$. *Suppose that there exists an* $\mathfrak{R}$*-function* $\varphi : [0, \infty) \to [0, 1)$ *such that for each* $\mu_1, \mu_2 \in \mathcal{CB}(X)$, *and for arbitrary positive integers* i *and* j, $i \neq j$,

$$d_\infty(F_i(\mu_1), F_j(\mu_2)) \leqslant \varphi(d_\infty(\mu_1, \mu_2)) M(\mu_1, \mu_2), \tag{2.1}$$

where

$$M(\mu_1, \mu_2) = g\big(d_\infty(\mu_1, \mu_2), \rho_\infty(\mu_1, F_i(\mu_1)), \rho_\infty(\mu_2, F_j(\mu_2)), \rho_\infty(\mu_2, F_i(\mu_1)), \rho_\infty(\mu_1, F_j(\mu_2))\big).$$

Then there exists a $\mu_* \in \mathcal{CB}(X)$ *such that* $\mu_* \subseteq F_i(\mu_*)$ *for all* $i \in \mathbf{N}$.

Proof. Let $\mu_0 \in \mathcal{CB}(X)$, and $\mu_1 \subseteq F_1(\mu_0)$, by Lemma 1.2 (ii), we can get

$$\rho_\infty(\mu_0, F_1(\mu_0)) \leqslant d_\infty(\mu_0, \mu_1).$$

By induction, we produce a sequence $\{\mu_n\}$ of points of $\mathcal{CB}(X)$ such that

$$\begin{cases} \mu_{n+1} \subseteq F_{n+1}(\mu_n), \\ \rho_\infty(\mu_n, F_{n+1}(\mu_n)) \leqslant d_\infty(\mu_n, \mu_{n+1}). \end{cases} \tag{2.2}$$

Let us define a function $k : [0, \infty) \to [0, 1)$ by

$$k(t) = \frac{1 + \varphi(t)}{2} \quad \text{for all} \quad t \in [0, \infty).$$

Note that we have $0 \leqslant \varphi(t) < k(t) < 1$ for all $t \in [0, \infty)$.

We will start by picking a fuzzy set $\mu_0 \in \mathcal{CB}(X)$. We subsequently choose $\mu_1 \subseteq F_1(\mu_0)$ and a positive real number ϵ_0 such that $\epsilon_0 \in (\frac{1-k(d_\infty(\mu_0,\mu_1))}{2}, 1 - k(d_\infty(\mu_0, \mu_1)))$. Next, by using this ϵ_0, we can find a positive real number β_0 such that $\beta_0 \in (1, \frac{1-\epsilon_0}{k(d_\infty(\mu_0,\mu_1))})$. Now, by Lemma 1.3, there exists $\mu_2 \in \mathcal{CB}(X)$ such that $\mu_2 \subseteq F_2(\mu_1)$ and

$$d_\infty(\mu_1, \mu_2) \leqslant \beta_0 d_\infty(F_1(\mu_0), F_2(\mu_1)).$$

Now, We choose $\mu_2 \subseteq F_2(\mu_1)$ and a positive real number ϵ_1 such that $\epsilon_1 \in (\frac{1-k(d_\infty(\mu_1,\mu_2))}{2}, 1 - k(d_\infty(\mu_1, \mu_2)))$. Next, by using this ϵ_1, we can find a positive real number β_1 such that $\beta_1 \in (1, \frac{1-\epsilon_1}{k(d_\infty(\mu_1,\mu_2))})$. Now, by Lemma 1.3, there exists $\mu_3 \in \mathcal{CB}(X)$ such that $\mu_3 \subseteq F_3(\mu_2)$ and

$$d_\infty(\mu_2, \mu_3) \leqslant \beta_1 d_\infty(F_2(\mu_1), F_3(\mu_2)).$$

By induction, we produce two sequences of points of $\{\epsilon_n\}$ and $\{\beta_n\}$ and a sequence $\{\mu_n\}$ in $\mathcal{CB}(X)$ such that

$$\begin{cases} \mu_{n+1} \subseteq F_{n+1}(\mu_n), \\ d_\infty(\mu_{n+1}, \mu_{n+2}) \leqslant \beta_n d_\infty(F_{n+1}(\mu_n), F_{n+2}(\mu_{n+1})), \\ \epsilon_n \in (\frac{1-k(d_\infty(\mu_n,\mu_{n+1}))}{2}, 1 - k(d_\infty(\mu_n, \mu_{n+1}))), \\ \beta_n \in (1, \frac{1-\epsilon_n}{k(d_\infty(\mu_n,\mu_{n+1}))}) \end{cases} \tag{2.3}$$

for all $n \in \mathbf{N}$.

Next, we prove that $\{\mu_n\}$ is a Cauchy sequence in $\mathcal{CB}(X)$. In fact, for arbitrary positive integer n, by inequalities (2.1), (2.2), and the formula (2.3), we have

$$\begin{aligned} d_\infty(\mu_{n+1}, \mu_{n+2}) &\leqslant \beta_n d_\infty(F_{n+1}(\mu_n), F_{n+2}(\mu_{n+1})) \\ &\leqslant \beta_n \varphi(d_\infty(\mu_n, \mu_{n+1}))M(\mu_n, \mu_{n+1}) \\ &< \beta_n k(d_\infty(\mu_n, \mu_{n+1}))M(\mu_n, \mu_{n+1}), \end{aligned} \tag{2.4}$$

where

$$\begin{aligned} M(\mu_n, \mu_{n+1}) = {} & g\big(d_\infty(\mu_n, \mu_{n+1}), \rho_\infty(\mu_n, F_{n+1}(\mu_n)) \\ & , \rho_\infty(\mu_{n+1}, F_{n+2}(\mu_{n+1})), \rho_\infty(\mu_{n+1}, F_{n+1}(\mu_n)), \rho_\infty(\mu_n, F_{n+2}(\mu_{n+1}))\big) \\ \leqslant {} & g\big(d_\infty(\mu_n, \mu_{n+1}), d_\infty(\mu_n, \mu_{n+1}), d_\infty(\mu_{n+1}, \mu_{n+2}) \\ & , d_\infty(\mu_{n+1}, \mu_{n+1}), \rho_\infty(\mu_n, F_{n+2}(\mu_{n+1}))\big) \\ \leqslant {} & g\big(d_\infty(\mu_n, \mu_{n+1}), d_\infty(\mu_n, \mu_{n+1}), d_\infty(\mu_{n+1}, \mu_{n+2}), 0, \rho_\infty(\mu_n, F_{n+2}(\mu_{n+1}))\big) \\ \leqslant {} & g\big(d_\infty(\mu_n, \mu_{n+1}), d_\infty(\mu_n, \mu_{n+1}), d_\infty(\mu_{n+1}, \mu_{n+2}), 0 \\ & , d_\infty(\mu_n, \mu_{n+1}) + \rho_\infty(\mu_{n+1}, F_{n+2}(\mu_{n+1}))\big) \\ \leqslant {} & g\big(d_\infty(\mu_n, \mu_{n+1}), d_\infty(\mu_n, \mu_{n+1}), d_\infty(\mu_{n+1}, \mu_{n+2}), 0 \\ & , d_\infty(\mu_n, \mu_{n+1}) + d_\infty(\mu_{n+1}, \mu_{n+2})\big). \end{aligned} \tag{2.5}$$

