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A note on impulsive control of nonlinear systems with impulse time window

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Abstract

In this paper, we present some sufficient conditions for the stability of nonlinear systems with impulse time window by using some inequality techniques and results of matrix analysis. The proposed results are simpler than ones shown by Feng et al. [Y.-M. Feng, C.-D. Li, T.-W. Huang, Neurocomputing, **193** (2016), 7–13]. Finally, several numerical examples are given to show the effectiveness of our results.

Keywords: Nonlinear systems, impulsive control, impulse time window.
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1. Introduction

In this note we mainly adopt the notation and terminology in [20]. For convenience, recall that, as usual, we use $P^T, \lambda_{max}(P)$, and $s_{max}(P)$ to denote the transpose, the maximum eigenvalue and maximum singular value of a square matrix P, respectively. The symbol $\|x\|$ is used to denote the Euclidean norm of the vector x. We use $P > 0$ (< 0, $\leqslant 0$, $\geqslant 0$) to denote a symmetrical positive (negative, semi-negative, semi-positive) definite matrix P. $f(x(t_1^-))$ is defined by $f(x(t_1^-)) = \lim\limits_{t \to t_1^-} f(x(t))$. I denotes the identity matrix of proper dimension.

Let $g(t)$ be a continuous real-valued function defined on a real interval Ω and H be a Hermitian matrix with eigenvalues in Ω, let

$$H = U\mathrm{diag}(\lambda_1, \cdots, \lambda_n) U^*,$$

be a spectral decomposition with U unitary, then the functional calculus for H is defined as

$$g(H) = U\mathrm{diag}(g(\lambda_1), \cdots, g(\lambda_n)) U^*.$$

For example, if $H \geqslant 0$, then

$$H^{1/2} = U\mathrm{diag}\left(\lambda_1^{1/2}, \cdots, \lambda_n^{1/2}\right) U^*.$$

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Feng et al. [4] discussed the following nonlinear impulsive control systems with impulse time window:

$$\begin{cases} \dot{x}(t) = Ax(t) + f(x(t)), & mT \leqslant t < mT + \tau_m, \\ x(t) = x(t^-) + Jx(t^-), & t = mT + \tau_m, \\ \dot{x}(t) = Ax(t) + f(x(t)), & mT + \tau_m < t < (m+1)T, \end{cases} \tag{1.1}$$

where $x \in R^n$ presents the state vector, $f : R^n \to R^n$ is a continuous nonlinear function satisfying $f(0) = 0$ and there exists a diagonal matrix $L = \mathrm{diag}(l_1, l_2, \cdots, l_n) \geqslant 0$ such that $\|f(x)\|^2 \leqslant x^T L x$ for any $x \in R^n$, $A \in R^{n\times n}$ is constant matrix. $T > 0$ denotes the control period, $0 \leqslant \tau_m < T$, τ_m is unknown within impulse time window $[mT, (m+1)T]$ and $J \in R^{n\times n}$.

In [4], the authors gave some conditions ([4, Theorem 1]) to ensure the system (1.1) to be stable. One of the conditions is to find constants $g > 0$, $\epsilon > 0$ and a symmetric and positive definite matrix $P \in R^{n\times n}$, such that

$$PA + A^T P + \epsilon P^2 + \epsilon^{-1} L - gP \leqslant 0, \tag{1.2}$$

is valid. Inequality (1.2) is equivalent to the following LMI

$$\begin{bmatrix} PA + A^T P + \epsilon^{-1} L - gP & -P \\ -P & -\epsilon^{-1} I \end{bmatrix} \leqslant 0.$$

Although LMIs can be solved in polynomial-time, the computation amount of solving LMIs is not very small [1], so methods of avoiding solving LMIs will be very helpful for solving the problem. Therefore, our target is to find proper T and J avoiding solving LMIs, such that the system (1.1) is stable, which implies that system (1.1) can be controlled by periodically single state-jumps impulsive control methods with impulse time window. For more results on this topic and its applications, readers are referred to [2, 3, 5, 7–9, 11, 13–19, 21–24, 26, 27] and the references therein.

In this paper, we use some inequality techniques and results of matrix analysis to simplify the condition (1.2).

2. Main results

We first need the following lemma [4].

