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Robust state estimation for neutral-type neural networks with mixed time delays

Bo Du[a,*], Wenbing Zhang[b], Qing Yang[a]

[a]Department of Mathematics, Huaiyin Normal University, Huaian Jiangsu, 223300, P. R. China.

[b]Department of Mathematics, Yangzhou University, Yangzhou Jiangsu, 225002, P. R. China.

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Abstract

In this paper, the state estimation problem is dealt with a class of neutral-type Markovian neural networks with mixed time delays. The network systems have a finite number of modes, and the modes may jump from one state to another according to a Markov chain. We are devoted to design a state estimator to estimate the neuron states, through available output measurements, such that the dynamics of the estimation error is globally asymptotically stable in the mean square. From the Lyapunov-Krasovskii functional and linear matrix inequality (LMI) approach, we establish sufficient conditions to guarantee the existence of the state estimators. Furthermore, it is shown that the traditional stability analysis issue for delayed neural networks with Markovian chains can be included as a special case of our main results. A simulation shows the usefulness of the derived LMI-based stability conditions.

Keywords: Neutral-type, Markovian jumping system, Lyapunov functional method, stability.
2010 MSC: 34B15.

1. Introduction

In the past two decades, the successful applications of cellular neural networks (CNNs) in a variety of areas (e.g. pattern recognition, associative memory and combinational optimization) have received a surge of research interests in the dynamical behaviors of the CNNs, see e.g. [1, 3, 4, 11, 25, 31, 32, 34–36]. We note that large-scale and high-order neural networks have shown their great capacities in learning and data control. For relatively research, however, it is often the case that only partial information about the neuron states can be available in the network outputs. Therefore, for using the neural networks in practice, it becomes necessary to estimate the neuron states by available measurements. The state estimation problems for neural networks have been received great attention, see e.g. [6, 8, 24, 28]. For example, in [28], the neuron state estimation problem has been studied for recurrent neural networks with time-varying delays, and an effective LMI approach has been used to verify the stability of the estimation error dynamics. Salam and Zhang [24] obtained an adaptive state estimator by using techniques of optimization theory, the calculus of variations and gradient descent dynamics.

*Corresponding author
Email addresses: dubo7307@163.com (Bo Du), zwb850506@126.com (Wenbing Zhang), yangqing3511115@163.com (Qing Yang)

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Generally speaking, time delays are often encountered in various biological, engineering and economic systems due to the finite signal propagation time in biological networks or the finite switching speed of amplifiers in electronic networks and so on (see e.g. [18, 20, 21]). For the dynamical behavior analysis of delayed neural networks, different types of time delays, have been taken into account by using a variety of techniques that include Lyapunov functional method, linear matrix inequality (LMI) approach, topological degree theory, M-matrix theory and techniques of inequality analysis, see e.g. [19, 29, 37].

Neutral functional differential equation (NFDE) is a class of equations depending on past as well as present values, but which involves derivatives with delays as well as the function itself. NFDEs are not only an extension of functional differential equations, but also provide good models in many fields including electronics, biology, mechanics and economics. Particularly, for engineering systems, the time delays occur not only in the system states but also in the derivatives of system states. Accordingly, CNNs with neutral terms have gained extensive research interests due to the fact that the neutral delays could exist during the implementation process of CNNs. The stability analysis issue of neutral CNNs has recently received much more research attention and a rich body of results has been obtained, see e.g. [22, 30, 33].

Markovian jumping systems firstly introduced in [12], are the hybrid systems with two components in the state. The first one referred to the mode which was described by a continuous-time finite-state Markovian process, and the second one referred to the state which was represented by a system of differential equations. The jumping systems have the advantage of modeling the dynamic systems subject to abrupt variation in their structures.

So far, to the best of our knowledge, there is few results for the state estimation problem to neutral-type Markovian neural networks with mixed time delays. The major challenges lie in the follows:

(1) in order to construct a feasible Lyapunov-Krasovskii functional, the neutral-type operator $\mathcal{A}$ (defined by (2.3)) needs to be considered. So, when the operator $\mathcal{A}$ exists in the neural system, how can we construct a feasible Lyapunov-Krasovskii functional;

(2) when the mixed delays and Markovian parameters exist in CNNs, the corresponding state estimation becomes more complicated since a new Lyapunov functional is required to reflect these influences; and

(3) it is non-trivial to establish a unified framework to handle the above system with the neutral terms, mixed delays and Markovian parameters. The main purpose of this paper is to make the first attempt to handle the listed challenges.

In this paper, we consider the state estimation problem for a generalized neutral-type Markovian neural networks with mixed delays. The purpose of this paper is to estimate the neuron states via available output measurements such that the estimation error converges to zero exponentially. A numerically efficient LMI approach is developed to solve the addressed problem, and the explicit expression of the set of desired estimators is characterized. A simulation example is used to verify the usefulness of the LMI method.

The main contributions of this paper are outlined as follows:

(1) from neural network (2.7), we find that neutral operator $\mathcal{A}$ reflects neutral influence in (2.7), which is different from the existing papers, see e.g. [30, 33]. Hence, when the neutral delay term is studied as a neutral operator $\mathcal{A}$, novel analysis technique should be developed since the conventional analysis tool no longer applies;

(2) until now, neither the state estimation problem nor the stability analysis problem have been studied in the literature for Markovian jumping CNNs with both mixed time-delays, this paper will shorten such the above gap;

(3) a unified framework is established to handle the Markovian jumping parameters, neutral terms and mixed delays.

Let us recall some related research, Cheng et al. [5] studied a class of neutral-type neural networks with delays:

$$x_i'(t)+\sum_{j=1}^{n} e_{ij}x_j'(t-\tau_j)=-d_i(x_i(t))\Big(c_i(x_i(t))-\sum_{j=1}^{n} a_{ij}f_j(x_j(t))-\sum_{j=1}^{n} b_{ij}f_j(x_j(t-\tau_j))\Big).$$

Lou and Cui [17] investigated stochastic stability for a class of delayed neural networks of neutral type with Markovian jumping parameters

$$u'(t)=-C(\eta_t)u(t)+A(\eta_t)g(u(t))+B(\eta_t)g(u(t-\tau(t)))+D(\eta_t)u'(t-\tau(t))+J.$$

Liu et al. [15] considered the following neutral-type neural networks with Markovvian jumping parameters and mixed delays:

$$\begin{aligned} x'(t)=\,&E(r(t))x'(t-\tau_{1,r(t)})-A(r(t))x(t)-B(r(t))f(x(t))\\ &+C(r(t))g(x(t-\tau_{2,r(t)}))+D(r(t))\int_{t-\tau_{3,r(t)}}^{t-\tau_{4,r(t)}} h(x(s))ds. \end{aligned}$$

Liu et al. [16] considered an uncertain neutral-type neural networks with time-varying delays:

$$\begin{aligned} y'(t)=\,&-(A+\Delta A(t))y(t)+(W_1+\Delta W_1(t))g(y(t))\\ &+(W_2+\Delta W_2(t))g(y(t-\tau(t)))+(W_3+\Delta W_3(t))y'(t-h(t))\\ &+(W_4+\Delta W_4(t))\int_{t-r(t)}^{t} g(y(s))ds+I. \end{aligned}$$

