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New homoclinic rogue wave solution for the coupled Schrödinger-Boussinesq equation

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Abstract

Exact homoclinic breather wave solution for the coupled Schrödinger-Boussinesq equation is obtained by using homoclinic test technique. Based on the homoclinic breather wave solution, rational homoclinic breather wave solution is generated by homoclinic breather limit method, rogue wave in the form of the rational homoclinic solution is derived when the period of homoclinic breather wave goes to infinite. This is a new way for generating rogue wave which is different from direct constructing method, Darboux dressing technique and ansätz with complexity of parameter. This result shows the homoclinic rogue wave can be generated from homoclinic breather wave, and it is useful for explaining some related nonlinear phenomenon.

Keywords: Schrödinger-Boussinesq equation, Hirota bilinear form, homoclinic breather limit method, rogue wave.
2010 MSC: 35Exx, 35Qxx.

1. Introduction

Nonlinear evolution equations (NLEEs) depict some physical scenarios that appear in many areas of physics, engineering, nonlinear science, and applied mathematics [1]. It is indeed important to investigate the methods for solutions of NLEEs, so many effective methods have been developed, such as the new technology combining the variational iterative method and an integral transform [28], the local fractional variational iteration method [27], the local fractional Riccati differential equation method [29], new integral transform operator for finding the analytical solution [26]. Rogue waves [2–5, 7, 8, 12–17, 21, 22, 25, 30, 31], as a special type of solitary waves, have been triggered much interest in various physical branches. Rogue wave is a kind of waves that seems abnormal which is first observed in the deep ocean. Recently, rogue wave solutions in other more complex systems have been sought by using the Darboux dressing technique or Hirota bilinear method [3, 13, 15, 16, 22, 30, 31]. Periodic breather, Akhmediev breather, Ma breather and rogue wave solutions are obtained for the coupled long-wave-short-wave system by using a Hirota two-soliton method with complex frequency and complex wave number [23]. Now

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we consider the following coupled Schrödinger-Boussinesq equation:

$$\begin{cases} iE_t + E_{xx} + \beta_1 E - NE = 0, \\ 3N_{tt} - N_{xxxx} + 3(N^2)_{xx} + \beta_2 N_{xx} - (|E|^2)_{xx} = 0, \end{cases} \tag{1.1}$$

with the periodic boundary condition

$$E(x,t) = E(x+l,t), \quad N(x,t) = N(x+l,t),$$

where l, β_1, β_2 are real constants, $E(x,t)$ is a complex-valued function, and $N(x,t)$ is a real-valued function. (1.1) is known to describe various physical processes in laser and plasma, such as formation, Langmuir field amplitude, intense electromagnetic waves, and modulational instabilities [6, 18–20], the complex-valued function E represents the short wave amplitude, the real-valued function N represents the long wave amplitude, and the subscripts t and x denote partial differentiation with respect to time and space. N-soliton solution, the complete integrability, homoclinic solutions and heteroclinic solutions of (1.1) have been studied [6, 9–11]. Previously, rogue wave solutions were reported by Mu and Qin [15] and Wang et al. [24].

The Peregrine method was applied to (1.1) in [15] and rogue wave solution was given

$$\begin{cases} E(x,t) = ae^{\beta_1 it}\left(\dfrac{4-4ibt}{1+bx^2+b^2t^2} - 1\right), \\ N(x,t) = \dfrac{4b(bx^2-b^2t^2-1)}{(1+bx^2+b^2t^2)^2}, \end{cases}$$

where $a = \frac{1}{2}\sqrt{2b(3b-\beta_2)}$, its existence condition is that the parameter b must satisfy $b(3b-\beta_2) > 0$. Obviously, N is independent of the coefficient β_1 of (1.1) and $(E(x,t), N(x,t))$ is not the homoclinic solution.