Since $0 < \beta_n k(d_\infty(\mu_n, \mu_{n+1})) < 1 - \epsilon_n < 1$ for all $n \in \mathbf{N}$, by (2.4) and (2.5), then for all n, we have

$$d_\infty(\mu_{n+1}, \mu_{n+2}) < g\big(d_\infty(\mu_n, \mu_{n+1}), d_\infty(\mu_n, \mu_{n+1}), d_\infty(\mu_{n+1}, \mu_{n+2}), 0, d_\infty(\mu_n, \mu_{n+1}) + d_\infty(\mu_{n+1}, \mu_{n+2})\big)$$

From Definition 2.1 (ii), we can conclude that

$$d_\infty(\mu_{n+1}, \mu_{n+2}) \leqslant h d_\infty(\mu_n, \mu_{n+1}),$$

where $0 < h < 1$. By iteration, we have

$$d_\infty(\mu_n, \mu_{n+1}) \leqslant h d(x_{n-1}, x_n) \leqslant \cdots \leqslant h^n d(\mu_0, \mu_1).$$

Furthermore, for $m > n$,

$$\begin{aligned} d(\mu_n, \mu_m) &\leqslant d_\infty(\mu_n, \mu_{n+1}) + d_\infty(\mu_{n+1}, \mu_{n+2}) + \cdots + d_\infty(\mu_{m-1}, \mu_m) \\ &\leqslant (h^n + h^{n-1} + \cdots + h^{m-1}) d_\infty(\mu_0, \mu_1) \leqslant \frac{h^n}{1-h} d_\infty(\mu_0, \mu_1). \end{aligned}$$

It follows that $\{\mu_n\}$ is a Cauchy sequence in $\mathcal{CB}(X)$. Since X is complete, it implies that $(\mathcal{CB}(X), d_\infty)$ is complete. Thus there exists a μ_* such that $\lim_{n\to\infty} \mu_n = x_*$. Next, we show that $\{\mu_*\} \subset F_i\mu_*$ for all $i \in \mathbf{N}$.

Let $i \in \mathbf{N}$ be arbitrary. By (ii) and (iii) in Lemma 1.2, we can get

$$\rho_\infty(\mu_*, F_i(\mu_*)) \leqslant d_\infty(\mu_*, \mu_j) + \rho_\infty(\mu_j, F_i(\mu_*)) \leqslant d_\infty(\mu_*, \mu_j) + d_\infty(F_j(\mu_{j-1}), F_i(\mu_*)), \tag{2.6}$$

since $\mu_j \in F_j(\mu_{j-1})$ for arbitrary natural numbers j such that $i \neq j$. Subsequently, by using (2.1), (2.6), and (iii) in Lemma 1.2, we have

$$\begin{aligned} d_\infty(F_j(\mu_{j-1}), F_i(\mu_*)) &\leqslant \varphi(d_\infty(\mu_{j-1}, \mu_*)) g\big(d_\infty(\mu_{j-1}, \mu_*), \rho_\infty(\mu_{j-1}, F_j(\mu_{j-1})) \\ &\quad , \rho_\infty(\mu_*, F_i(\mu_*)), \rho_\infty(\mu_*, F_j(\mu_{j-1})), \rho_\infty(\mu_{j-1}, F_i(\mu_*))\big) \\ &\leqslant \varphi(d_\infty(\mu_{j-1}, \mu_*)) g\big(d_\infty(\mu_{j-1}, \mu_*), \rho_\infty(\mu_{j-1}, F_j(\mu_{j-1})) \\ &\quad , d_\infty(\mu_*, \mu_j) + \rho_\infty(\mu_j, F_i(\mu_*)) \\ &\quad , d_\infty(\mu_*, \mu_j) + \rho_\infty(\mu_j, F_j(\mu_{j-1})), d_\infty(\mu_{j-1}, \mu_*) + \rho_\infty(\mu_*, F_i(\mu_*))\big) \\ &< g\big(d_\infty(\mu_{j-1}, \mu_*), d_\infty(\mu_{j-1}, \mu_j), d_\infty(\mu_*, \mu_j) + \rho_\infty(\mu_j, F_i(\mu_*)) \\ &\quad , d_\infty(\mu_*, \mu_j) + d_\infty(\mu_j, \mu_j), d_\infty(\mu_{j-1}, \mu_*) + \rho_\infty(\mu_*, F_i(\mu_*))\big). \end{aligned} \tag{2.7}$$

Letting $j \to \infty$ in inequalities (2.6) and (2.7), we obtain

$$\rho_\infty(\mu_*, F_i(\mu_*)) < g\big(0, 0, \rho_\infty((\mu_*), F_i(\mu_*)), 0, \rho_\infty(\mu_*, F_i(\mu_*))\big).$$

Using (ii) in Definition 2.1, we can get $\rho_\infty(\mu_*, F_i\mu_*) = 0$. Therefore, we have $\{\mu_*\} \subset F_i\mu_*$. □

Remark 2.3.

(1) If we choose $\varphi(t) = \lambda (0 < \lambda < 1)$ and $g(x_1, x_2, x_3, x_4, x_5) = \delta \max\left\{x_1, x_2, x_3, \frac{x_4+x_5}{2}\right\} (0 < \delta < 1)$, then by Theorem 2.2, As long as we take $0 < q = \lambda\delta < 1$, then we can get Theorem 1.6.

(2) Since we are considering a larger class of G-distance function, Theorem 2.2 improves Theorem 1.7.

Next, we want to give some results by using a G-distance function in compact metric spaces.

Theorem 2.4. *Let* (X, d) *be a compact metric space and* g *be a* G*-distance function and* $\{F_i\}_{i=1}^{\infty}$ *a sequence of self-mappings of* $\mathcal{C}(X)$*. Suppose that there exist* $\phi \in \Psi$ *and* $L \geqslant 0$ *such that for each* $\mu_1, \mu_2 \in \mathcal{C}(X)$*, and for arbitrary positive integers* i *and* j, $i \neq j$,

$$\phi(d_\infty(F_i(\mu_1), F_j(\mu_2))) \leqslant \phi(M(\mu_1, \mu_2)) + LN(\mu_1, \mu_2), \tag{2.8}$$

where

$$M(\mu_1,\mu_2)=g\big(d_\infty(\mu_1,\mu_2),\rho_\infty(\mu_1,F_i(\mu_1)),\rho_\infty(\mu_2,F_j(\mu_2)),\rho_\infty(\mu_2,F_i(\mu_1)),\rho_\infty(\mu_1,F_j(\mu_2))\big),$$
$$N(\mu_1,\mu_2)=\min\big\{d_\infty(\mu_1,\mu_2),\rho_\infty(\mu_1,F_i(\mu_1)),\rho_\infty(\mu_2,F_j(\mu_2)),\rho_\infty(\mu_2,F_i(\mu_1)),\rho_\infty(\mu_1,F_j(\mu_2))\big\}.$$

Then there exists a $\mu_* \in \mathcal{C}(X)$ *such that* $\mu_* \subseteq F_i(\mu_*)$ *for all* $i \in \boldsymbol{N}$.