Lemma 2.1. *Let $x, y \in R^n$ and $\varepsilon > 0$. Then*

$$2x^T y \leqslant \varepsilon x^T x + \frac{1}{\varepsilon} y^T y.$$

Now we are going to give the main results.

Theorem 2.2. *Let $g, \varepsilon, \delta > 0$ such that*

(1) $\frac{g^2}{4} I - 2\left(A^T A + L\right) > 0$;

(2) $gT + \ln\lambda < 0$,

where

$$\lambda = \lambda_{\max}\left(P^{-1}(I+J)^T P (I+J)\right),$$

with

$$P = \frac{1}{\sqrt{\varepsilon}}\left(\left(\frac{g^2}{4\varepsilon} - \delta\right) I - \frac{2}{\varepsilon}\left(A^T A + L\right)\right)^{1/2} + \frac{g}{2\varepsilon} I,$$

then the origin of system (1.1) is exponentially stable.

Proof. Since

$$\frac{g^2}{4}I-2\left(A^TA+L\right)>0,$$

we have

$$\frac{g^2}{4\varepsilon}I-\frac{2}{\varepsilon}\left(A^TA+L\right)>0.$$

So, there exists $\delta>0$ such that

$$\left(\frac{g^2}{4\varepsilon}-\delta\right)I-\frac{2}{\varepsilon}\left(A^TA+L\right)\geqslant 0.$$

Let

$$X=\left(\left(\frac{g^2}{4\varepsilon}-\delta\right)I-\frac{2}{\varepsilon}\left(A^TA+L\right)\right)^{1/2}\geqslant 0,$$

and

$$P=\frac{1}{\sqrt{\varepsilon}}X+\frac{g}{2\varepsilon}I>0,$$

then, we have

$$X^2\leqslant\frac{g^2}{4\varepsilon}I-\frac{2}{\varepsilon}\left(A^TA+L\right),$$

and so

$$\left(\sqrt{\varepsilon}P-\frac{g}{2\sqrt{\varepsilon}}I\right)^2\leqslant\frac{g^2}{4\varepsilon}I-\frac{2}{\varepsilon}\left(A^TA+L\right),$$

which is equivalent to

$$\varepsilon P^2+\frac{2}{\varepsilon}\left(A^TA+L\right)-gP\leqslant 0. \tag{2.1}$$

Now, let us construct the following Lyapunov function:

$$V\left(x\left(t\right)\right)=x^T\left(t\right)Px\left(t\right).$$

If $mT\leqslant t<mT+\tau_m$, then by Lemma 2.1 and inequality (2.1), we have

$$\begin{aligned}
D^+\left(V\left(x\left(t\right)\right)\right)&=2x^TP\left(Ax+f\left(x\right)\right)\\
&\leqslant\varepsilon x^TP^2x+\frac{1}{\varepsilon}\left(Ax+f\left(x\right)\right)^T\left(Ax+f\left(x\right)\right)\\
&=\varepsilon x^TP^2x+\frac{1}{\varepsilon}\left(x^TA^TAx+f^T\left(x\right)f\left(x\right)\right)\\
&\quad+\frac{1}{\varepsilon}\left(x^TA^Tf\left(x\right)+f^T\left(x\right)Ax\right)\\
&\leqslant\varepsilon x^TP^2x+\frac{1}{\varepsilon}\left(x^TA^TAx+f^T\left(x\right)f\left(x\right)\right)\\
&\quad+\frac{1}{\varepsilon}\left(x^TA^TAx+f^T\left(x\right)f\left(x\right)\right)\\
&=\varepsilon x^TP^2x+\frac{2}{\varepsilon}\left(x^TA^TAx+f^T\left(x\right)f\left(x\right)\right)\\
&\leqslant\varepsilon x^TP^2x+\frac{2}{\varepsilon}\left(x^TA^TAx+x^TLx\right)\\
&=x^T\left(\varepsilon P^2+\frac{2}{\varepsilon}\left(A^TA+L\right)\right)x\\
&=x^T\left(\varepsilon P^2+\frac{2}{\varepsilon}\left(A^TA+L\right)\right)x-gx^TPx+gx^TPx
\end{aligned}$$

$$
\begin{aligned}
&= x^T \left(\varepsilon P^2 + \frac{2}{\varepsilon} \left(A^T A + L \right) - gP \right) x + gV(x) \\
&\leqslant gV(x(t)),
\end{aligned}
$$