Rakkiyappan et al. [23] studied the following impulsive neutral-type neural system:

$$\begin{aligned} &y'(t)=-Ay(t)+Bg(y(t))+Cg(y(t-\tau(t)))+Dy'(t-h(t)),\quad t\neq t_k,\\ &\Delta y(t)=I_k(y(t)),\quad t=t_k,\\ &y(t_0^+ + s)=\phi(s),\quad s\in[t_0-\rho,t_0],\ k\in\mathbb{N}. \end{aligned}$$

Their neutral terms in the above systems are $y'(t-h(t))$, $u'(t-\tau(t))$, $x_j'(t-\tau_j)$ and $x'(t-\tau_{1,r(t)})$. As was point by Hale [9] that the properties of operator $\mathcal{A}$ (defined by (2.3)) are important for studying NFDEs. Hence system (2.7) has significant theoretical value for research of FDEs and neural networks. Furthermore, in order to obtain stochastic stability results for the above systems, Lyapunov functional method is necessary. But, in the present paper, based on new stochastic analysis and mathematical analysis technique, we can derive the conditions for the existence of the desired estimators for the system (2.7). We also parameterize the explicit expression of the set of desired estimators.

Throughout the manuscript, $\mathbb{R}^n$ and $\mathbb{R}^{n\times m}$ denote, respectively, the n-dimensional Euclidean space and the set of all $n\times m$ real matrices. The superscript "T" denotes the matrix transposition. Let $h>0$ and $C([-h,0];\mathbb{R}^n)$ with the norm $\|\phi\|=\sup_{-h\leqslant\theta\leqslant0}|\phi(\theta)|$ denote the family of continuous functions ϕ from $[-h,0]$. We will use the notation $A>0$ (or $A<0$) to denote that A is a symmetric and positive definite (or negative definite) matrix. If A,B are symmetric matrices, $A>B$ ($A\geqslant B$), then $A-B$ is a positive definite (positive semi-definite). $|z|$ denotes the Euclidean norm of a vector z and $\|A\|$ denotes the induced norm of the matrix A, that is $\|A\|=\sqrt{\lambda_{\max}(A^\top A)}$ where $\lambda_{\max}(\cdot)$ means the largest eigenvalue of A. Let $(\Omega,\mathfrak{F},\{\mathfrak{F}_t\}_{t\geqslant0},P)$ be a complete probability space with a filtration $\{\mathfrak{F}_t\}_{t\geqslant0}$. Denote by $L^p_{\mathfrak{F}_t}([-h,0],\mathbb{R}^n)$ the family of all $\mathfrak{F}_0$-measurable $C([-h,0];\mathbb{R}^n)$-value random variables $\xi=\{\xi(\theta):-h\leqslant\theta\leqslant0\}$ such that $\sup_{-h\leqslant\theta\leqslant0}\mathbb{E}|\xi(\theta)|^p<\infty$ stands for the mathematical expectation operator with respect to the given probability measure P. Matrices, if their dimensions are not explicitly stated, assumed to be compatible for algebraic operations.

The following sections are organized as follows. The state estimation problem is formulated in Section 2 for Markovian jumping mixed delayed neural networks. In Section 3, we design a state estimator for the neural network described by (2.7) such that for every mode, the dynamics of the system (2.9) is globally asymptotically stable in the mean square. In Section 4 a numerical example is given to show the feasibility of our results. Finally, some conclusions are given about this paper.

2. Problem formulation and main lemmas

Consider the following neutral-type delayed neural network with Markovian jumping:

$$
\begin{aligned}
(\mathcal{A}_i x_i)'(t) = &-a_i x_i(t) + \sum_{j=1}^{n} b_{ij} f_j(x_j(t)) + \sum_{j=1}^{n} d_{ij} g_j(x_j(t-\tau(t))) \\
&+ \int_{t-\delta(t)}^{t} \sum_{j=1}^{n} \omega_{ij}(t) h_j(x_j(s)) ds + I_i(t),
\end{aligned} \tag{2.1}
$$

where $\mathcal{A}_i$ is a difference operator defined by

$$
(\mathcal{A}_i x_i)(t) = x_i(t) - c_i x_i(t-\gamma), \quad |c_i| \neq 1, \quad i = 1, 2, \cdots, n,
$$

$\gamma > 0$ is a constant, $x_i(t)$ and I_i represent the activation and external input of the ith neuron in the I-layer, respectively, f_j, g_j and h_j are the activation functions of the j-th neuron with $f_j(0) = g_j(0) = h_j(0) = 0$, a_i represents the rate with which the i-th unit will reset its potential to the resting state when disconnected from the network and external inputs at time t, $\tau(t) > 0$ corresponds to the finite speed of the axonal transmission of signal, $\delta(t) > 0$ describes the distributed time delay, b_{ij} denotes the strength of the j-th unit on the i-th unit at time t, d_{ij} denotes the strength of the j-th unit on the i-th unit at time $t-\tau(t)$, w_{ij} denotes the distributively delayed connection weights of the j-th neuron on the i neuron.

The neural network (2.1) can be rewritten as the following matrix-vector form:

$$
(\mathcal{A}x)'(t) = -Ax(t) + BF(x(t)) + DG(x(t-\tau(t))) + W\int_{t-\delta(t)}^{t} H(x(s)) ds + I, \tag{2.2}
$$

where

$$
\begin{aligned}
&\mathcal{A}x(t) = x(t) - Cx(t-\gamma), \quad C = \operatorname{diag}(c_1, c_2, \cdots, c_n), \qquad (2.3)\\
&\mathcal{A}x(t) = (\mathcal{A}_1 x_1(t), \mathcal{A}_2 x_2(t), \cdots, \mathcal{A}_n x_n(t))^\top, \quad A = \operatorname{diag}(a_1, a_2, \cdots, a_n), \\
&B = (b_{ij})_{n\times n}, \quad D = (d_{ij})_{n\times n}, \quad W = (w_{ij})_{n\times n}, \quad I(t) = (I_1(t), I_2(t), \cdots, I_n(t))^\top, \\
&F(x(t)) = (f_1(x_1(t)), f_2(x_2(t)), \cdots, f_n(x_n(t)))^\top, \\
&G(x(t-\tau(t))) = (g_1(x_1(t-\tau(t))), g_2(x_2(t-\tau(t))), \cdots, g_n(x_n(t-\tau(t))))^\top, \\
&H(x(t)) = (h_1(x_1(t)), h_2(x_2(t)), \cdots, h_n(x_n(t)))^\top.
\end{aligned}
$$

Remark 2.1. We find that neutral-type model (2.2) shows the neutral character by the $\mathcal{A}$ operator, which is different from the corresponding ones in other papers, see e.g. [17, 22, 30, 33].

We give two assumptions for the proof.

