Available online at www.isr-publications.com/jnsa
J. Nonlinear Sci. Appl., 10 (2017), 3201–3216
Research Article
ISSN: 2008-1898
Journal of Nonlinear Sciences and Applications
Journal Homepage: www.tjnsa.com - www.isr-publications.com/jnsa

Common tripled fixed point theorem in two rectangular b-metric spaces and applications

Feng Gu*, Liya Liu

Institute of Applied Mathematics and Department of Mathematics, Hangzhou Normal University, Hangzhou, Zhejiang 310036, China.

Communicated by C. Alaca

Abstract

In this paper, we establish some new common tripled fixed point theorems for mappings defined on a set equipped with two rectangular b-metrics. We also provide illustrative examples in support of our new results. In the end of the paper, we give an existence and uniqueness theorem for a class of nonlinear integral equations by using the obtained result. The results presented in this paper generalize the well-known comparable results in the literature.

Keywords: Rectangular b-metric space, contractive mappings, tripled coincidence point, common tripled fixed point, ω-compatible mapping pairs.
2010 MSC: 47H10, 54H25.

1. Introduction and preliminaries

The concept of b-metric space was introduced by Czerwik [10] which is defined as:

Definition 1.1 ([10])**.** Let X be a nonempty set and $s \geqslant 1$ be a given real number. A function $d : X \times X \to [0, \infty)$ is a b-metric on X, if for all $x, y, z \in X$, the following conditions hold:

(bM1) $d(x, y) = 0$ if and only if $x = y$;

(bM2) $d(x, y) = d(y, x)$;

(bM3) $d(x, z) \leqslant s[d(x, y) + d(y, z)]$.

In this case, the pair (X, d) is called a b-metric space, and the number s is called the coefficient of (X, d).

As an important generalizations of usual metric spaces, Branciari [8] introduced the concept of rectangular metric space as follows:

Definition 1.2 ([8])**.** Let X be a nonempty set, and let $d : X \times X \to [0, \infty)$ be a mapping such that for all $x, y \in X$, the following conditions hold:

*Corresponding author
Email addresses: mathgufeng@163.com (Feng Gu), 846883245@qq.com (Liya Liu)

doi:10.22436/jnsa.010.06.33

Received 2017-02-01

(RM1) $d(x,y) = 0$ if and only if $x = y$;

(RM2) $d(x,y) = d(y,x)$;

(RM3) $d(x,z) \leqslant d(x,u) + d(u,v) + d(v,z)$ for all $u, v \in X \setminus \{x,y\}$ and $u \neq v$.

Then (X,d) is called a rectangular or generalized metric space.

After that, fixed point results in rectangular metric space have been studied by many authors (see e.g. [1–3, 6, 7, 9, 11, 12, 15, 16, 18–25, 27, 28, 30]).

Recently, George et al. [17] and Roshan et al. [26] introduced the notion of rectangular b-metric space as follows:

Definition 1.3 ([17, 26])**.** Let X be a nonempty set, $s \geqslant 1$ be a given real number and let $d : X \times X \to [0,\infty)$ be a mapping such that for all $x, y \in X$, the following conditions hold:

(RbM1) $d(x,y) = 0$ if and only if $x = y$;

(RbM2) $d(x,y) = d(y,x)$;

(RbM3) $d(x,z) \leqslant s[d(x,u) + d(u,v) + d(v,z)]$ for all $u, v \in X \setminus \{x,y\}$ and $u \neq v$.

Then (X,d) is called a rectangular b-metric space or a generalized b-metric space, and the number s is called the coefficient of (X,d).

Remark 1.4 ([14]). Every metric space is a rectangular metric space and every rectangular metric space is a rectangular b-metric space (with coefficient $s = 1$). However the converse is not necessarily true ([17, Examples 1.4. and 1.5.]). Also, every metric space is a b-metric space and every b-metric space with coefficient s is a rectangular b-metric space with coefficient s^2 but the converse is not necessarily true ([17, Examples 1.7]).