In [24], the rogue wave solution of (1.1) was obtained by ansätz with complexity of parameters.

$$\begin{cases} E(x,t) = E_0 e^{-i(\xi+\theta_0)}\left(1 - \dfrac{4(1+i((\Re(\Omega_1^2) - 2k\Im(\Omega_1))t + x(\Im(\Omega_1)+2k)))}{((x-\Im(\Omega_1)t)^2 + t^2\Re^2(\Omega_1) + \frac{1}{\Re^2(\Omega_1)})|2k - i\Omega_1|^2}\right), \\ N(x,t) = N_0 - \dfrac{4}{(x-\Im(\Omega_1)t)^2 + t^2\Re^2(\Omega_1) + \frac{1}{\Re^2(\Omega_1)}} + \dfrac{8(x-\Im(\Omega_1)t)^2}{((x-\Im(\Omega_1)t)^2 + t^2\Re^2(\Omega_1) + \frac{1}{\Re^2(\Omega_1)})^2}, \end{cases}$$

where $\xi = kx + lt$, Ω_1 satisfies $2E_0^2 + (3\Omega_1^2 - \beta_2 + 6(k^2 - \beta_1 - l))(i\Omega_1 - 2k)^2 = 0$, and $\Re(\Omega_1) \neq 0$, $k \neq -\frac{\Im(\Omega_1)}{2}$, $\Re(\Omega_1)$ and $\Im(\Omega_1)$ represent the real and imaginary part of complex Ω_1 respectively. This is a multi-parameter rogue wave solution, but the solution is not homoclinic solution of (1.1).

In this work, Homoclinic solutions with oscillatory structure for the coupled Schrödinger-Boussinesq equation is constructed by using the Hirota bilinear form and extended homoclinic test method, and then rogue solution is obtained by taking the limit of period of the homoclinic solution with the periodic condition approaching infinite (homoclinic breather limit method). The rogue wave solution obtained here is still a homoclinic solution.

2. Linear stability analysis

It is easy to see that $(e^{-ait}, a+\beta_1)$ is a fixed cycle of (1.1), we consider a small perturbation of the form

$$\begin{cases} E(x,t) = e^{-ait}(1+\epsilon(x,t)), \\ N(x,t) = (a+\beta_1)(1+\phi(x,t)), \end{cases} \tag{2.1}$$

where $|\epsilon(x,t)| \ll 1$, $\phi(x,t) \ll 1$, a is an arbitrary constant, and $a \neq 0$. Substituting (2.1) into (1.1), we get the lineared equation

$$\begin{cases} i\epsilon_t + \epsilon_{xx} - (a+\beta_1)\phi = 0, \\ 3(a+\beta_1)\phi_{tt} - (a+\beta_1)\phi_{xxxx} + (a+\beta_1)(6a+6\beta_1+\beta_2)\phi_{xx} - \epsilon_{xx} - \bar{\epsilon}_{xx} = 0. \end{cases} \tag{2.2}$$

Assume that $\epsilon(x,t)$ and $\phi(x,t)$ have the following forms:

$$\begin{cases} \epsilon(x,t) = Ge^{i\mu_n x+\sigma_n t} + He^{-i\mu_n x+\sigma_n t}, \\ \phi(x,t) = C(e^{i\mu_n x+\sigma_n t} + e^{-i\mu_n x+\sigma_n t}), \end{cases} \tag{2.3}$$

where G, H are complex constants, and C is a real number, $\mu_n = 2\pi n/l$, and σ_n is the growth rate. Substituting (2.3) into (2.2), we have

$$\begin{cases} H(i\sigma_n - \mu_n^2) = (a+\beta_1)C, \\ G(i\sigma_n - \mu_n^2) = (a+\beta_1)C, \\ (3(a+\beta_1)\sigma_n^2 + (a+\beta_1)\mu_n^4 - (a+\beta_1)(6a+6\beta_1+\beta_2)\mu_n^2)C = -(H+\bar{G})\mu_n^2, \\ (3(a+\beta_1)\sigma_n^2 + (a+\beta_1)\mu_n^4 - (a+\beta_1)(6a+6\beta_1+\beta_2)\mu_n^2)C = -(G+\bar{H})\mu_n^2. \end{cases} \tag{2.4}$$