Assumption 2.2. There exist constants τ_0 and δ_0 such that

$$
\dot{\tau}(t) \leqslant \tau_0 < 1, \quad \dot{\delta}(t) \leqslant \delta_0 < 1.
$$

Assumption 2.3. The neuron activation functions in (2.2), $F(\cdot)$, $G(\cdot)$ and $H(\cdot)$ satisfy the following Lipschitz condition:

$$\begin{aligned} |F(x)-F(y)| &\leqslant |G_1(x-y)|, \\ |G(x)-G(y)| &\leqslant |G_2(x-y)|, \\ |H(x)-H(y)| &\leqslant |G_3(x-y)|, \end{aligned} \tag{2.4}$$

where $G_i \in \mathbb{R}^{n\times n}$ $(i=1,2,3)$ are known constant matrices. The type of activation functions in (2.4) is not necessarily monotonic and smooth, and have been used in numerous papers, see e.g. [13, 14] and references therein.

Remark 2.4. In general, the error state for estimation is similar to synchronization problems, see e.g. [27, 38]. However, the purpose of state estimators is that choosing a proper estimator K so that $\hat{x}(t)$ approaches $x(t)$ asymptotically or exponentially. Furthermore, the estimator K can be obtained, which is different from synchronization problems.

Suppose that the output from the neural network (2.2) is of the form:

$$y(t) = Rx(t) + Q(t, x(t)), \tag{2.5}$$

where $y(t) = (y_1(t), y_2(t), \cdots, y_m(t))^\top \in \mathbb{R}^m$ is the measurement output of the neural network, $R \in \mathbb{R}^{m\times n}$ is a known constant matrix with appropriate dimension.

$$Q(t, x(t)) = (q_1(t, x(t)), \cdots, q_m(t, x(t)))^\top \in \mathbb{R}^m,$$

is the nonlinear disturbance dependent on the neuron state that satisfies the following Lipschitz condition:

$$|Q(t,x) - Q(t,y)| \leqslant |L(x-y)|, \tag{2.6}$$

where $L \in \mathbb{R}^{n\times n}$ is a known constant matrix.

Let $\{r(t),\ t \geqslant 0\}$ be a right-continuous Markov process on the probability space which takes values in the finite space $\rho = \{1, 2, \cdots, N\}$ with generator $\Gamma = (\gamma_{ij})$ $(i, j \in \rho)$ given by

$$P\{r(t+\Delta) = j | r(t) = i\} = \begin{cases} \gamma_{ij}\Delta + o(\Delta) & \text{if } i \neq j, \\ 1 + \gamma_{ij}\Delta + o(\Delta) & \text{if } i = j, \end{cases}$$

where $\Delta > 0$ and $\lim_{\Delta\to 0} o(\Delta)/\Delta = 0$, $\gamma_{ij} \geqslant 0$ is the transition rate from i to j and $\gamma_{ii} = -\sum_{j\neq i} \gamma_{ij}$.

In this paper, we will focus on the following neural network with Markovian jumping parameters, which can be seen as a variation of the model (2.2), (2.3), (2.4), (2.5):

$$\begin{aligned} (\mathcal{A}x)'(t) &= -A(r(t))x(t) + B(r(t))F(x(t)) + D(r(t))G(x(t-\tau(t))) \\ &\quad + W(r(t)) \int_{t-\delta(t)}^{t} H(x(s))ds + I(r(t)), \\ y(t) &= R(r(t))x(t) + Q(t, x(t)), \end{aligned} \tag{2.7}$$

where $x(t)$ and $y(t)$ have the same meanings as those in (2.2) and (2.5), $F(\cdot)$, $G(\cdot), H(\cdot)$ and $Q(t, x(t))$ satisfy (2.4) and (2.6), respectively. For fixed system mode, $A(r(t))$, $B(r(t))$, $D(r(t))$ and $I(r(t))$ are known constant matrices with appropriate dimensions.

The main objective of this paper is to develop an efficient algorithm to estimate the neuron states $x(t)$ in (2.2) from the available network outputs in (2.5). From now on we shall work on the network mode $r(t) = i$, for all $i \in \rho$. The full-order state estimator of (2.7) is of the form

$$\begin{aligned} (\mathcal{A}\hat{x})'(t) = {} & -A(i)\hat{x}(t) + B(i)F(\hat{x}(t)) + D(i)G(\hat{x}(t-\tau(t))) + W(i)\int_{t-\delta(t)}^{t} H(\hat{x}(s))ds \\ & + I(r(t)) + K(i)[y(t) - R(i)\hat{x}(t) - Q(t, \hat{x}(t))], \end{aligned} \tag{2.8}$$

where $\hat{x}(t)$ is the estimation of the neuron state, and $K(i) \in \mathbb{R}^{n\times m}$ is the estimator gain matrix to be designed.

Let

$$\mathcal{E}(t) = (\varepsilon_1(t), \cdots, \varepsilon_n(t))^\top = \hat{x}(t) - x(t),$$

be the state estimation error. In view of (2.2) and (2.8), the state error $\mathcal{E}(t)$ satisfies the following equation

$$\begin{aligned}(\mathcal{A}\mathcal{E})'(t) &= (-A(i) - K(i)R(i))\mathcal{E}(t) + B(i)\hat{F}(\mathcal{E}(t)) + D(i)\hat{G}(\mathcal{E}(t-\tau(t))) \\ &\quad + W(i)\int_{t-\delta(t)}^{t} \hat{H}(\mathcal{E}(s))ds - K(i)\hat{Q}(t, \mathcal{E}(t)),\end{aligned} \tag{2.9}$$

where

$$\begin{aligned}&\mathcal{A}\mathcal{E}(t) = [\mathcal{A}_1\varepsilon_1(t), \mathcal{A}_2\varepsilon_2(t), \cdots, \mathcal{A}_n\varepsilon_n(t)]^\top = \mathcal{A}\hat{x}(t) - \mathcal{A}x(t),\\ &\hat{F}(\mathcal{E}(t)) = [\hat{f}_1(\varepsilon_1(t)), \hat{f}_2(\varepsilon_2(t)), \cdots, \hat{f}_n(\varepsilon_n(t))]^\top = F(\hat{x}(t)) - F(x(t)),\\ &\hat{G}(\mathcal{E}(t)) = [\hat{g}_1(\varepsilon_1(t)), \hat{g}_2(\varepsilon_2(t)), \cdots, \hat{g}_n(\varepsilon_n(t))]^\top = G(\hat{x}(t)) - G(x(t)),\\ &\hat{H}(\mathcal{E}(t)) = [\hat{h}_1(\varepsilon_1(t)), \hat{h}_2(\varepsilon_2(t)), \cdots, \hat{h}_n(\varepsilon_n(t))]^\top = H(\hat{x}(t)) - H(x(t)),\\ &\hat{Q}(t, \mathcal{E}(t)) = Q(t, \hat{x}(t)) - Q(t, x(t)).\end{aligned}$$

Let $\mathcal{E}(t;\xi)$ denote the state trajectory of the error-state system (2.9) with the initial data condition $\mathcal{E}(\theta) = \xi(\theta)$ on $-h \leqslant \theta \leqslant 0$ in $L^2_{\mathcal{F}_0}([-h, 0]; \mathbb{R}^n)$. It can be easily seen that the system (2.9) admits a trivial solution (equilibrium point) $e(t; 0) \equiv 0$ corresponding to the initial data $\xi = 0$.