Very recently, Ding et al. [13, 14] and Aydi et al. [5] also discussed the fixed point and common fixed point problems of different contractive mapping for rectangular b-metric spaces. However, so far, no one discussed tripled fixed point problem in rectangular b-metric space.

The purpose of this paper is to prove some new common tripled fixed point theorems for mappings defined on a set equipped with two rectangular b-metrics. We also provide an existence and uniqueness theorem of solution for a class of nonlinear integral equations by using the obtained result.

Now, we give some basic notions before introducing some main results.

Definition 1.5 ([17])**.** Let (X,d) be a rectangular b-metric space, $\{x_n\}$ be a sequence in X and $x \in X$. Then

(a) The sequence $\{x_n\}$ is said to be convergent in X and converges to x, if for every $\epsilon > 0$ there exists $n_0 \in \mathbb{N}$ such that $d(x_n, x) < \epsilon$ for all $n > n_0$ and this fact is represented by $\lim_{n\to\infty} x_n = x$ or $x_n \to x$ as $n \to \infty$.

(b) The sequence $\{x_n\}$ is said to be Cauchy sequence in (X,d) if for every $\epsilon > 0$ there exists $n_0 \in \mathbb{N}$ such that $d(x_n, x_{n+p}) < \epsilon$ for all $n > n_0$, $p > 0$ or equivalently, if $\lim_{n\to\infty} d(x_n, x_{n+p}) = 0$ for all $p > 0$.

(c) (X,d) is said to be a complete rectangular b-metric space if every Cauchy sequence in X converges to some $x \in X$.

Note that limit of sequence in a rectangular b-metric space is not necessarily unique and also every convergent sequence in a rectangular b-metric space is not necessarily Cauchy sequence ([17, Examples 1.7]).

Definition 1.6 ([29])**.** An element $(x,y,z) \in X \times X \times X$ is called a tripled fixed point of the mapping $F : X \times X \times X \to X$, if $F(x,y,z) = x$, $F(y,z,x) = y$ and $F(z,x,y) = z$.

Definition 1.7 ([4]). An element $(x, y, z) \in X \times X \times X$ is called a tripled coincidence point of mappings $F : X \times X \times X \to X$ and $g : X \to X$, if $F(x, y, z) = gx$, $F(y, z, x) = gy$ and $F(z, x, y) = gz$. In this case, (gx, gy, gz) is called a triple point of coincidence of the mappings g and F.

Definition 1.8 ([4]). An element $(x, y, z) \in X \times X \times X$ is called a common tripled fixed point of mappings $F : X \times X \times X \to X$ and $g : X \to X$, if $F(x, y, z) = gx = x$, $F(y, z, x) = gy = y$ and $F(z, x, y) = gz = z$.

Definition 1.9 ([4]). Let X be a nonempty set. A pair of mappings $F : X \times X \times X \to X$ and $g : X \to X$ are called to be w-compatible, if $gF(x, y, z) = F(gx, gy, gz)$ whenever $F(x, y, z) = gx$ and $F(y, z, x) = gy$.

2. Main results

Theorem 2.1. *Let d_1 and d_2 be two rectangular b-metrics on X such that $d_2(x, y) \leqslant d_1(x, y)$ for all $x, y \in X$, (X, d_1) with coefficient $s \geqslant 1$, and $F : X \times X \times X \to X$ and $g : X \to X$ be two mappings. Suppose that there exist k_1, k_2 and k_3 in $[0, 1)$ with $0 \leqslant k_1 + k_2 + k_3 < 1$ and $0 \leqslant sk_3 < 1$ such that the condition*

$$\begin{aligned} d_1(F(x,y,z),F(u,v,w)) + d_1(F(y,z,x),F(v,w,u)) + d_1(F(z,x,y),F(w,u,v)) \\ \leqslant k_1[d_2(gx,gu) + d_2(gy,gv) + d_2(gz,gw)] \\ + k_2[d_2(gx,F(x,y,z)) + d_2(gy,F(y,z,x)) + d_2(gz,F(z,x,y))] \\ + k_3[d_2(gu,F(u,v,w)) + d_2(gv,F(v,w,u)) + d_2(gw,F(w,u,v))], \end{aligned} \tag{2.1}$$

holds for all $(x, y, z), (u, v, w) \in X \times X \times X$.