Solving (2.4), we obtain that

$$\sigma_n^2 = \frac{(6a+6\beta_1+\beta_2)\mu_n^2 - 2\mu_n^4 \pm \sqrt{((6a+6\beta_1+\beta_2)\mu_n^2 - 2\mu_n^4)^2 + 24\mu_n^2 + 12(6a+6\beta_1+\beta_2)\mu_n^6 + 12\mu_n^8}}{6}.$$

Obviously, the above formula implies that $(6a+6\beta_1+\beta_2)\mu_n^2 - 2\mu_n^4 > 0$, then

$$\mu_n^2 < \frac{6a+6\beta_1+\beta_2}{2}.$$

3. Rogue wave solution for the coupled Schrödinger-Boussinesq equation

Make the transformation

$$E(x,t) = e^{-ait}u(x,t), \quad N(x,t) = v_0 + v(x,t),$$

$a \neq 0$, then (1.1) can be reduced into the following form

$$\begin{cases} iu_t + u_{xx} + (a+\beta_1-v_0)u - uv = 0, \\ 3v_{tt} - v_{xxxx} + (6v_0+\beta_2)v_{xx} + 3(v^2)_{xx} - (|u|^2)_{xx} = 0, \end{cases} \tag{3.1}$$

when $a+\beta_1-v_0 = 0$, using the following transformation

$$u(x,t) = \frac{g(x,t)}{f(x,t)}, v(x,t) = -2(\ln f(x,t))_{xx}.$$

Equation (3.1) may be rewritten as the following coupled bilinear form:

$$\begin{cases} (iD_t + D_x^2)g \cdot f = 0, \\ (3D_t^2 + (6v_0+\beta_2)D_x^2 - D_x^4 - \lambda)f \cdot f + gg^* = 0, \end{cases}$$

where λ is an integration constant, g^* denotes the complex conjugation of g. The Hirota bilinear operator $D_x^m D_t^n$ is defined by $(n, m \geqslant 0)$

$$D_x^m D_t^n f(x,t) \cdot g(x,t) = (\frac{\partial}{\partial x} - \frac{\partial}{\partial x'})^m (\frac{\partial}{\partial t} - \frac{\partial}{\partial t'})^n f(x,t)g(x',t')|_{x'=x,t'=t}.$$

Taking the following test function

$$\begin{cases} g(x,t) = 1 + b_1\cos(px)e^{\Omega t+\gamma} + b_2 e^{2\Omega t+2\gamma}, \\ f(x,t) = 1 + b_3\cos(px)e^{\Omega t+\gamma} + b_4 e^{2\Omega t+2\gamma}, \end{cases}$$

the parameters Ω, γ, $b_s(s = 1,2,3,4,5,6)$ will be determined later, $b_s(s = 1,2)$ are complex numbers, b_3 and b_4 are real numbers.

So the solution for (1.1) as

$$\begin{cases} E(x,t) = e^{-ait}\dfrac{e^{-\Omega t-\gamma} + b_1\cos(px) + b_2 e^{\Omega t+\gamma}}{\sqrt{b_4}(2\cosh(\Omega t+\gamma+\ln\sqrt{b_4}) + b_3\cos(px))}, \\ N(x,t) = \beta_1 + \dfrac{2b_3p^2(2\sqrt{b_4}\cos(px)\cosh(\Omega t+\gamma+\ln\sqrt{b_4}) + b_3)}{b_4(2\cosh(\Omega t+\gamma+\ln\sqrt{b_4}) + b_3\cos(px))^2}, \end{cases} \tag{3.2}$$