We need the following definition to go ahead to design the desired estimators.

Definition 2.5. For the system (2.9) and each $\xi \in L^2_{\mathcal{F}_0}([-h, 0]; \mathbb{R}^n)$, the equilibrium point is asymptotically stable in the mean square, if for every network mode,

$$\lim_{t\to\infty} \mathbb{E}|\mathcal{E}(t;\xi)|^2 = 0.$$

We shall design a state estimator for the neural network described by (2.2) such that for every mode, the dynamics of the system (2.2) is globally asymptotically stable in the mean square for the nonlinear activation function and the nonlinear disturbance in (2.2).

Lemma 2.6 ([13])**.** *Let X, Y be any n-dimensional real vectors, and $\varepsilon > 0$. Then the following matrix inequality holds:*

$$2X^\top Y \leqslant \varepsilon X^\top X + \varepsilon^{-1} Y^\top Y.$$

Lemma 2.7 ([7])**.** *For any positive definite matrix $M > 0$, scalar $\gamma > 0$, vector function $\omega : [0, \gamma] \to \mathbb{R}^n$ such that the integrations concerned are well-defined, the following inequality holds:*

$$\left(\int_0^\gamma \omega(s)ds\right)^\top M\left(\int_0^\gamma \omega(s)ds\right) \leqslant \gamma\left(\int_0^\gamma \omega^\top(s)M\omega(s)ds\right).$$

Lemma 2.8 ([2])**.** *Given constant matrices $\Omega_1, \Omega_2, \Omega_3$ where $\Omega_1 = \Omega_1^\top$ and $\Omega_2 > 0$, then*

$$\Omega_1 + \Omega_3^\top \Omega_2^{-1}\Omega_3 < 0,$$

if and only if

$$\begin{pmatrix} \Omega_1 & \Omega_3^\top \\ \Omega_3 & -\Omega_2 \end{pmatrix} < 0 \quad \text{or} \quad \begin{pmatrix} -\Omega_2 & \Omega_3 \\ \Omega_3^\top & -\Omega_1 \end{pmatrix} < 0.$$

Lemma 2.9. *If $\sqrt{n}c_i^+ < 1$, $i = 1, 2, \cdots, n$, then the inverse of difference operator $\mathcal{A}$ denoted by $\mathcal{A}^{-1}$, exists and*

$$|\mathcal{A}^{-1}| \leqslant \frac{1}{1 - \sqrt{n}c_i^+},$$

where $c_i^+ = \max\{|c_1|, |c_2|, \cdots, |c_n|\}$.

Proof. Let $Bx(t) = Cx(t-\gamma)$, then $|B| = \sqrt{n}c_i^+ < 1$. Thus, $\mathcal{A}^{-1} = (I-B)^{-1}$ exists and $|\mathcal{A}^{-1}| = |(I-B)^{-1}| \leqslant \frac{1}{1-\sqrt{n}c_i^+}$. □

We are now ready to deal with the stability analysis problem, that is, deriving the conditions under which the error dynamics of the estimation process (2.9) is globally asymptotically stable in the mean square. The following theorem shows that such conditions can be obtained if a quadratic matrix inequality involving several scalar parameters is feasible.

3. Main results

Theorem 3.1. *Suppose that Assumption 2.2 and Assumption 2.3 hold, $\sqrt{n}c_i^+ < 1$, $i = 1, 2, \cdots, n$. Let the estimator gain $K(i)$ be given. If there exist four sequences of positive scalars $\varepsilon_{1i}, \varepsilon_{2i}, \cdots, \varepsilon_{4i}$, $i \in \rho$ and a symmetric positive sequence define matrices $P(i)$ $(i \in \rho)$ such that the following inequalities*

$$\Pi_1 = 2P(i)\left[(-A(i) - K(i)R(i)) + \sum_{j=1}^{N} \gamma_{ij}P_j\right] = A_{ki}^\top P(i) + P(i)A_{ki} + \sum_{j=1}^{N} \gamma_{ij}P_j < 0,$$

where $A_{ki} = -A(i) - K(i)R(i)$,

$$\begin{aligned}
\Pi_2 &= \varepsilon_{1i}G_1^\top G_1 + \varepsilon_{2i}G_2^\top G_2 + \varepsilon_{3i}\delta_0^2 G_3^\top G_3 + \varepsilon_{4i}L^\top L + (2+\tau_0)Q_1 + 3\delta_0 Q_2 < 0,\\
\Pi_3 &= \varepsilon_{1i}^{-1}P(i)B(i)B^\top(i)P(i) + \varepsilon_{2i}^{-1}P(i)D(i)D^\top(i)P(i) + \varepsilon_{3i}^{-1}P(i)W(i)W^\top(i)P(i)\\
&\quad + \varepsilon_{4i}^{-1}P(i)K(i)K^\top(i)P(i) < 0,
\end{aligned}$$

hold, where Q_1 and Q_2 are defined by (3.4), then the error-state system (2.9) of the neural network (2.7) is globally asymptotically stable in the mean square.

Proof. It follows immediately from (2.4) that

$$\begin{aligned}
\hat{F}^\top(\cdot)\hat{F}(\cdot) &= |F(\hat{x}(\cdot)) - F(x(\cdot))| \leqslant |G_1\mathcal{E}(\cdot)|^2 = \mathcal{E}^\top(\cdot)G_1^\top G_1\mathcal{E}(\cdot),\\
\hat{G}^\top(\cdot)\hat{G}(\cdot) &= |G(\hat{x}(\cdot)) - G(x(\cdot))| \leqslant |G_2\mathcal{E}(\cdot)|^2 = \mathcal{E}^\top(\cdot)G_2^\top G_2\mathcal{E}(\cdot),\\
\hat{H}^\top(\cdot)\hat{H}(\cdot) &= |H(\hat{x}(\cdot)) - H(x(\cdot))| \leqslant |G_3\mathcal{E}(\cdot)|^2 = \mathcal{E}^\top(\cdot)G_3^\top G_3\mathcal{E}(\cdot),
\end{aligned} \tag{3.1}$$

$$\hat{Q}^\top(t,\cdot)\hat{Q}(t,\cdot) = |Q(t,\hat{x}(\cdot)) - Q(t,x(\cdot))| \leqslant |L\mathcal{E}(\cdot)|^2 = \mathcal{E}^\top(\cdot)L^\top L\mathcal{E}(\cdot). \tag{3.2}$$