Proof. Let $\mu_0 \in \mathcal{C}(X)$, and $\mu_1 \subseteq F_1(\mu_0)$, by Lemma 1.4, there exists a μ_2 such that $\mu_2 \subseteq F_2(\mu_1)$ and

$$d_\infty(\mu_1,\mu_2) \leqslant d_\infty(F_1(\mu_0),F_2(\mu_1)).$$

Again by Lemma 1.4, we can find $\mu_3 \in \mathcal{C}(X)$ such that $\mu_3 \subseteq F_3(\mu_2)$,

$$d_\infty(\mu_2,\mu_3) \leqslant d_\infty(F_2(\mu_1),F_3(\mu_2)).$$

By induction, we produce a sequence $\{\mu_n\}$ of points of $\mathcal{C}(X)$ such that

$$\begin{cases}\mu_{n+2} \subseteq F_{n+2}(\mu_{n+1}),\\ d_\infty(\mu_{n+1},\mu_{n+2}) \leqslant d_\infty(F_{n+1}(\mu_n),F_{n+2}(\mu_{n+1})).\end{cases} \tag{2.9}$$

Next, we prove that $\{\mu_n\}$ is a Cauchy sequence in $\mathcal{C}(X)$. In fact, for arbitrary positive integer n, by inequality (2.8), formula (2.9), and the properties of ϕ, we have

$$\phi(d_\infty(\mu_{n+1},\mu_{n+2})) \leqslant \phi(d_\infty(F_{n+1}(\mu_n),F_{n+2}(\mu_{n+1}))) \leqslant \phi(M(\mu_n,\mu_{n+1}))+LN(\mu_n,\mu_{n+1}), \tag{2.10}$$

where

$$\begin{aligned}M(\mu_n,\mu_{n+1}) &= g\big(d_\infty(\mu_n,\mu_{n+1}),\rho_\infty(\mu_n,F_{n+1}(\mu_n))\\&\quad,\rho_\infty(\mu_{n+1},F_{n+2}(\mu_{n+1})),\rho_\infty(\mu_{n+1},F_{n+1}(\mu_n)),\rho_\infty(\mu_n,F_{n+2}(\mu_{n+1}))\big)\\&\leqslant g\big(d_\infty(\mu_n,\mu_{n+1}),d_\infty(\mu_n,\mu_{n+1}),d_\infty(\mu_{n+1},\mu_{n+2})\\&\quad,d_\infty(\mu_{n+1},\mu_{n+1}),\rho_\infty(\mu_n,F_{n+2}(\mu_{n+1}))\big)\\&\leqslant g\big(d_\infty(\mu_n,\mu_{n+1}),d_\infty(\mu_n,\mu_{n+1}),d_\infty(\mu_{n+1},\mu_{n+2}),0,\rho_\infty(\mu_n,F_{n+2}(\mu_{n+1}))\big)\\&\leqslant g\big(d_\infty(\mu_n,\mu_{n+1}),d_\infty(\mu_n,\mu_{n+1}),d_\infty(\mu_{n+1},\mu_{n+2}),0\\&\quad,d_\infty(\mu_n,\mu_{n+1})+\rho_\infty(\mu_{n+1},F_{n+2}(\mu_{n+1}))\big)\\&\leqslant g\big(d_\infty(\mu_n,\mu_{n+1}),d_\infty(\mu_n,\mu_{n+1}),d_\infty(\mu_{n+1},\mu_{n+2}),0\\&\quad,d_\infty(\mu_n,\mu_{n+1})+d_\infty(\mu_{n+1},\mu_{n+2})\big),\end{aligned} \tag{2.11}$$

$$\begin{aligned}N(\mu_n,\mu_{n+1}) &= \min\big\{d_\infty(\mu_n,\mu_{n+1}),\rho_\infty(\mu_n,F_{n+1}(\mu_n))\\&\quad,\rho_\infty(\mu_{n+1},F_{n+2}(\mu_{n+1})),\rho_\infty(\mu_{n+1},F_{n+1}(\mu_n)),\rho_\infty(\mu_n,F_{n+2}(\mu_{n+1}))\big\}\\&\leqslant \min\big\{d_\infty(\mu_n,\mu_{n+1}),d_\infty(\mu_n,\mu_{n+1}),d_\infty(\mu_{n+1},\mu_{n+2})\\&\quad,d_\infty(\mu_{n+1},\mu_{n+1}),\rho_\infty(\mu_n,F_{n+2}(\mu_{n+1}))\big\}\\&\leqslant \min\big\{d_\infty(\mu_n,\mu_{n+1}),d_\infty(\mu_n,\mu_{n+1}),d_\infty(\mu_{n+1},\mu_{n+2}),0,\rho_\infty(\mu_n,F_{n+2}(\mu_{n+1}))\big\}=0.\end{aligned} \tag{2.12}$$

Thus, from (2.12), we can get

$$N(\mu_n,\mu_{n+1})=0. \tag{2.13}$$

By (2.10), (2.11), (2.13) and the nondecreasing character of ϕ, we have

$$d_\infty(\mu_{n+1},\mu_{n+2}) \leqslant g\big(d_\infty(\mu_n,\mu_{n+1}),d_\infty(\mu_n,\mu_{n+1}),d_\infty(\mu_{n+1},\mu_{n+2}),0,d_\infty(\mu_n,\mu_{n+1})+d_\infty(\mu_{n+1},\mu_{n+2})\big).$$

From Definition 2.1 (ii), we can conclude that

$$d_\infty(\mu_{n+1}, \mu_{n+2}) \leqslant h d_\infty(\mu_n, \mu_{n+1}),$$

where $0 < h < 1$. By iteration, we have

$$d_\infty(\mu_n, \mu_{n+1}) \leqslant h d(x_{n-1}, x_n) \leqslant \cdots \leqslant h^n d(\mu_0, \mu_1).$$

Furthermore, for $m > n$,

$$\begin{aligned} d(\mu_n, \mu_m) &\leqslant d_\infty(\mu_n, \mu_{n+1}) + d_\infty(\mu_{n+1}, \mu_{n+2}) + \cdots + d_\infty(\mu_{m-1}, \mu_m) \\ &\leqslant (h^n + h^{n-1} + \cdots + h^{m-1}) d_\infty(\mu_0, \mu_1) \leqslant \frac{h^n}{1-h} d_\infty(\mu_0, \mu_1). \end{aligned}$$

It follows that $\{\mu_n\}$ is a Cauchy sequence in $\mathcal{C}(X)$. Since X is compact, it implies that X is complete. Thus there exists a μ_* such that $\lim\limits_{n\to\infty} \mu_n = x_*$. Next, we show that $\{\mu_*\} \subset F_i \mu_*$ for all $i \in \mathbf{N}$.