which implies that

$$V(x(t)) \leqslant V(x(mT)) e^{g(t-mT)}. \tag{2.2}$$

If $t = mT + \tau_m$, then we have

$$
\begin{aligned}
V(x(t)) &= \left(x(t^-) + Jx(t^-)\right)^T P \left(x(t^-) + Jx(t^-)\right) \\
&= x^T(t^-) \left(I + I\right)^T P \left(I + J\right) x(t^-) \\
&= x^T(t^-) P^{1/2} P^{-1/2} \left(I + I\right)^T P \left(I + J\right) P^{-1/2} P^{1/2} x(t^-) \\
&\leqslant \lambda V\left(x\left(t^-\right)\right).
\end{aligned} \tag{2.3}
$$

Similarly, if $mT + \tau_m < t \leqslant (m+1)T$, we also have

$$D^+ \left(V(x(t))\right) \leqslant gV(x(t)),$$

which implies that

$$V(x(t)) \leqslant V(x(mT + \tau_m)) e^{g(t-mT-\tau_m)}. \tag{2.4}$$

It follows from (2.3) and (2.4) that

$$V(x(t)) \leqslant \lambda V\left(x\left((mT + \tau_m)^-\right)\right) e^{g(t-mT-\tau_m)}, \tag{2.5}$$

where

$$mT + \tau_m \leqslant t < (m+1)T.$$

For $m = 0$:

1) If $0 \leqslant t < \tau_0$, then by (2.2) we have

$$V(x(t)) \leqslant V(x(0)) e^{gt}. \tag{2.6}$$

2) If $\tau_0 \leqslant t < T$, then combining (2.5) and (2.6), we have

$$
\begin{aligned}
V(x(t)) &\leqslant \lambda V\left(x\left(\tau_0^-\right)\right) e^{g(t-\tau_0)} \\
&\leqslant \lambda V(x(0)) e^{gt}.
\end{aligned} \tag{2.7}
$$

For $m = 1$:

1) If $T \leqslant t < T + \tau_1$, then it follows from (2.2) and (2.7) that

$$
\begin{aligned}
V(x(t)) &\leqslant V(x(T)) e^{g(t-T)} \\
&\leqslant \lambda V(x(0)) e^{gt}.
\end{aligned} \tag{2.8}
$$

2) If $T + \tau_1 \leqslant t < 2T$, then combining (2.5) and (2.8), we obtain

$$
\begin{aligned}
V(x(t)) &\leqslant \lambda V\left(x\left((T + \tau_1)^-\right)\right) e^{g(t-T-\tau_1)} \\
&\leqslant \lambda^2 V(x(0)) e^{gt}.
\end{aligned} \tag{2.9}
$$

For $m = 2$:

1) If $2T \leqslant t < 2T + \tau_2$, then it follows from (2.2) and (2.9) that

$$\begin{aligned} V(x(t)) &\leqslant V(x(2T))\, e^{g(t-2T)} \\ &\leqslant \lambda^2 V(x(0))\, e^{gt}. \end{aligned} \tag{2.10}$$

2) If $2T + \tau_2 \leqslant t < 3T$, then combining (2.5) and (2.10), we get

$$\begin{aligned} V(x(t)) &\leqslant \lambda V\left(x\left((2T+\tau_2)^-\right)\right) e^{g(t-2T-\tau_2)} \\ &\leqslant \lambda^3 V(x(0))\, e^{gt}. \end{aligned}$$

By induction, for $m = k$:

1) If $kT \leqslant t < kT + \tau_k$, then we have

$$V(x(t)) \leqslant \lambda^k V(x(0))\, e^{gt}. \tag{2.11}$$

2) If $kT + \tau_k \leqslant t < (k+1)T$, then we obtain

$$V(x(t)) \leqslant \lambda^{k+1} V(x(0))\, e^{gt}. \tag{2.12}$$

From (2.11), we know that if $kT \leqslant t < kT + \tau_k$, then

$$V(x(t)) \leqslant V(x(0))\, e^{gT + k(gT + \ln\lambda)}. \tag{2.13}$$

It is from (2.12) that if $kT + \tau_k \leqslant t < (k+1)T$, then

$$V(x(t)) \leqslant V(x(0))\, e^{(k+1)(gT + \ln\lambda)}. \tag{2.14}$$

It follows from (2.13), (2.14), and $gT + \ln\lambda < 0$ that

$$\lim_{t\to\infty} V(x(t)) = 0.$$

This completes the proof. □

Remark 2.3. For any symmetric matrix, by Schur's theorem, we know that there exists $g > 0$ such that $gI - A > 0$. So, we can always find the symmetric and positive definite matrix P in Theorem 2.2.