the following relations among the parameters:

$$\begin{cases} \nu_0 = \beta_1 + a, \\ \lambda = 1, \quad b_1 = \frac{i\Omega+p^2}{i\Omega-p^2}b_3, \\ b_2 = (\frac{i\Omega+p^2}{i\Omega-p^2})^2 b_4, \quad b_3^2 = \frac{4\Omega^2}{\Omega^2+p^4}, \\ (3\Omega^2 - p^4 - (6a+6\beta_1+\beta_2)p^2)(\Omega^2+p^4) = 2p^4. \end{cases} \tag{3.3}$$

From $\Omega^2 > 0$, we have

$$\Omega = \pm p\sqrt{\frac{(6a+6\beta_1+\beta_2) - 2p^2 + \sqrt{24 + (6a+6\beta_1+\beta_2)^2 + 8(6a+6\beta_1+\beta_2)p^2 + 16p^4}}{6}}.$$

Equation (3.2) can be rewritten as

$$\begin{cases} E(x,t) = e^{-ait}\dfrac{2\sqrt{b_2}\cosh(\Omega t+\gamma+\theta) + b_1\cos(px)}{2\sqrt{b_4}\cosh(\Omega t+\gamma+\theta_1) + b_3\cos(px)}, \\ N(x,t) = a + \beta_1 + \dfrac{2b_3p^2(2\sqrt{b_4}\cos(px)\cosh(\Omega t+\gamma+\theta_1) + b_3)}{b_4(2\cosh(\Omega t+\gamma+\theta_1) + b_3\cos(px))^2}, \end{cases}$$

where $\theta = \ln\sqrt{b_2}, \theta_1 = \ln\sqrt{b_4}$. This is a solution of Abs type [2]. The trajectory of this solution is defined explicitly by $t = -\frac{\gamma+\theta}{\Omega}$. That is, this solution evolves periodically along the straight line parallel to the x axis. So this solution is an Akhmediev breather (space periodic breather solutions) as well.

Let $b_4 = 1,\ \gamma = 0$, we get

$$\begin{cases} E(x,t) = e^{-ait}\dfrac{e^{-\Omega t} + b_1\cos(px) + b_2 e^{\Omega t}}{2\cosh(\Omega t) + b_3\cos(px)}, \\ N(x,t) = a + \beta_1 + \dfrac{2b_3p^2(2\cos(px)\cosh(\Omega t) + b_3)}{(2\cosh(\Omega t) + b_3\cos(px))^2}, \end{cases} \tag{3.4}$$

substitute $\Omega = \pm p\sqrt{\frac{(6a+6\beta_1+\beta_2)-2p^2+\sqrt{24+(6a+6\beta_1+\beta_2)^2+8(6a+6\beta_1+\beta_2)p^2+16p^4}}{6}}$ into (3.3) and (3.4), then we take the limit of p approaching zero (period $\frac{2\pi}{p} \to \infty$), the following rational solution can be obtained.

$$\begin{cases} E(x,t) = e^{-ait}\dfrac{A}{B}, \\ N(x,t) = a + \beta_1 + \dfrac{C}{D}, \end{cases}$$