Let $C^{2,1}(\mathbb{R}^n \times \mathbb{R}_+ \times \rho, \mathbb{R}_+)$ denote the family of all nonnegative functions $\Phi(\mathcal{E}, t, i)$ on $\mathbb{R}^n \times \mathbb{R}_+ \times \rho$ which are continuously twice differentiable in $\mathcal{E}$ and differentiable in t. Fix $\xi \in L^2_{\mathcal{F}_0}([-h, 0]; \mathbb{R}^n)$. Define a Lyapunov functional candidate $\Phi(\mathcal{E}, t, i) \in C^{2,1}(\mathbb{R}^n \times \mathbb{R}_+ \times \rho, \mathbb{R}_+)$ by

$$\begin{aligned}
\Phi(\mathcal{E}, t, i) &= [\mathcal{A}\mathcal{E}(t)]^\top P(i)[\mathcal{A}\mathcal{E}(t)]\\
&\quad + \int_{t-\tau(t)}^{t} \mathcal{E}^\top(s)Q_1\mathcal{E}(s)ds + \int_0^{\delta(t)}\int_{t-s}^{t} \mathcal{E}^\top(\eta)Q_2\mathcal{E}(\eta)d\eta ds,
\end{aligned} \tag{3.3}$$

where $P(i) > 0$ is the positive definite solution to $\Pi_i < 0$ $(i = 1, 2, 3)$, $Q_1 \geqslant 0$ and $Q_2 \geqslant 0$ are defined by

$$Q_1 = \varepsilon_{1i}G_1^\top G_1, \quad Q_2 = \varepsilon_{3i}\delta_0 G_3^\top G_3. \tag{3.4}$$

By [26] we know that $\{\mathcal{E}(t), r(t)\}(t \geqslant 0)$ is a $C([-h, 0]; \mathbb{R}^n) \times \rho$-valued Markov process. Along the trajectory of (3.3), the weak infinitesimal operator $\mathcal{L}$ (see Ji and Chizeck [10]) of the stochastic process

$$\{r(t), x(t)\} \quad (t \geqslant 0),$$

is given by

$$\begin{aligned}
\mathcal{L}\Phi(\mathcal{E}(t), r(t)) &= \lim_{\Delta \to 0^+} \frac{1}{\Delta}[\mathbb{E}\{\Phi(\mathcal{E}(t+\Delta), r(t+\Delta))|r(t)=i\} - \Phi(\mathcal{E}(t), r(t)=i)] \\
&= 2[\mathcal{A}\mathcal{E}(t)]^\top P(i)\Bigg[(-A(i) - K(i)R(i))\mathcal{E}(t) \\
&\quad + \sum_{j=1}^{N}\gamma_{ij}P_j\mathcal{E}(t) + B(i)\hat{F}(\mathcal{E}(t)) + D(i)\hat{G}(\mathcal{E}(t-\tau(t))) \\
&\quad + W(i)\int_{t-\delta(t)}^{t}\hat{H}(\mathcal{E}(s))ds - K(i)\hat{Q}(t, \mathcal{E}(t))\Bigg] \\
&\quad + [\mathcal{E}(t)]^\top(Q_1 + \delta(t)Q_2)\mathcal{E}(t) - (1-\tau'(t))[\mathcal{E}(t-\tau(t))]^\top Q_1\mathcal{E}(t-\tau(t)) \\
&\quad - \int_{t-\delta(t)}^{t}[\mathcal{E}(s)]^\top Q_2\mathcal{E}(s)ds \\
&\quad + \sum_{j=1}^{N}\gamma_{ij}\int_{t-\tau(t)}^{t}\mathcal{E}^\top(s)Q_1\mathcal{E}(s)ds + \sum_{j=1}^{N}\gamma_{ij}\int_{0}^{\delta(t)}\int_{t-s}^{t}\mathcal{E}^\top(\eta)Q_2\mathcal{E}(\eta)d\eta ds.
\end{aligned} \tag{3.5}$$

It follows from Lemma 2.6, (3.1) and (3.2) that

$$\begin{aligned}
2[\mathcal{A}\mathcal{E}(t)]^\top P(i)B(i)\hat{F}(\mathcal{E}(t)) &= 2\hat{F}^\top(\mathcal{E}(t))B^\top(i)P(i)\mathcal{A}\mathcal{E}(t) \\
&\leqslant \varepsilon_{1i}\hat{F}^\top(\mathcal{E}(t))\hat{F}(\mathcal{E}(t)) + \varepsilon_{1i}^{-1}[\mathcal{A}\mathcal{E}(t)]^\top P(i)B(i)B^\top(i)P(i)[\mathcal{A}\mathcal{E}(t)] \\
&\leqslant \mathcal{E}^\top(t)(\varepsilon_{1i}G_1^\top G_1)\mathcal{E}(t) + [\mathcal{A}\mathcal{E}(t)]^\top\varepsilon_{1i}^{-1}P(i)B(i)B^\top(i)P(i)[\mathcal{A}\mathcal{E}(t)],
\end{aligned} \tag{3.6}$$

$$\begin{aligned}
2[\mathcal{A}\mathcal{E}(t)]^\top P(i)D(i)\hat{G}(\mathcal{E}(t-\tau(t))) &= 2\hat{G}^\top(\mathcal{E}(t-\tau(t)))D^\top(i)P(i)\mathcal{A}\mathcal{E}(t) \\
&\leqslant \varepsilon_{2i}\hat{G}^\top(\mathcal{E}(t-\tau(t)))\hat{G}(\mathcal{E}(t-\tau(t))) \\
&\quad + \varepsilon_{2i}^{-1}[\mathcal{A}\mathcal{E}(t)]^\top P(i)D(i)D^\top(i)P(i)[\mathcal{A}\mathcal{E}(t)] \\
&\leqslant \mathcal{E}^\top(t-\tau(t))(\varepsilon_{2i}G_2^\top G_2)\mathcal{E}(t-\tau(t)) \\
&\quad + [\mathcal{A}\mathcal{E}(t)]^\top\varepsilon_{2i}^{-1}P(i)D(i)D^\top(i)P(i)[\mathcal{A}\mathcal{E}(t)],
\end{aligned} \tag{3.7}$$

$$\begin{aligned}
2[\mathcal{A}\mathcal{E}(t)]^\top P(i)W(i)\int_{t-\delta(t)}^{t}\hat{H}(\mathcal{E}(s))ds &= 2\left[\int_{t-\delta(t)}^{t}\hat{H}(\mathcal{E}(s))ds\right]^\top W^\top(i)P(i)\mathcal{A}\mathcal{E}(t) \\
&\leqslant \varepsilon_{3i}\left[\int_{t-\delta(t)}^{t}\hat{H}(\mathcal{E}(s))ds\right]^\top\left[\int_{t-\delta(t)}^{t}\hat{H}(\mathcal{E}(s))ds\right] \\
&\quad + \varepsilon_{3i}^{-1}[\mathcal{A}\mathcal{E}(t)]^\top P(i)W(i)W^\top(i)P(i)[\mathcal{A}\mathcal{E}(t)],
\end{aligned} \tag{3.8}$$

$$\begin{aligned}
-2[\mathcal{A}\mathcal{E}(t)]^\top P(i)K(i)\hat{Q}(t, \mathcal{E}(t)) &= -2\left[\hat{Q}(t, \mathcal{E}(t))\right]^\top K^\top(i)P(i)\mathcal{A}\mathcal{E}(t) \\
&\leqslant \varepsilon_{4i}\left[\hat{Q}(t, \mathcal{E}(t))\right]^\top\left[\hat{Q}(t, \mathcal{E}(t))\right] \\
&\quad + \varepsilon_{4i}^{-1}[\mathcal{A}\mathcal{E}(t)]^\top P(i)K(i)K^\top(i)P(i)[\mathcal{A}\mathcal{E}(t)], \\
&\leqslant [\mathcal{E}(t)]^\top\varepsilon_{4i}L^\top L[\mathcal{E}(t)] + \varepsilon_{4i}^{-1}[\mathcal{A}\mathcal{E}(t)]^\top P(i)K(i)K^\top(i)P(i)[\mathcal{A}\mathcal{E}(t)].
\end{aligned} \tag{3.9}$$