Remark 2.4. If we choose $J = aI$, then we have

$$\begin{aligned} \lambda &= \lambda_{max}\left(P^{-1}(I+J)^T P (I+J)\right) \\ &= \lambda_{max}\left(P^{-1}(1+a) IP (1+a) I\right) \\ &= (1+a)^2, \end{aligned}$$

and so condition (2) becomes

$$gT + 2\ln(1+a) < 0,$$

which can be easily calculated.

Corollary 2.5. *Let* $g > 0$ *such that*

(1) $\frac{g^2}{4} I - 2\left(A^T A + L\right) > 0$;

(2) $gT + 2\ln s_{max}(I+J) < 0$,

then the origin of system (1.1) *is exponentially stable.*

Proof. It is known that

$$\begin{aligned}
\lambda &= \lambda_{\max}\left(P^{-1}(I+J)^T P(I+J)\right)\\
&= \lambda_{\max}\left(P^{-1/2}(I+J)^T P(I+J)P^{-1/2}\right)\\
&= s_{\max}\left(P^{-1/2}(I+J)^T P(I+J)P^{-1/2}\right)\\
&\leqslant s_{\max}\left(P^{-1/2}\right)s_{\max}\left((I+J)^T\right)s_{\max}(P)\,s_{\max}(I+J)\,s_{\max}\left(P^{-1/2}\right)\\
&= s_{\max}^{-1/2}(P)\,s_{\max}(I+J)\,s_{\max}(P)\,s_{\max}(I+J)\,s_{\max}^{-1/2}(P)\\
&= s_{\max}^2(I+J).
\end{aligned}$$

From the inequality above, we know that if

$$gT + 2\ln s_{\max}(I+J) < 0,$$

then

$$gT + \ln\lambda < 0.$$

This completes the proof. □

Remark 2.6. The calculations of $2\ln s_{\max}(I+J)$ are much simpler than ones of $\ln\lambda$, because the former does not employ the matrix P.

According to the proof of Theorem 2.2, a simple design strategy for controlling chaotic systems can be obtained as follows.

Step 1. Finding a diagonal matrix $L = \mathrm{diag}(l_1, l_2, \cdots, l_n) \geqslant 0$ such that $\|f(x)\|^2 \leqslant x^T L x$ for any $x \in R^n$.

Step 2. Finding $g > 0$ such that $\frac{g^2}{4}I - 2\left(A^T A + L\right) > 0$, by using Geršgorin disk theorem [6] or other results about bounds for eigenvalues such as presented in [28].

Step 3. Choosing suitable ε, δ and calculating the matrix P according to the following formula,

$$P = \frac{1}{\sqrt{\varepsilon}}\left(\left(\frac{g^2}{4\varepsilon} - \delta\right)I - \frac{2}{\varepsilon}\left(A^T A + L\right)\right)^{1/2} + \frac{g}{2\varepsilon}I.$$

Step 4. Choosing a J according to the needs of the actual applications and computing the bound on T by applying the following inequality,

$$gT + \ln\lambda < 0.$$

If we want to use Corollary 2.5 for controlling chaotic systems, the following steps can be considered.

Step 1. Finding a diagonal matrix $L = \mathrm{diag}(l_1, l_2, \cdots, l_n) \geqslant 0$ such that $\|f(x)\|^2 \leqslant x^T L x$, for any $x \in R^n$.

Step 2. Finding $g > 0$ such that $\frac{g^2}{4}I - 2\left(A^T A + L\right) > 0$ by using Geršgorin disk theorem [6] or other results about bounds for eigenvalues such as presented in [28].

Step 3. Choosing a J according to the needs of the actual applications and computing the bound on T by applying the following inequality,

$$gT + 2\ln s_{\max}(I+J) < 0.$$

3. Numerical examples

In this section, we illustrate the effectiveness of the foregoing results by showing several simulation results employing the Chua's, Lorenz's, and Rössler's system. Throughout this section we assume that $\mathbf{x} = [x, y, z]^T$.