Let $i \in \mathbf{N}$ be arbitrary. By (ii) and (iii) in Lemma 1.2, let us notice that

$$\rho_\infty(\mu_*, F_i(\mu_*)) \leqslant d_\infty(\mu_*, \mu_j) + \rho_\infty(\mu_j, F_i(\mu_*)) \leqslant d_\infty(\mu_*, \mu_j) + d_\infty(F_j(\mu_{j-1}), F_i(\mu_*)),$$

since $\mu_j \in F_j(\mu_{j-1})$ for arbitrary natural numbers j such that $i \neq j$. From

$$|\rho_\infty(\mu_*, F_i(\mu_*)) - d_\infty(\mu_*, \mu_j)| \leqslant d_\infty(F_j(\mu_{j-1}), F_i(\mu_*)),$$

(2.8), and the nondecreasing character of ϕ, we have

$$\phi(|\rho_\infty(\mu_*, F_i(\mu_*)) - d_\infty(\mu_*, \mu_j)|) \leqslant \phi(d_\infty(F_j(\mu_{j-1}), F_i(\mu_*))) \leqslant \phi(M(\mu_{j-1}, \mu_*)) + LN(\mu_{j-1}, \mu_*), \tag{2.14}$$

where

$$\begin{aligned} M(\mu_{j-1}, \mu_*) &= g\big(d_\infty(\mu_{j-1}, \mu_*), \rho_\infty(\mu_{j-1}, F_j(\mu_{j-1})) \\ &\quad, \rho_\infty(\mu_*, F_i(\mu_*)), \rho_\infty(\mu_*, F_j(\mu_{j-1})), \rho_\infty(\mu_{j-1}, F_i(\mu_*))\big) \\ &\leqslant g\big(d_\infty(\mu_{j-1}, \mu_*), d_\infty(\mu_{j-1}, \mu_j), d_\infty(\mu_*, \mu_j) + \rho_\infty(\mu_j, F_i(\mu_*)) \\ &\quad, d_\infty(\mu_*, \mu_j) + d_\infty(\mu_j, \mu_j)), d_\infty(\mu_{j-1}, \mu_*) + \rho_\infty(\mu_*, F_i(\mu_*))\big) \end{aligned} \tag{2.15}$$

and

$$\begin{aligned} N(\mu_{j-1}, \mu_*) &= \min\big\{d_\infty(\mu_{j-1}, \mu_*), \rho_\infty(\mu_{j-1}, F_j(\mu_{j-1})) \\ &\quad, \rho_\infty(\mu_*, F_i(\mu_*)), \rho_\infty(\mu_*, F_j(\mu_{j-1})), \rho_\infty(\mu_{j-1}, F_i(\mu_*))\big\} \\ &\leqslant \min\big\{d_\infty(\mu_{j-1}, \mu_*), d_\infty(\mu_{j-1}, \mu_j)), d_\infty(\mu_*, \mu_j) + \rho_\infty(\mu_j, F_i(\mu_*)) \\ &\quad, d_\infty(\mu_*, \mu_j) + d_\infty(\mu_j, \mu_j)), d_\infty(\mu_{j-1}, \mu_*) + \rho_\infty(\mu_*, F_i(\mu_*))\big\}. \end{aligned} \tag{2.16}$$

Letting $j \to \infty$ in inequalities (2.14)-(2.16), we obtain

$$\phi(\rho_\infty(\mu_*, F_i(\mu_*))) \leqslant \phi\big(g(0, 0, \rho_\infty(\mu_*, F_i(\mu_*)), 0, \rho_\infty(\mu_*, F_i(\mu_*)))\big).$$

By the nondecreasing character of ϕ, we have

$$\rho_\infty(\mu_*, F_i(\mu_*)) < g\big(0, 0, \rho_\infty(\mu_*, F_i(\mu_*)), 0, \rho_\infty(\mu_*, F_i(\mu_*))\big).$$

Using (ii) in Definition 2.1, we can get $\rho_\infty(\mu_*, F_i \mu_*) = 0$. Therefore, we have $\{\mu_*\} \subset F_i \mu_*$. □

If in Theorem 2.4 we choose $\phi(t) = t$, we can get the following corollary.

Corollary 2.5. *Let* (X, d) *be a compact metric space and* g *be a* G*-distance function and* $\{F_i\}_{i=1}^{\infty}$ *be a sequence of self-mappings of* $\mathcal{C}(X)$*. Suppose that there exists an* $L \geqslant 0$ *such that for each* $\mu_1, \mu_2 \in \mathcal{C}(X)$*, and for arbitrary positive integers* i *and* j*,* $i \neq j$*,*

$$d_\infty(F_i(\mu_1), F_j(\mu_2)) \leqslant M(\mu_1, \mu_2) + LN(\mu_1, \mu_2),$$

where

$$M(\mu_1, \mu_2) = g\big(d_\infty(\mu_1, \mu_2), \rho_\infty(\mu_1, F_i(\mu_1)), \rho_\infty(\mu_2, F_j(\mu_2)), \rho_\infty(\mu_2, F_i(\mu_1)), \rho_\infty(\mu_1, F_j(\mu_2))\big),$$
$$N(\mu_1, \mu_2) = \min\big\{d_\infty(\mu_1, \mu_2), \rho_\infty(\mu_1, F_i(\mu_1)), \rho_\infty(\mu_2, F_j(\mu_2)), \rho_\infty(\mu_2, F_i(\mu_1)), \rho_\infty(\mu_1, F_j(\mu_2))\big\}.$$

Then there exists a $\mu_* \in \mathcal{C}(X)$ *such that* $\mu_* \subseteq F_i(\mu_*)$ *for all* $i \in \boldsymbol{N}$*.*

Corollary 2.6. *Let* (X, d) *be a compact metric space and* g *be a* G*-distance function and* $\{F_i\}_{i=1}^{\infty}$ *be a sequence of self-mappings of* $\mathcal{C}(X)$*. Suppose that there exist* $\phi \in \Psi$ *and* $\mathfrak{R}$*-function* $\varphi : [0, \infty) \to [0, 1)$ *such that for each* $\mu_1, \mu_2 \in \mathcal{C}(X)$*, and for arbitrary positive integers* i *and* j*,* $i \neq j$*,*

$$\phi(d_\infty(F_i(\mu_1), F_j(\mu_2))) \leqslant \phi(M(\mu_1, \mu_2)) - \varphi(d_\infty(\mu_1, \mu_2)),$$

where

$$M(\mu_1, \mu_2) = g\big(d_\infty(\mu_1, \mu_2), \rho_\infty(\mu_1, F_i(\mu_1)), \rho_\infty(\mu_2, F_j(\mu_2)), \rho_\infty(\mu_2, F_i(\mu_1)), \rho_\infty(\mu_1, F_j(\mu_2))\big).$$

Then there exists a $\mu_* \in \mathcal{C}(X)$ *such that* $\mu_* \subseteq F_i(\mu_*)$ *for all* $i \in \boldsymbol{N}$*.*

Proof. Since

$$\phi(d_\infty(F_i(\mu_1), F_j(\mu_2))) \leqslant \phi(M(\mu_1, \mu_2)) - \varphi(d_\infty(\mu_1, \mu_2)) \leqslant \phi(M(\mu_1, \mu_2)) + LN(\mu_1, \mu_2),$$

Hence, by using Theorem 2.4, there exists a point μ_* in $\mathcal{C}(X)$ such that $\{\mu_*\} \subset F_i(\mu_*)$. □

If in Corollary 2.6 we chose $\phi(t) = t$, we can obtain the following corollary.