By resorting to Lemma 2.7 and the definition of Q_2 in (3.4), we have

$$\begin{aligned}
&\varepsilon_{3i}\left[\int_{t-\delta(t)}^{t}\hat{H}(\mathcal{E}(s))ds\right]^{\top}\left[\int_{t-\delta(t)}^{t}\hat{H}(\mathcal{E}(s))ds\right]\\
&\qquad\leqslant \varepsilon_{3i}\delta_0\int_{t-\delta(t)}^{t}\hat{H}^{\top}(\mathcal{E}(s))\hat{H}(\mathcal{E}(s))dsds\\
&\qquad\leqslant \varepsilon_{3i}\delta_0\int_{t-\delta(t)}^{t}\mathcal{E}^{\top}(s)G_3^{\top}G_3\mathcal{E}(s)ds=\delta_0\int_{t-\delta(t)}^{t}\mathcal{E}^{\top}(s)Q_2\mathcal{E}(s)ds,
\end{aligned}$$

and hence

$$\begin{aligned}
&2\left[\int_{t-\delta(t)}^{t}\hat{H}(\mathcal{E}(s))ds\right]^{\top}W^{\top}(i)P(i)\mathcal{A}\mathcal{E}(t)\\
&\qquad\leqslant\int_{t-\delta(t)}^{t}\mathcal{E}^{\top}(s)Q_2\mathcal{E}(s)ds+\varepsilon_{3i}^{-1}[\mathcal{A}\mathcal{E}(t)]^{\top}P(i)W(i)W^{\top}(i)P(i)[\mathcal{A}\mathcal{E}(t)].
\end{aligned}\tag{3.10}$$

Furthermore, it follows from $\sum_{j=1}^{N}\gamma_{ij}=0$ that

$$\sum_{j=1}^{N}\gamma_{ij}\int_{t-\tau(t)}^{t}\mathcal{E}^{\top}(s)Q_1\mathcal{E}(s)ds=\sum_{j=1}^{N}\gamma_{ij}\int_{0}^{\delta(t)}\int_{t-s}^{t}\mathcal{E}^{\top}(\eta)Q_2\mathcal{E}(\eta)d\eta ds=0.\tag{3.11}$$

In the view of (3.5), (3.6), (3.7), (3.8), (3.9), (3.10), and (3.11), we obtain from Lemma 2.9 that

$$\begin{aligned}
\mathcal{L}\Phi(\mathcal{E}(t),r(t))&\leqslant 2[\mathcal{A}\mathcal{E}(t)]^{\top}P(i)\left[(-A(i)-K(i)R(i))+\sum_{j=1}^{N}\gamma_{ij}P_j\right]\mathcal{E}(t)\\
&\quad+[\mathcal{E}(t)]^{\top}\Big(\varepsilon_{1i}G_1^{\top}G_1+\varepsilon_{2i}G_2^{\top}G_2+\varepsilon_{3i}\delta_0^2G_3^{\top}G_3+\varepsilon_{4i}L^{\top}L\\
&\quad+(2+\tau_0)Q_1+3\delta_0Q_2\Big)\mathcal{E}(t)\\
&\quad+[\mathcal{A}\mathcal{E}(t)]^{\top}\Big(\varepsilon_{1i}^{-1}P(i)B(i)B^{\top}(i)P(i)+\varepsilon_{2i}^{-1}P(i)D(i)D^{\top}(i)P(i)\\
&\quad+\varepsilon_{3i}^{-1}P(i)W(i)W^{\top}(i)P(i)+\varepsilon_{4i}^{-1}P(i)K(i)K^{\top}(i)P(i)\Big)[\mathcal{A}\mathcal{E}(t)]\\
&=[\mathcal{A}\mathcal{E}(t)]^{\top}\Pi_1\mathcal{E}(t)+[\mathcal{E}(t)]^{\top}\Pi_2\mathcal{E}(t)+[\mathcal{A}\mathcal{E}(t)]^{\top}\Pi_3[\mathcal{A}\mathcal{E}(t)]\\
&\leqslant[\mathcal{A}\mathcal{E}(t)]^{\top}\left(\frac{1}{1-\sqrt{n}c_i^{+}}\Pi_1+\frac{1}{(1-\sqrt{n}c_i^{+})^2}\Pi_2+\Pi_3\right)[\mathcal{A}\mathcal{E}(t)].
\end{aligned}\tag{3.12}$$

Taking the mathematical expectation of both sides of (3.12), we have

$$\begin{aligned}
\mathcal{L}\mathbb{E}\Phi(\mathcal{E}(t),r(t))&\leqslant\mathbb{E}\left\{[\mathcal{A}\mathcal{E}(t)]^{\top}\left(\frac{1}{1-\sqrt{n}c_i^{+}}\Pi_1+\frac{1}{(1-\sqrt{n}c_i^{+})^2}\Pi_2+\Pi_3\right)[\mathcal{A}\mathcal{E}(t)]\right\}\\
&\leqslant-\tilde{\lambda}\mathbb{E}|\mathcal{A}\mathcal{E}(t)|^2,
\end{aligned}$$

where $\tilde{\lambda}=\min\{\frac{1}{1-\sqrt{n}c_i^{+}}\lambda_{\min}(-\Pi_1),\frac{1}{(1-\sqrt{n}c_i^{+})^2}\lambda_{\min}(-\Pi_2),\lambda_{\min}(-\Pi_3)\}$. Then we have

$$\lim_{t\to\infty}\mathbb{E}|\mathcal{A}\mathcal{E}(t)|^2=0,$$

together with $|\mathcal{E}(t)| \leqslant \frac{1}{1-\sqrt{n c_i^+}}|\mathcal{A}\mathcal{E}(t)|$ yields

$$\lim_{t\to\infty} \mathbb{E}|\mathcal{E}(t)|^2 = 0.$$

Therefore, from Definition 2.5, we arrive at the conclusion that the error-state system (2.9) is globally asymptotically stable in the mean square. This completes the proof of Theorem 3.1. □

Remark 3.2. In general, when $|c_i| \geqslant 1$ $(i = 1, 2, \cdots, n)$ in (2.3), the operator $\mathcal{A}$ has no inverse operator and Lemma 2.9 can not hold. Hence, it is very difficult for obtaining stability results to the error system (2.9). The above case will be studied by us in the future.