Example 3.1. The original and dimensionless form of a Chua's oscillator [12] is given by

$$\begin{cases} \dot{x} = \alpha(y - x - g(x)), \\ \dot{y} = x - y + z, \\ \dot{z} = -\beta y, \end{cases} \tag{3.1}$$

where α and β are parameters and $g(x)$ is the piecewise linear characteristics of the Chua's diode, which is defined by

$$g(x) = bx + 0.5(a - b)(|x + 1| - |x - 1|),$$

where $a < b < 0$ are two constants.

We rewrite the system (3.1) as follows

$$\dot{\mathbf{x}} = A\mathbf{x} + f(\mathbf{x}),$$

where

$$A = \begin{bmatrix} -\alpha - \alpha b & \alpha & 0 \\ 1 & -1 & 1 \\ 0 & -\beta & 0 \end{bmatrix},$$

$$f(\mathbf{x}) = \begin{bmatrix} -0.5\alpha(a - b)(|x + 1| - |x - 1|) \\ 0 \\ 0 \end{bmatrix}.$$

So, simple calculations show that

$$\begin{aligned} ||f(\mathbf{x})||^2 &= 0.25\alpha^2(a - b)^2[(x + 1)^2 \\ &\quad + (x - 1)^2 - 2|(x + 1)(x - 1)|] \\ &= 0.5\alpha^2(a - b)^2(x^2 + 1 - |x^2 - 1|) \\ &= \begin{cases} \alpha^2(a - b)^2, & x^2 > 1, \\ \alpha^2(a - b)^2 x^2, & x^2 \leqslant 1, \end{cases} \\ &\leqslant \alpha^2(a - b)^2 x^2. \end{aligned}$$

Thus we can choose $L = \mathrm{diag}(\alpha^2(a - b)^2, 0, 0)$.

In this example, we set the system parameters as

$$\alpha = 9.2156, \quad \beta = 15.9946, \quad a = -1.24905, \quad b = -0.75735,$$

which make Chua's circuit (3.1) chaotic [12]. Figure 1 shows the chaotic phenomenon of Chua's oscillator with the initial condition $x(0) = (5, 1, -3)^T$.

It is easy to see that

$$A^T A + L = \begin{bmatrix} 26.5332 & -21.6076 & 1.0000 \\ -21.6076 & 341.7545 & -1.0000 \\ 1.0000 & -1.0000 & 1.0000 \end{bmatrix}.$$

By [28, Theorem 2.1], we choose $g = 55$.

Let $\varepsilon = 1$ and $\delta = 60$. Then, we have

$$P = \begin{bmatrix} 52.8158 & 1.5139 & -0.0408 \\ 1.5139 & 30.7318 & 0.0697 \\ -0.0408 & 0.0697 & 53.8485 \end{bmatrix}.$$

Meanwhile, we set

$$J = \text{diag}\,(-0.5, -0.6, -0.4)\,,$$

and then $\lambda = 0.3600$. From the inequality

$$gT + \ln \lambda < 0,$$

we can choose $T = 0.0180$. Thus by Theorem 2.2 we obtain that the origin of system (3.1) is exponentially stable. The time response curves of Chua's oscillator by using such method is shown in Figure 2.

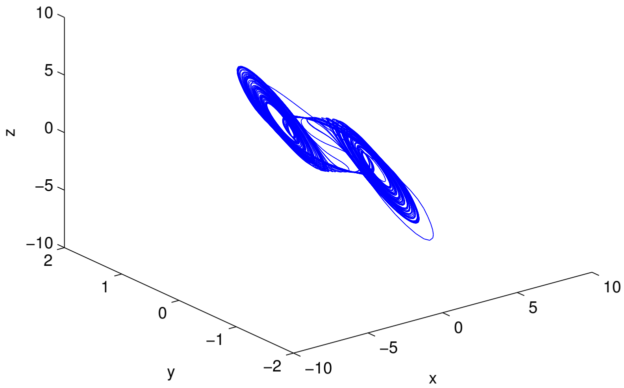

Figure 1: The chaotic phenomenon of Chua's oscillator with the initial condition $x(0) = (5, 1, -3)^T$.