Corollary 2.7. *Let* (X, d) *be a compact metric space and* g *be a* G*-distance function and* $\{F_i\}_{i=1}^{\infty}$ *be a sequence of self-mappings of* $\mathcal{C}(X)$*. Suppose that there exists an* $\mathfrak{R}$*-function* $\varphi : [0, \infty) \to [0, 1)$ *such that for each* $\mu_1, \mu_2 \in \mathcal{C}(X)$*, and for arbitrary positive integers* i *and* j*,* $i \neq j$*,*

$$d_\infty(F_i(\mu_1), F_j(\mu_2)) \leqslant M(\mu_1, \mu_2) - \varphi(d_\infty(\mu_1, \mu_2)),$$

where

$$M(\mu_1, \mu_2) = g\big(d_\infty(\mu_1, \mu_2), \rho_\infty(\mu_1, F_i(\mu_1)), \rho_\infty(\mu_2, F_j(\mu_2)), \rho_\infty(\mu_2, F_i(\mu_1)), \rho_\infty(\mu_1, F_j(\mu_2))\big).$$

Then there exists a $\mu_* \in \mathcal{C}(X)$ *such that* $\mu_* \subseteq F_i(\mu_*)$ *for all* $i \in \boldsymbol{N}$*.*

3. Fuzzy fixed point theorems under a G'-distance function

In this section, inspired by Constantin [17] and Chen et al. [11], we will show some fuzzy fixed point theorems on a space of fuzzy sets via a G'-distance function. In what follows, we give the definition of G'-distance functions which are introduced by Chen et al. [11].

Definition 3.1. A function g is said to be a G'-distance function if $g : [0, \infty)^5 \to [0, \infty)$ is continuous function with the following properties hold:

(i) g is increasing in each co-ordinate variable;
(ii) $g(t, t, t, at, bt) \leqslant t$ for every $t \in [0, \infty)$, where $a + b = 2$.

Now, we establish and prove the following fixed point theorem.

Theorem 3.2. *Let (X, d) be a compact metric space and g be a G'-distance function and $\{F_i\}_{i=1}^{\infty}$ be a sequence of self-mappings of $\mathcal{C}(X)$. Suppose that there exist $\psi \in \Omega$, $\eta \in \Upsilon$ and $L \geqslant 0$ such that for each $\mu_1, \mu_2 \in \mathcal{C}(X)$, and for arbitrary positive integers i and j, $i \neq j$,*

$$d_\infty(F_i(\mu_1), F_j(\mu_2)) \leqslant \psi(M(\mu_1, \mu_2)) + L\eta(N(\mu_1, \mu_2)), \tag{3.1}$$

where

$$\begin{aligned} M(\mu_1, \mu_2) &= g\big(d_\infty(\mu_1, \mu_2), \rho_\infty(\mu_1, F_i(\mu_1)), \rho_\infty(\mu_2, F_j(\mu_2)), \rho_\infty(\mu_2, F_i(\mu_1)), \rho_\infty(\mu_1, F_j(\mu_2))\big), \\ N(\mu_1, \mu_2) &= \min\big\{d_\infty(\mu_1, \mu_2), \rho_\infty(\mu_1, F_i(\mu_1)), \rho_\infty(\mu_2, F_j(\mu_2)), \rho_\infty(\mu_2, F_i(\mu_1)), \rho_\infty(\mu_1, F_j(\mu_2))\big\}. \end{aligned}$$

Then there exists a $\mu_* \in \mathcal{C}(X)$ such that $\mu_* \subseteq F_i(\mu_*)$ for all $i \in \boldsymbol{N}$.

Proof. Let $\mu_0 \in \mathcal{C}(X)$ and $\mu_1 \subseteq F_1(\mu_0)$, by Lemma 1.4, there exists a μ_2 such that $\mu_2 \subseteq F_2(\mu_1)$ and

$$d_\infty(\mu_1, \mu_2) \leqslant d_\infty(F_1(\mu_0), F_2(\mu_1)).$$

Again by Lemma 1.4, we can find $\mu_3 \in \mathcal{C}(X)$ such that $\mu_3 \subseteq F_3(\mu_2)$,

$$d_\infty(\mu_2, \mu_3) \leqslant d_\infty(F_2(\mu_1), F_3(\mu_2)).$$

By induction, we produce a sequence $\{\mu_n\}$ of points of $\mathcal{C}(X)$ such that

$$\begin{cases} \mu_{n+2} \subseteq F_{n+2}(\mu_{n+1}), \\ d_\infty(\mu_{n+1}, \mu_{n+2}) \leqslant d_\infty(F_{n+1}(\mu_n), F_{n+2}(\mu_{n+1})). \end{cases} \tag{3.2}$$

Next, we prove that $\{\mu_n\}$ is a Cauchy sequence in $\mathcal{C}(X)$. In fact, for arbitrary positive integer n, by the inequality (3.1), the formula (3.2) and the properties of ψ, we have

$$d_\infty(\mu_{n+1}, \mu_{n+2}) \leqslant d_\infty(F_{n+1}(\mu_n), F_{n+2}(\mu_{n+1})) \leqslant \psi(M(\mu_n, \mu_{n+1})) + L\eta(N(\mu_n, \mu_{n+1})), \tag{3.3}$$