where

$$\begin{cases} A = -54 + 12t^2 + 3(x^2 - 4it)(6a + 6\beta_1 + \beta_2) + t^2(6a + 6\beta_1 + \beta_2)^2 + (t^2(6a + 6\beta_1 + \beta_2) \\ \quad +3(x^2 - 4it))\sqrt{24 + (6a + 6\beta_1 + \beta_2)^2}, \\ B = 18 + 12t^2 + 3x^2(6a + 6\beta_1 + \beta_2) + t^2(6a + 6\beta_1 + \beta_2)^2 \\ \quad +(t^2(6a + 6\beta_1 + \beta_2) + 3x^2)\sqrt{24 + (6a + 6\beta_1 + \beta_2)^2}, \\ C = -24(-36x^2 + 9(1 + 2t^2)(6a + 6\beta_1 + \beta_2) - 3x^2(6a + 6\beta_1 + \beta_2)^2 + t^2(6a + 6\beta_1 + \beta_2)^3 \\ \quad +(9 + 6t^2 - 3x^2(6a + 6\beta_1 + \beta_2) + t^2(6a + 6\beta_1 + \beta_2)^2)\sqrt{24 + (6a + 6\beta_1 + \beta_2)^2}), \\ D = (18 + 12t^2 + 3x^2(6a + 6\beta_1 + \beta_2) + t^2(6a + 6\beta_1 + \beta_2)^2 \\ \quad +(t^2(6a + 6\beta_1 + \beta_2) + 3x^2)\sqrt{24 + (6a + 6\beta_1 + \beta_2)^2})^2. \end{cases} \tag{3.5}$$

The spatial structures of the function $E(x,t)$, $N(x,t)$ have similar structures of the rogue waves, Figure 1 shows that the function $E(x,t)$ has bright-dark rogue wave features, the bright-dark rogue wave turns into a bright rogue wave with an eye-shaped distribution (a hump and two holes). While $N(x,t)$ shows dark rogue wave features in Figure 2. In Figures 1 and 2 we can see that an obvious feature of these solutions is localized in both space and time. It is a singular breather and describes a single wave. The rogue waves of $|E|$, N are first-order rogue waves and concentrated around $(0,0)$. we can observe the changes of $(|E(x,0)|, N(x,0))$ in the direction of the x axes and see that the maximal amplitude of $|E(x,t)|$ occurs at point $(0,0)$ and the maximum amplitude of this rogue wave solution is equal to 3, the minimum amplitudes of $|E(x,t)|$ occurs at two points

$$(x = \pm\frac{\sqrt{3}}{2}\sqrt{-(6a + 6\beta_1 + \beta_2) + \sqrt{24 + (6a + 6\beta_1 + \beta_2)^2}}, t = 0),$$

but the maximum amplitudes of $N(x,t)$ occur at two points

$$(x = \pm\frac{\sqrt{3}}{2}\sqrt{-(6a + 6\beta_1 + \beta_2) + \sqrt{24 + (6a + 6\beta_1 + \beta_2)^2}}, t = 0),$$

the minimum amplitude of N occurs at $(0,0)$ and the minimum amplitude of this rogue wave solution is equal to -4. $(E(x,t), N(x,t))$ contains two waves with different velocities and directions. Moreover, $(E(x,t), N(x,t))$ is not only the rational homoclinic solution and homoclinic to the fixed cycle $(e^{-iat}, a + \beta_1)$ as $t \to \infty$ or $x \to \infty$. In fact,

$$(E, N) \to (e^{-iat}, a + \beta_1) \ \text{ as } \ t \to \pm\infty.$$

$(E(x,t), N(x,t))$ is also a rogue wave solution which has two to three times amplitude higher than its surrounding waves and generally forms in a short time (see Figure 1 and Figure 2). It is a new discovery that the rogue wave solution can come from homoclinic breather solution for coupled Schrödinger-Boussinesq equation. One may think whether the energy collection and superposition of homoclinic breather wave in many periods lead to a rogue wave or not. Moreover, it follows from Figure 1 and Figure 2 that the amplitudes of rogue waves become more and more short as time goes, and approach a non-zero constant background finally. It is shown that the rogue waves arise from the non-zero constant background and then disappear into the non-zero constant background again. Obviously, the solution in this work is different from the solution in [15] or [24]. Giving some special parameters in (3.5), the shape of the rogue wave can be exhibited (Figure 1 and Figure 2).