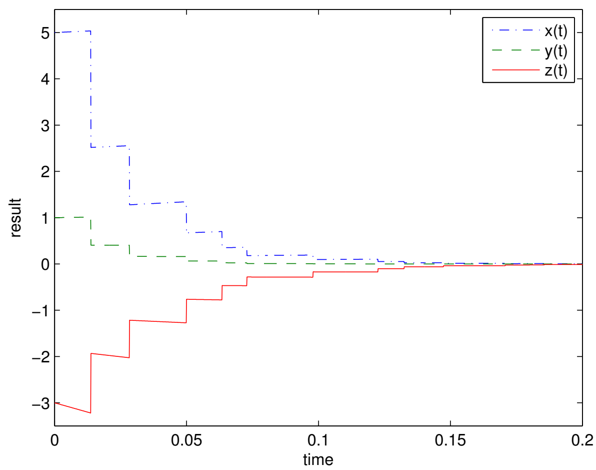

Figure 2: Time response curves of controlled Chua's system.

Example 3.2. The Lorenz system [10] is given by

$$\begin{cases} \dot{x} = -\sigma x + \sigma y, \\ \dot{y} = rx - y - xz, \\ \dot{z} = xy - bz, \end{cases} \tag{3.2}$$

where σ, r and b are three real positive parameters. Assume that $x \in [-d, d]$ and $d > 0$.

The system (3.2) can be easily rewritten as

$$\dot{\mathbf{x}} = A\mathbf{x} + f(\mathbf{x}),$$

where

$$A = \begin{bmatrix} -\sigma & \sigma & 0 \\ r & -1 & 0 \\ 0 & 0 & -b \end{bmatrix}, \quad f(\mathbf{x}) = \begin{bmatrix} 0 \\ -xz \\ xy \end{bmatrix}.$$

Thus

$$\|f(\mathbf{x})\|^2 = x^2y^2 + x^2z^2 \leqslant d^2y^2 + d^2z^2.$$

So we can choose $L = \mathrm{diag}(0, d^2, d^2)$.

In this example, we set the system parameters as

$$\sigma = 10, \quad r = 28, \quad b = \frac{8}{3}, \quad d = 20,$$

which make Lorenz system (3.2) chaotic [10]. Figure 3 shows the chaotic phenomenon of Lorenz system with the initial condition $x(0) = (5, 1, -3)^T$.

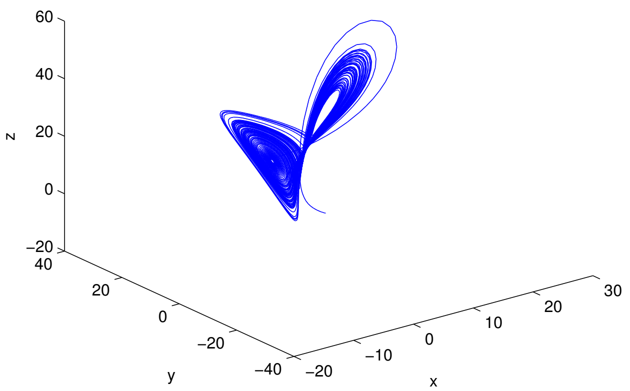

Figure 3: The chaotic phenomenon of Lorenz system with the initial condition $x(0) = (5, 1, -3)^T$.

Simple calculations show that

$$A^TA + L = \begin{bmatrix} 884.0000 & -128.0000 & 0.0000 \\ -128.0000 & 501.0000 & 0.0000 \\ 0.0000 & 0.0000 & 407.1111 \end{bmatrix}.$$

By [28, Theorem 2.1], we choose $g = 90$.

Meanwhile, we set

$$J = \mathrm{diag}\left(-0.6, -0.7, -0.8\right),$$

and then $s_{max}(I+J) = 0.4000$. From the inequality

$$gT + 2\ln s_{max}(I+J) < 0,$$

we can choose $T = 0.0200$. Thus by Theorem 2.2 we obtain that the origin of system (3.2) is exponentially stable. The time response curves of Lorenz system by using such method is shown in Figure 4.