where

$$\begin{aligned} M(\mu_n, \mu_{n+1}) &= g\big(d_\infty(\mu_n, \mu_{n+1}), \rho_\infty(\mu_n, F_{n+1}(\mu_n)) \\ &\quad, \rho_\infty(\mu_{n+1}, F_{n+2}(\mu_{n+1})), \rho_\infty(\mu_{n+1}, F_{n+1}(\mu_n)), \rho_\infty(\mu_n, F_{n+2}(\mu_{n+1}))\big) \\ &\leqslant g\big(d_\infty(\mu_n, \mu_{n+1}), d_\infty(\mu_n, \mu_{n+1}), d_\infty(\mu_{n+1}, \mu_{n+2}) \\ &\quad, d_\infty(\mu_{n+1}, \mu_{n+1}), \rho_\infty(\mu_n, F_{n+2}(\mu_{n+1}))\big) \\ &\leqslant g\big(d_\infty(\mu_n, \mu_{n+1}), d_\infty(\mu_n, \mu_{n+1}), d_\infty(\mu_{n+1}, \mu_{n+2}), 0, \rho_\infty(\mu_n, F_{n+2}(\mu_{n+1}))\big) \\ &\leqslant g\big(d_\infty(\mu_n, \mu_{n+1}), d_\infty(\mu_n, \mu_{n+1}), d_\infty(\mu_{n+1}, \mu_{n+2}), 0 \\ &\quad, d_\infty(\mu_n, \mu_{n+1}) + \rho_\infty(\mu_{n+1}, F_{n+2}(\mu_{n+1}))\big) \\ &\leqslant g\big(d_\infty(\mu_n, \mu_{n+1}), d_\infty(\mu_n, \mu_{n+1}), d_\infty(\mu_{n+1}, \mu_{n+2}), 0 \\ &\quad, d_\infty(\mu_n, \mu_{n+1}) + d_\infty(\mu_{n+1}, \mu_{n+2})\big), \end{aligned} \tag{3.4}$$

$$\begin{aligned} N(\mu_n, \mu_{n+1}) &= \min\big\{d_\infty(\mu_n, \mu_{n+1}), \rho_\infty(\mu_n, F_{n+1}(\mu_n)) \\ &\quad, \rho_\infty(\mu_{n+1}, F_{n+2}(\mu_{n+1})), \rho_\infty(\mu_{n+1}, F_{n+1}(\mu_n)), \rho_\infty(\mu_n, F_{n+2}(\mu_{n+1}))\big\} \\ &\leqslant \min\big\{d_\infty(\mu_n, \mu_{n+1}), d_\infty(\mu_n, \mu_{n+1}), d_\infty(\mu_{n+1}, \mu_{n+2}) \\ &\quad, d_\infty(\mu_{n+1}, \mu_{n+1}), \rho_\infty(\mu_n, F_{n+2}(\mu_{n+1}))\big\} \\ &\leqslant \min\big\{d_\infty(\mu_n, \mu_{n+1}), d_\infty(\mu_n, \mu_{n+1}), d_\infty(\mu_{n+1}, \mu_{n+2}), 0, \rho_\infty(\mu_n, F_{n+2}(\mu_{n+1}))\big\} = 0. \end{aligned} \tag{3.5}$$

Thus, from (3.5) and the properties of η, we can get

$$\eta(N(\mu_n, \mu_{n+1})) = 0. \tag{3.6}$$

By (3.3), (3.4), (3.6), and the nondecreasing character of ψ, we have

$$\begin{aligned} d_\infty(\mu_{n+1}, \mu_{n+2}) \leqslant \psi\big(g\big(&d_\infty(\mu_n, \mu_{n+1}), d_\infty(\mu_n, \mu_{n+1}), d_\infty(\mu_{n+1}, \mu_{n+2}), 0 \\ &, d_\infty(\mu_n, \mu_{n+1}) + d_\infty(\mu_{n+1}, \mu_{n+2})\big)\big). \end{aligned} \tag{3.7}$$

Now, we prove that $d_\infty(\mu_{n+1}, \mu_{n+2}) \leqslant d_\infty(\mu_n, \mu_{n+1})$. If $d_\infty(\mu_{n+1}, \mu_{n+2}) > d_\infty(\mu_n, \mu_{n+1})$, then from (3.7) and the nondecreasing character of ψ and g, we can get

$$d_\infty(\mu_{n+1}, \mu_{n+2}) \leqslant \psi\big(g\big(d_\infty(\mu_{n+1}, \mu_{n+2}), d_\infty(\mu_{n+1}, \mu_{n+2}), d_\infty(\mu_{n+1}, \mu_{n+2}), 0, 2d_\infty(\mu_{n+1}, \mu_{n+2})\big)\big).$$

Since g is a G'-distance function, by (ii) of Definition 3.1, we can conclude that

$$\begin{aligned} d_\infty(\mu_{n+1}, \mu_{n+2}) \leqslant \psi\big(g\big(&d_\infty(\mu_{n+1}, \mu_{n+2}), d_\infty(\mu_{n+1}, \mu_{n+2}), d_\infty(\mu_{n+1}, \mu_{n+2}), 0 \times d_\infty(\mu_{n+1}, \mu_{n+2}) \\ &, 2d_\infty(\mu_{n+1}, \mu_{n+2})\big)\big) \leqslant \psi(d_\infty(\mu_{n+1}, \mu_{n+2})) < d_\infty(\mu_{n+1}, \mu_{n+2}), \end{aligned} \tag{3.8}$$

which is a contradiction. Hence, we have $d_\infty(\mu_{n+1}, \mu_{n+2}) \leqslant d_\infty(\mu_n, \mu_{n+1})$. By (3.8) and the nondecreasing character of ψ, we have $d_\infty(\mu_{n+1}, \mu_{n+2}) \leqslant \psi(d_\infty(\mu_{n+1}, \mu_{n+2})) \leqslant \psi(d_\infty(\mu_n, \mu_{n+1}))$. Therefore, for all n, we can conclude that $d_\infty(\mu_n, \mu_{n+1}) \leqslant d_\infty(\mu_{n-1}, \mu_n)$. Therefore, for positive integers m, n $(n > m)$, we get

$$\begin{aligned} d_\infty(\mu_m, \mu_n) &\leqslant d_\infty(\mu_m, \mu_{m+1}) + \cdots + d_\infty(\mu_{n-1}, \mu_n) \\ &< \psi^m(d_\infty(\mu_0, \mu_1)) + \cdots + \psi^{n-1}(d_\infty(\mu_0, \mu_1)) \\ &= \sum_{k=m}^{n-1} \psi^n(d_\infty(\mu_0, \mu_1)). \end{aligned} \tag{3.9}$$

In (3.9), as $m, n \to \infty$, we have $d_\infty(\mu_m, \mu_n) \to 0$. It follows that $\{\mu_n\}$ is a Cauchy sequence in $\mathcal{C}(X)$. Since X is compact, it implies that X is complete. Thus there exists a μ_* such that $\lim\limits_{n\to\infty} \mu_n = x_*$. Next, we show that $\{\mu_*\} \subset F_i\mu_*$ for all $i \in \mathbf{N}$.