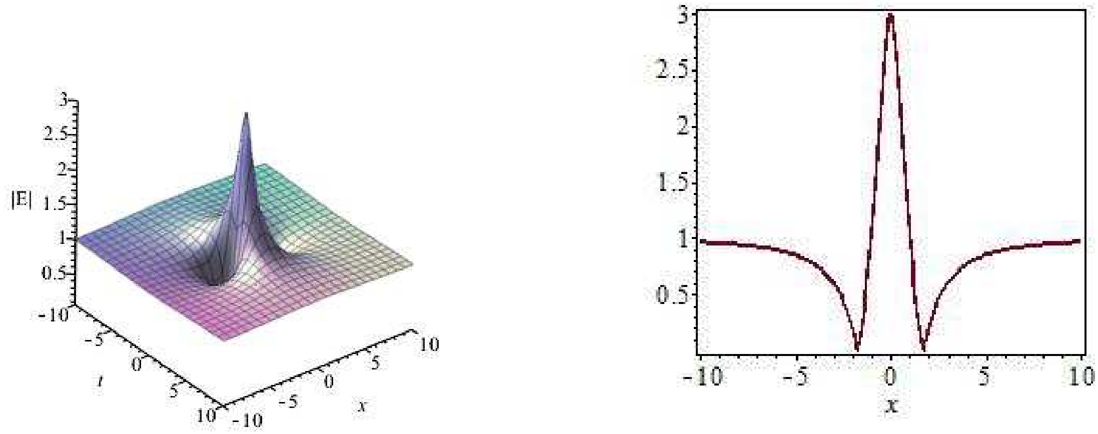

Figure 1: (1): Behavior of $|E|$ as $6\alpha + 6\beta_1 + \beta_2 = 1$. (2): Rogue wave variation in $x - |E|$ plane.

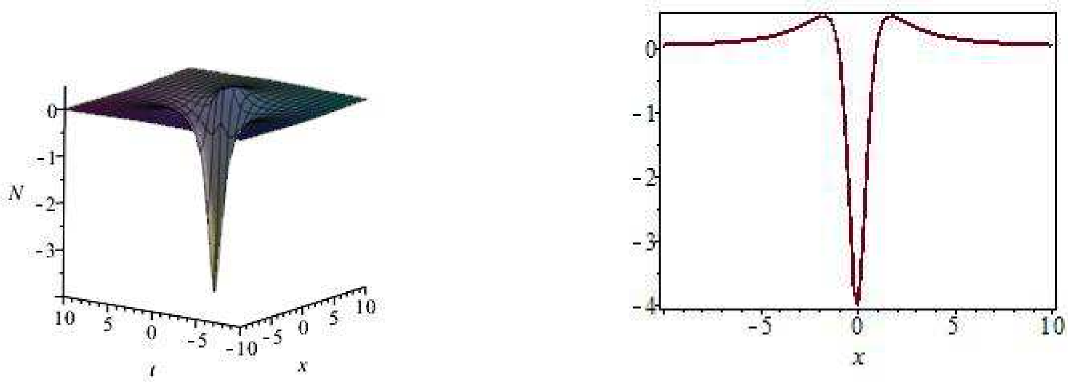

Figure 2: (1): Behavior of N as $6\alpha + 6\beta_1 + \beta_2 = 1$. (2): Rogue wave variation in $x - N$ plane.

4. Conclusion

In summary, applying the Hirota bilinear form and homoclinic breather limit method to the coupled Schrödinger-Boussinesq equation, exact rational homoclinic wave solution is obtained, it is rogue wave solution in the form of the rational homoclinic wave solution. Some features of rogue wave are presented, the bright rogue wave and the dark rogue wave with special structure are exhibited. Results show the complexity of dynamical behavior and the variety of structure for rogue wave solutions of the coupled Schrödinger-Boussinesq equation. At the same time, the way to generate rogue wave solution is various. The problems needed to be further studied is whether the other types of nonlinear evolution equations have this kind of homoclinic solution or not, and whether (1.1) has other type of specially spatiotemporal structure of solutions.

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