Now, let us consider the conditions for the estimation error-state system (2.9) of the neural network (2.7) is globally asymptotically stable in the mean square. Furthermore, we will give a practical design procedure for the estimator gain by LMIs.

Theorem 3.3. *If there exist sequences $\varepsilon_{1i}, \varepsilon_{2i}, \cdots, \varepsilon_{4i}, i \in \rho$ and a symmetric positive sequence define matrices $P(i) \in \mathbb{R}^{n\times n}$ $(i \in \rho)$ such that following LMI holds:*

$$\Psi = \left(\begin{array}{cccccc} \Gamma_{1i} & P(i)B(i) & \frac{\varepsilon_{1i}}{(1-\sqrt{n c_i^+})^2}G_1^\top & P(i)D(i) & \frac{\varepsilon_{2i}}{(1-\sqrt{n c_i^+})^2}G_2^\top & P(i)W(i) \\ B^\top(i)P^\top(i) & -\varepsilon_{1i}I & 0 & 0 & 0 & 0 \\ \frac{\varepsilon_{1i}}{(1-\sqrt{n c_i^+})^2}G_1 & 0 & -\varepsilon_{1i}I & 0 & 0 & \\ D^\top(i)P^\top(i) & 0 & 0 & -\varepsilon_{2i}I & 0 & 0 \\ \frac{\varepsilon_{2i}}{(1-\sqrt{n c_i^+})^2}G_2 & 0 & 0 & 0 & -\varepsilon_{2i}I & 0 \\ W^\top(i)P^\top(i) & 0 & 0 & 0 & 0 & -\varepsilon_{3i}I \\ \frac{\varepsilon_{3i}}{(1-\sqrt{n c_i^+})^2}\delta_0^2 G_3 & 0 & 0 & 0 & 0 & 0 \\ R_i^\top & 0 & 0 & 0 & 0 & 0 \\ \frac{\varepsilon_{4i}}{(1-\sqrt{n c_i^+})^2}L & 0 & 0 & 0 & 0 & 0 \end{array}\right.$$

$$\left.\begin{array}{ccc} P(i)D(i) & \frac{\varepsilon_{2i}}{(1-\sqrt{n c_i^+})^2}G_2^\top & P(i)W(i) \\ 0 & 0 & 0 \\ -\varepsilon_{1i}I & 0 & 0 \\ -\varepsilon_{2i}I & 0 & 0 \\ 0 & -\varepsilon_{2i}I & 0 \\ 0 & 0 & -\varepsilon_{3i}I \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{array}\right) < 0,$$

where

$$\Gamma_{1i} := \frac{1}{1-\sqrt{n c_i^+}}A_{ki}^\top + P(i) + \frac{1}{1-\sqrt{n c_i^+}}P(i)A_{ki} + \frac{1}{1-\sqrt{n c_i^+}}\sum_{j=1}^{N}\gamma_{ij}P_j.$$

In this case, the estimator gain matrix $K(i)$ can be taken as

$$K(i) = P(i)^{-1}R(i).$$

Then the error-state system (2.9) of the neural network (2.7) is globally asymptotically stable in the mean square.

Proof. Pre- and post-multiplying the inequality $\Psi < 0$ by the block diagonal matrix

$$\mathrm{diag}\{I, \varepsilon_{1i}^{-1/2}I, \varepsilon_{1i}^{-1/2}I, \varepsilon_{2i}^{-1/2}I, \varepsilon_{2i}^{-1/2}I, \varepsilon_{3i}^{-1/2}I, \varepsilon_{3i}^{-1/2}I, \varepsilon_{4i}^{-1/2}I, \varepsilon_{4i}^{-1/2}I\},$$

yields the following inequality:

$$\Phi = \begin{pmatrix} \Gamma_{1i} & \varepsilon_{1i}^{-1/2}P(i)B(i) & \frac{\varepsilon_{1i}^{1/2}}{1-c_i^+}G_1^\top & \varepsilon_{2i}^{-1/2}P(i)D(i) & \frac{\varepsilon_{2i}^{1/2}}{1-c_i^+}G_2^\top & \varepsilon_{3i}^{-1/2}P(i)W(i) & \frac{\varepsilon_{3i}^{1/2}}{1-c_i^+}\delta_0^2G_3^\top & \varepsilon_{4i}^{-1/2}R_i & \frac{\varepsilon_{4i}^{1/2}}{1-c_i^+}L^\top \\ \varepsilon_{1i}^{-1/2}B^\top(i)P^\top(i) & -I & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ \frac{\varepsilon_{1i}^{1/2}}{1-c_i^+}G_1 & 0 & -I & 0 & 0 & 0 & 0 & 0 & 0 \\ \varepsilon_{2i}^{-1/2}D^\top(i)P^\top(i) & 0 & 0 & -I & 0 & 0 & 0 & 0 & 0 \\ \frac{\varepsilon_{2i}}{1-c_i^+}G_2 1/2 & 0 & 0 & 0 & -I & 0 & 0 & 0 & 0 \\ \varepsilon_{3i}^{-1/2}W^\top(i)P^\top(i) & 0 & 0 & 0 & 0 & -I & 0 & 0 & 0 \\ \frac{\varepsilon_{3i}^{1/2}\delta_0^2}{1-c_i^+}G_3 & 0 & 0 & 0 & 0 & 0 & -I & 0 & 0 \\ \varepsilon_{4i}^{-1/2}R_i^\top & 0 & 0 & 0 & 0 & 0 & 0 & -I & 0 \\ \frac{\varepsilon_{4i}^{-1/2}}{1-c_i^+}\varepsilon_{4i}L & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -I \end{pmatrix} < 0,$$

or

$$\begin{pmatrix} \Gamma_{1i} & \Gamma_{3i}^\top \\ \Gamma_{3i} & -\Gamma_{2i} \end{pmatrix} < 0, \tag{3.13}$$

where $\Gamma_{2i} = I$,

$$\Gamma_{3i} = [\varepsilon_{1i}^{-1/2}P(i)B(i)\ \frac{\varepsilon_{1i}^{1/2}}{1-c_i^+}G_1^\top\ \varepsilon_{2i}^{-1/2}P(i)D(i)\ \frac{\varepsilon_{2i}^{1/2}}{1-c_i^+}G_2^\top\ \varepsilon_{3i}^{-1/2}P(i)W(i)\ \frac{\varepsilon_{3i}^{1/2}}{1-c_i^+}\delta_0^2G_3^\top\ \varepsilon_{4i}^{-1/2}R_i\ \frac{\varepsilon_{4i}^{1/2}}{1-c_i^+}L^\top]^\top.$$