Let $i \in \mathbf{N}$ be arbitrary. By (ii) and (iii) in Lemma 1.2, let us notice that

$$\begin{aligned} \rho_\infty(\mu_*, F_i(\mu_*)) &\leqslant d_\infty(\mu_*, \mu_j) + \rho_\infty(\mu_j, F_i(\mu_*)) \\ &\leqslant d_\infty(\mu_*, \mu_j) + d_\infty(F_j(\mu_{j-1}), F_i(\mu_*)) \\ &\leqslant d_\infty(\mu_*, \mu_j) + \psi(M(\mu_{j-1}, \mu_*)) + L\eta(N(\mu_{j-1}, \mu_*)), \end{aligned} \tag{3.10}$$

since $\mu_j \in F_j(\mu_{j-1})$ for arbitrary natural numbers j such that $i \neq j$, where

$$\begin{aligned} M(\mu_{j-1}, \mu_*) = g\big(&d_\infty(\mu_{j-1}, \mu_*), \rho_\infty(\mu_{j-1}, F_j(\mu_{j-1})), \\ &, \rho_\infty(\mu_*, F_i(\mu_*)), \rho_\infty(\mu_*, F_j(\mu_{j-1})), \rho_\infty(\mu_{j-1}, F_i(\mu_*))\big) \\ \leqslant g\big(&d_\infty(\mu_{j-1}, \mu_*), d_\infty(\mu_{j-1}, \mu_j), d_\infty(\mu_*, \mu_j) + \rho_\infty(\mu_j, F_i(\mu_*)) \\ &, d_\infty(\mu_*, \mu_j) + d_\infty(\mu_j, \mu_j), d_\infty(\mu_{j-1}, \mu_*) + \rho_\infty(\mu_*, F_i(\mu_*))\big) \end{aligned} \tag{3.11}$$

and

$$\begin{aligned} N(\mu_{j-1}, \mu_*) = \min\big\{&d_\infty(\mu_{j-1}, \mu_*), \rho_\infty(\mu_{j-1}, F_j(\mu_{j-1})) \\ &, \rho_\infty(\mu_*, F_i(\mu_*)), \rho_\infty(\mu_*, F_j(\mu_{j-1})), \rho_\infty(\mu_{j-1}, F_i(\mu_*))\big\} \\ \leqslant \min\big\{&d_\infty(\mu_{j-1}, \mu_*), d_\infty(\mu_{j-1}, \mu_j), d_\infty(\mu_*, \mu_j) + \rho_\infty(\mu_j, F_i(\mu_*)) \\ &, d_\infty(\mu_*, \mu_j) + d_\infty(\mu_j, \mu_j), d_\infty(\mu_{j-1}, \mu_*) + \rho_\infty(\mu_*, F_i(\mu_*))\big\}. \end{aligned} \tag{3.12}$$

Letting $j \to \infty$ in inequalities (3.10)-(3.12), we obtain

$$\rho_\infty(\mu_*, F_i(\mu_*)) \leqslant 0 + \psi\big(g(0, 0, \rho_\infty(\mu_*, F_i(\mu_*)), 0, \rho_\infty(\mu_*, F_i(\mu_*)))\big) + L\eta(0).$$

By the nondecreasing character of ψ, we have

$$\begin{aligned}\rho_\infty(\mu_*, F_i(\mu_*)) &\leqslant g\big(0, 0, \rho_\infty(\mu_*, F_i(\mu_*)), 0, \rho_\infty(\mu_*, F_i(\mu_*))\big)\\ &\leqslant \psi\big(g\big(\rho_\infty(\mu_*, F_i(\mu_*)), \rho_\infty(\mu_*, F_i(\mu_*)), \rho_\infty(\mu_*, F_i(\mu_*)), \rho_\infty(\mu_*, F_i(\mu_*)), \rho_\infty(\mu_*, F_i(\mu_*))\big)\big)\\ &\leqslant \psi\big(\rho_\infty(\mu_*, F_i(\mu_*))\big) < \rho_\infty(\mu_*, F_i(\mu_*)),\end{aligned}$$

which is a contradiction. Hence, we can get $\rho_\infty(\mu_*, F_i\mu_*) = 0$. Therefore, we have $\{\mu_*\} \subset F_i\mu_*$. □

If in Theorem 3.2 we choose $L = 0$, then we can get the following corollary.

Corollary 3.3. *Let (X, d) be a compact metric space and g be a G'-distance function and $\{F_i\}_{i=1}^{\infty}$ be a sequence of self-mappings of $\mathcal{C}(X)$. Suppose that there exists a $\psi \in \Omega$ such that for each $\mu_1, \mu_2 \in \mathcal{C}(X)$, and for arbitrary positive integers i and j, $i \neq j$,*

$$d_\infty(F_i(\mu_1), F_j(\mu_2)) \leqslant \psi(M(\mu_1, \mu_2)),$$

where

$$M(\mu_1, \mu_2) = g\big(d_\infty(\mu_1, \mu_2), \rho_\infty(\mu_1, F_i(\mu_1)), \rho_\infty(\mu_2, F_j(\mu_2)), \rho_\infty(\mu_2, F_i(\mu_1)), \rho_\infty(\mu_1, F_j(\mu_2))\big).$$

Then there exists a $\mu_* \in \mathcal{C}(X)$ such that $\mu_* \subseteq F_i(\mu_*)$ for all $i \in \boldsymbol{N}$.

Remark 3.4. If in Corollary 3.3, we choose $g(x_1, x_2, x_3, x_4, x_5) = \max\left\{x_1, x_2, x_3, \frac{x_4+x_5}{2}\right\}$, then we can get Theorem 1.8.

If in Corollary 3.3, we choose $\psi(t) = qt$ where $0 < q < 1$, then we can get the following corollary.

Corollary 3.5. *Let (X, d) be a compact metric space and g be a G'-distance function and $\{F_i\}_{i=1}^{\infty}$ be a sequence of self-mappings of $\mathcal{C}(X)$. Suppose that there exists a $q \in (0, 1)$ such that for each $\mu_1, \mu_2 \in \mathcal{C}(X)$, and for arbitrary positive integers i and j, $i \neq j$,*

$$d_\infty(F_i(\mu_1), F_j(\mu_2)) \leqslant q(M(\mu_1, \mu_2)),$$

where

$$M(\mu_1, \mu_2) = g\big(d_\infty(\mu_1, \mu_2), \rho_\infty(\mu_1, F_i(\mu_1)), \rho_\infty(\mu_2, F_j(\mu_2)), \rho_\infty(\mu_2, F_i(\mu_1)), \rho_\infty(\mu_1, F_j(\mu_2))\big).$$

Then there exists a $\mu_* \in \mathcal{C}(X)$ such that $\mu_* \subseteq F_i(\mu_*)$ for all $i \in \boldsymbol{N}$.

If in Corollary 3.5, we choose $g(x_1, x_2, x_3, x_4, x_5) = \max\left\{x_1, x_2, x_3, \frac{x_4+x_5}{2}\right\}$, then we can get the following corollary.