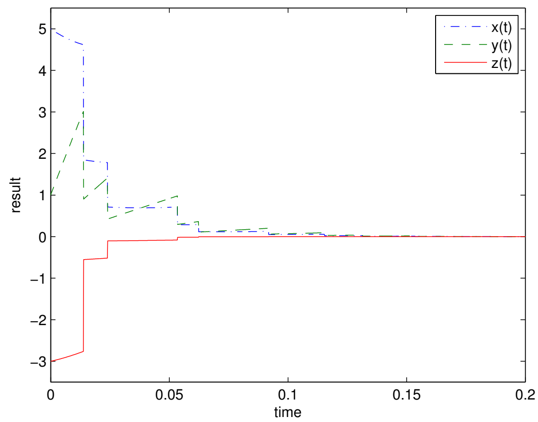

Figure 4: Time response curves of controlled Lorenz system.

Example 3.3. The Rössler system [25] is given by

$$\begin{cases} \dot{x} = -y - z, \\ \dot{y} = x + ay, \\ \dot{z} = bx - cz + xz, \end{cases} \tag{3.3}$$

where a, b and c are three real positive parameters. Assume that $x \in [-d, d]$ and $d > 0$. System (3.3) can be easily rewritten as

$$\dot{\mathbf{x}} = A\mathbf{x} + f(\mathbf{x}),$$

where

$$A = \begin{bmatrix} 0 & -1 & -1 \\ 1 & a & 0 \\ b & 0 & -c \end{bmatrix}, \quad f(\mathbf{x}) = \begin{bmatrix} 0 \\ 0 \\ xz \end{bmatrix}.$$

Thus

$$||f(\mathbf{x})||^2 = x^2z^2 \leqslant d^2z^2.$$

Thus we can choose $L = \text{diag}(0, 0, d^2)$.

In this example, we set the system parameters as

$$a = 0.34, \quad b = 0.4, \quad c = 4.5, \quad d = 20,$$

which make Rössler attractor (3.3) chaotic [25]. Figure 5 shows the chaotic phenomenon of Rössler attractor with the initial condition $x(0) = (-1, 3, -2)^T$.

Simple calculations show that

$$A^TA + L = \begin{bmatrix} 1.1600 & 0.3400 & -1.8000 \\ 0.3400 & 1.1156 & 1.0000 \\ -1.8000 & 1.0000 & 421.2500 \end{bmatrix}.$$

By [28, Theorem 2.1], we choosing $g = 60$. Meanwhile, we set

$$J = \mathrm{diag}\left(-0.5, -0.6, -0.5\right),$$

and then $s_{max}(I + J) = 0.5000$. From the inequality

$$gT + 2\ln s_{max}(I + J) < 0,$$

we can choose $T = 0.0220$. Thus by Theorem 2.2 we obtain that the origin of system (3.3) is exponentially stable. The time response curves of Rössler attractor by using such method is shown in Figure 6.

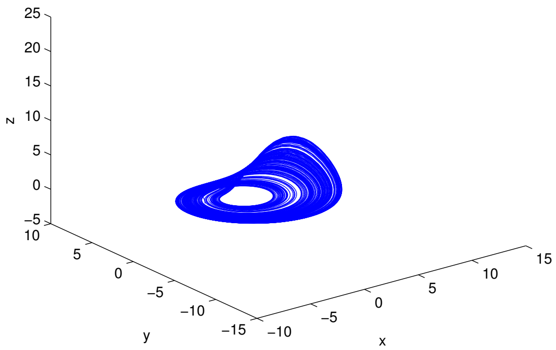

Figure 5: The chaotic phenomenon of Rössler attractor with the initial condition $x(0) = (-1, 3, -2)^T$.

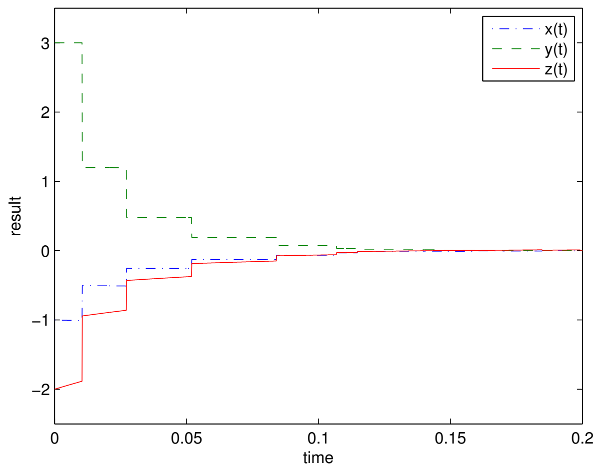

Figure 6: Time response curves of controlled Rössler system.

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