From Lemma 2.8, we can find that (3.13) holds if and only if

$$\Gamma_{1i} + \Gamma_{3i}^\top\Gamma_{2i}^{-1}\Gamma_{3i} < 0,$$

or

$$\begin{aligned} &\frac{1}{1-\sqrt{n}c_i^+}A_{ki}^\top P(i) + \frac{1}{1-\sqrt{n}c_i^+}P(i)A_{ki} + \frac{1}{1-\sqrt{n}c_i^+}\sum_{j=1}^N \gamma_{ij}P_j \\ &+ \frac{1}{(1-\sqrt{n}c_i^+)^2}\Big(\varepsilon_{1i}G_1^\top G_1 + \varepsilon_{2i}G_2^\top G_2 + \varepsilon_{3i}\delta_0^2G_3^\top G_3 + \varepsilon_{4i}L^\top L + (2+\tau_0)Q_1 + 3\delta_0Q_2\Big) \\ &+ \varepsilon_{1i}^{-1}P(i)B(i)B^\top(i)P(i) + \varepsilon_{2i}^{-1}P(i)D(i)D^\top(i)P(i) \\ &+ \varepsilon_{3i}^{-1}P(i)W(i)W^\top(i)P(i) + \varepsilon_{4i}^{-1}R(i)R^\top(i) < 0. \end{aligned} \tag{3.14}$$

Noticing $R(i) = P(i)K(i)$. Obviously, (3.14) is the same as

$$\frac{1}{1-\sqrt{n}c_i^+}\Pi_1 + \frac{1}{(1-\sqrt{n}c_i^+)^2}\Pi_2 + \Pi_3 < 0.$$

Hence, from Theorem 3.1 and the estimator gain $K(i) = P^{-1}(i)R(i)$ in Theorem 3.3, the system (2.9) is globally asymptotically stable in the mean square of the neural network (2.7). □

Remark 3.4. A meaningful approach to tackle state estimation problems is to convert the nonlinearly coupled matrix inequalities into linear matrix inequalities (LMIs), while the estimator gain is designed simultaneously. It should be mentioned that, in the past decade, LMIs have gained much attention for their computational tractability and usefulness in many areas, including the stability testing for neural networks. Hence, LMIs method is very crucial for obtaining the main results in the present paper.

4. Numerical example

In this section, we present a simulation example so as to illustrate the usefulness of our main results.

Example 4.1. Consider a 3-neuron neural network (2.7) with the following parameters:

$$\delta(t) = \tau(t) = \frac{1}{2}\sin t, \quad \gamma = 2, \quad C = \mathrm{diag}\{\frac{1}{3}, \frac{1}{3}, \frac{1}{3}\}, \quad A_{k1} = \mathrm{diag}\{1.5, 1.7, 1.8\}, \quad A_{k2} = \mathrm{diag}\{1.4, 1.5, 1.8\},$$

$$G_1 = \mathrm{diag}\{0.01, 0.02, 0.03\}, \quad G_2 = \mathrm{diag}\{0.03, 0.04, 0.06\}, \quad G_3 = \mathrm{diag}\{0.05, 0.07, 0.09\},$$

$$L = \mathrm{diag}\{0.08, 0.08, 0.08\},\ B_1 = [0.06\ \ -0.085\ 0.09], \quad B_2 = \begin{pmatrix} 0.02 & 0.004 & 0.001 \\ 0.04 & 0.001 & 0.002 \\ 0.001 & 0.04 & 0.03 \end{pmatrix},$$

$$D_1 = \begin{pmatrix} 0.02 & 0.004 & 0.001 \\ 0.04 & 0.001 & 0.002 \\ 0.001 & 0.04 & 0.03 \end{pmatrix}, \quad D_2 = \begin{pmatrix} 0.02 & 0.004 & 0.001 \\ 0.04 & 0.001 & 0.002 \\ 0.001 & 0.04 & 0.03 \end{pmatrix},$$

$$W_1 = \begin{pmatrix} 0.03 & 0.005 & 0.002 \\ 0.04 & 0.002 & 0.002 \\ 0.002 & 0.02 & 0.03 \end{pmatrix}, \quad W_2 = \begin{pmatrix} 0.03 & 0.005 & 0.004 \\ 0.04 & 0.004 & 0.002 \\ 0.004 & 0.02 & 0.03 \end{pmatrix},$$

$$\Gamma = \begin{pmatrix} -3 & 5 & 4 \\ 5 & -4 & 2 \\ 4 & 2 & -3 \end{pmatrix}, \qquad I_1 = I_2 = [0.3\ 0.3\ 0.3]^\top.$$

Then we have $\tau_0 = 0.5$, $\delta_0 = 0.5$, $c_i^+ = \frac{1}{3}$, $\sqrt{n}c_i^+ = \frac{\sqrt{3}}{3} < 1$. Using the Matlab LMI toolbox to solve the LMI Ψ, we obtain

$$P_1 = \begin{pmatrix} 62.5121 & 0.3235 & 0.2986 \\ 0.3235 & 61.4512 & 0.1527 \\ 0.2986 & 0.1527 & 56.4822 \end{pmatrix}, \qquad P_2 = \begin{pmatrix} 46.2110 & 0.2218 & 0.3416 \\ 0.2218 & 48.2945 & 0.2011 \\ 0.3416 & 0.2011 & 52.2713 \end{pmatrix},$$

$$R_1 = \begin{pmatrix} -190.1256 & 46.0211 & 42.0219 \\ 205.0301 & 1.1419 & 0.8904 \\ 165.3009 & 0.7804 & 0.9980 \end{pmatrix}, \quad R_2 = \begin{pmatrix} 190.1256 & -46.0211 & -40.0021 \\ -205.0301 & -1.1419 & -0.7598 \\ -162.2514 & -0.6855 & -0.8925 \end{pmatrix},$$

and we have

$$K_1 = P_1^{-1}R_1 = \begin{pmatrix} -3.0727 & 0.7361 & 0.6721 \\ 3.3454 & 0.0147 & 0.0109 \\ 2.9338 & 0.0099 & 0.0141 \end{pmatrix}, \quad K_2 = P_2^{-1}R_2 = \begin{pmatrix} 4.1577 & -0.9958 & -0.8655 \\ -4.2515 & -0.0190 & -0.0117 \\ -3.1148 & -0.0065 & -0.0114 \end{pmatrix}.$$

From Theorem 3.1 and estimator gain K_i $(i = 1, 2)$, the error dynamics for the neutral-type neural network converges to zero asymptotically in the mean square.

Remark 4.2. In [13], Liu et al. studied the state estimation problem of a class of non-neutral type neural networks with mixed delays

$$x'(t) = -Dx(t) + AF(x(t)) + BG(x(t-\tau_1)) + W\int_{t-\tau_2}^{t} H(x(s))ds + I(t). \tag{4.1}$$

We can find that both the existence conditions and the explicit expression of the desired estimator can be characterized in terms of the solution to LMI method in [13]. It is clear that system (4.1) is a special case when $C = 0$ and $\tau(t) = \tau_1$, $\delta(t) = \tau_2$ in system (2.2). Hence, the numerical results of (4.1) can be easily deduced by the corresponding results of (2.2).

5. Conclusions

In this paper, some sufficient conditions for stochastic stability of a class of neutral-type neural networks with Markovian jumping parameters have been obtained by using LMI. In addition, the existence of the expected estimators have been derived in terms of the positive definite solution to an LMI involving several scalar parameters, and the analytical expression characterizing the desired estimators has been obtained. Finally, a simulation example has been provided to show the usefulness of the derived LMI-based stability conditions.

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