Corollary 3.6. *Let (X, d) be a compact metric space and g be a G'-distance function and $\{F_i\}_{i=1}^{\infty}$ be a sequence of self-mappings of $\mathcal{C}(X)$. Suppose that there exists a $q \in (0, 1)$ such that for each $\mu_1, \mu_2 \in \mathcal{C}(X)$, and for arbitrary positive integers i and j, $i \neq j$,*

$$d_\infty(F_i(\mu_1), F_j(\mu_2)) \leqslant q\Big(\max\Big\{d_\infty(\mu_1, \mu_2), \rho_\infty(\mu_1, F_i(\mu_1)), \rho_\infty(\mu_2, F_j(\mu_2)), \frac{\rho_\infty(\mu_2, F_i(\mu_1)) + \rho_\infty(\mu_1, F_j(\mu_2))}{2}\Big\}\Big).$$

Then there exists a $\mu_* \in \mathcal{C}(X)$ such that $\mu_* \subseteq F_i(\mu_*)$ for all $i \in \boldsymbol{N}$.

4. Applications

In this section, we mainly want to give some applications by using our mainly results. Firstly, we give an application to illustrate the usefulness of Theorem 2.2.

Theorem 4.1. *Let* (X, d) *be a compact metric space and* $\{F_i\}_{i=1}^{\infty}$ *a sequence of self-mappings of* $\mathcal{C}(X)$ *satisfying the following conditions:*

$$[d_\infty(F_i(\mu_1), F_j(\mu_2))]^2 \leqslant \alpha_1[d_\infty(\mu_1, \mu_2)]^2 + \alpha_2\rho(\mu_1, F_i(\mu_1))\rho(\mu_2, F_j(\mu_2)) + \alpha_3\rho(\mu_2, F_i(\mu_1))\rho(\mu_1, F_j(\mu_2)) + \alpha_4 d_\infty(\mu_1, \mu_2)\rho(\mu_1, F_i(\mu_1)) + \alpha_5 d_\infty(\mu_1, \mu_2)\rho(\mu_2, F_j(\mu_2)),$$

where $\alpha_i > 0$ $(i = 1, 2, 3, 4, 5)$, $\alpha_1 + \alpha_2 + \alpha_4 + \alpha_5 < 1$, $\alpha_1 + \alpha_3 < 1$. *Then there exists a point* μ_* *in* $\mathcal{C}(X)$ *such that* $\{\mu_*\} \subset F_i(\mu_*)$.

Proof. We can consider the function $g : [0, \infty)^5 \to [0, \infty)$ defined by

$$g(x_1, x_2, x_3, x_4, x_5) = \left[\alpha_1 {x_1}^2 + \alpha_2 x_2 x_3 + \alpha_3 x_4 x_5 + \alpha_4 x_1 x_2 + \alpha_5 x_1 x_3\right]^{\frac{1}{2}}.$$

Next, we prove g is a G-distance function. Firstly, obviously, g is nondecreasing in the 2^{nd}, 3^{rd}, 4^{th}, and 5^{th} variables. Secondly,

$$u \leqslant g(v, u, v, 0, u + v) = \left[\alpha_1 v^2 + \alpha_2 uv + \alpha_4 uv + \alpha_5 v^2\right]^{\frac{1}{2}} = \left[(\alpha_1 + \alpha_5)v^2 + (\alpha_2 + \alpha_4)uv\right]^{\frac{1}{2}}. \tag{4.1}$$

If $u \leqslant v$, from (4.1), we can get

$$u^2 \leqslant (\alpha_1 + \alpha_5)v^2 + (\alpha_2 + \alpha_4)uv \leqslant (\alpha_1 + \alpha_2 + \alpha_4 + \alpha_5)v^2.$$

Hence, there exists $h = \sqrt{\alpha_1 + \alpha_2 + \alpha_4 + \alpha_5} < 1$ such that $u \leqslant hv$, where $0 < h < 1$. If $u > v$, from (4.1), we can get

$$u^2 \leqslant (\alpha_1 + \alpha_5)v^2 + (\alpha_2 + \alpha_4)uv < (\alpha_1 + \alpha_2 + \alpha_4 + \alpha_5)u^2 < u^2,$$

which is a contradiction. Therefore, (ii) of Definition 2.1 holds. Thirdly, since $u \leqslant g(u, 0, 0, u, u) = [(\alpha_1 + \alpha_3)u^2]^{\frac{1}{2}} = \sqrt{\alpha_1 + \alpha_3}u < u$, which is a contradiction. Hence, $u = 0$. If in Theorem 2.2 we choose $L = 0$ and $\phi(t) = t$, Theorem 4.1 is satisfied with all conditions of Theorem 2.2. Hence, there exists a point μ_* in $\mathcal{C}(X)$ such that $\{\mu_*\} \subset F_i\mu^*$. □

Secondly, we give an application to illustrate the usefulness of Theorem 3.2 by slightly modifying Theorem 4.1.

Theorem 4.2. *Let* (X, d) *be a compact metric space and* $\{F_i\}_{i=1}^{\infty}$ *is a sequence of self-mappings of* $\mathcal{C}(X)$ *satisfying the following conditions:*

$$[d_\infty(F_i(\mu_1), F_j(\mu_2))]^2 \leqslant q\big\{\alpha_1[d_\infty(\mu_1, \mu_2)]^2 + \alpha_2\rho(\mu_1, F_i(\mu_1))\rho(\mu_2, F_j(\mu_2)) + \alpha_3 d_\infty(\mu_1, \mu_2)\rho(\mu_1, F_i(\mu_1)) + \alpha_4 d_\infty(\mu_1, \mu_2)\rho(\mu_2, F_j(\mu_2))\big\},$$

where $q \in (0, 1)$, $\alpha_i > 0$ $(i = 1, 2, 3, 4)$, *and* $\alpha_1 + \alpha_2 + \alpha_3 + \alpha_4 \leqslant 1$. *Then there exists a point* μ_* *in* $\mathcal{C}(X)$ *such that* $\{\mu_*\} \subset F_i(\mu_*)$.

Proof. We can consider the function $g : [0, \infty)^5 \to [0, \infty)$ defined by

$$g(x_1, x_2, x_3, x_4, x_5) = \left[\alpha_1 {x_1}^2 + \alpha_2 x_2 x_3 + \alpha_3 x_1 x_2 + \alpha_4 x_1 x_3\right]^{\frac{1}{2}}.$$

Next, we prove g is a G'-distance function. Firstly, obviously, g is nondecreasing in the each co-ordinate variable. Secondly,

$$g(t, t, t, at, bt) = \left[\alpha_1 v^2 + \alpha_2 t^2 + \alpha_3 t^2 + \alpha_4 t^2\right]^{\frac{1}{2}} = \left[(\alpha_1 + \alpha_2 + \alpha_3 + \alpha_4)t^2\right]^{\frac{1}{2}} = (\alpha_1 + \alpha_2 + \alpha_3 + \alpha_4)^{\frac{1}{2}} t \leqslant t.$$

If in Theorem 3.2, we choose $L = 0$ and $\psi(t) = qt$ $(0 < q < 1)$, Theorem 4.2 is satisfied with all conditions of Theorem 3.2. Hence, there exists a point μ_* in $\mathcal{C}(X)$ such that $\{\mu_*\} \subset F_i\mu^*$. □

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