If $F(X \times X \times X) \subseteq g(X)$ and $g(X)$ is d_1-complete, then g and F have a tripled coincidence point $(x, y, z) \in X \times X \times X$, satisfying that $gx = F(x, y, z) = gy = F(y, z, x) = gz = F(z, x, y)$.

Moreover, if g and F are w-compatible, then g and F have a unique common tripled fixed point of the form (u, u, u), which satisfies that $u = gu = F(u, u, u)$.

Proof. Let $(x_0, y_0, z_0) \in X \times X \times X$, by making the use of $F(X \times X \times X) \subseteq g(X)$, then there exist $x_1, y_1, z_1 \in X$ such that $gx_1 = F(x_0, y_0, z_0)$, $gy_1 = F(y_0, z_0, x_0)$ and $gz_1 = F(z_0, x_0, y_0)$. By similar arguments as above we can show that there exist $x_2, y_2, z_2 \in X$ such that $gx_2 = T(x_1, y_1, z_1)$, $gy_2 = T(y_1, z_1, x_1)$, $gz_2 = T(z_1, x_1, y_1), \cdots$. Repeating the above procedure, we can construct three sequences $\{x_n\}$, $\{y_n\}$ and $\{z_n\}$ such that

$$gx_{n+1} = F(x_n, y_n, z_n), \quad gy_{n+1} = F(y_n, z_n, x_n), \quad gz_{n+1} = F(z_n, x_n, y_n), \quad \forall n \geqslant 0.$$

Without loss of generality, we can assume that $gx_n \neq gx_{n+1}$, $gy_n \neq gy_{n+1}$ and $gz_n \neq gz_{n+1}$, for all $n \geqslant 0$.

By taking $(x, y, z) = (x_n, y_n, z_n)$ and $(u, v, w) = (x_{n+1}, y_{n+1}, z_{n+1})$ in (2.1), we obtain

$$\begin{aligned} d_1(gx_{n+1},gx_{n+2}) &+ d_1(gy_{n+1}, gy_{n+2}) + d_1(gz_{n+1}, gz_{n+2}) \\ &= d_1(F(x_n, y_n, z_n), F(x_{n+1}, y_{n+1}, z_{n+1})) + d_1(F(y_n, z_n, x_n), F(y_{n+1}, z_{n+1}, x_{n+1})) \\ &\quad + d_1(F(z_n, x_n, y_n), F(z_{n+1}, x_{n+1}, y_{n+1})) \\ &\leqslant k_1[d_2(gx_n, gx_{n+1}) + d_2(gy_n, gy_{n+1}) + d_2(gz_n, gz_{n+1})] \\ &\quad + k_2[d_2(gx_n, F(x_n, y_n, z_n)) + d_2(gy_n, F(y_n, z_n, x_n)) + d_2(gz_n, F(z_n, x_n, y_n))] \\ &\quad + k_3[d_2(gx_{n+1}, F(x_{n+1}, y_{n+1}, z_{n+1})) \\ &\quad + d_2(gy_{n+1}, F(y_{n+1}, z_{n+1}, x_{n+1})) + d_2(gz_{n+1}, F(z_{n+1}, x_{n+1}, y_{n+1}))] \\ &= k_1[d_2(gx_n, gx_{n+1}) + d_2(gy_n, gy_{n+1}) + d_2(gz_n, gz_{n+1})] \\ &\quad + k_2[d_2(gx_n, gx_{n+1}) + d_2(gy_n, gy_{n+1}) + d_2(gz_n, gz_{n+1})] \\ &\quad + k_3[d_2(gx_{n+1}, gx_{n+2}) + d_2(gy_{n+1}, gy_{n+2}) + d_2(gz_{n+1}, gz_{n+2})] \\ &\leqslant k_1[d_1(gx_n, gx_{n+1}) + d_1(gy_n, gy_{n+1}) + d_1(gz_n, gz_{n+1})] \end{aligned} \tag{2.2}$$

$$+ k_2[d_1(gx_n, gx_{n+1}) + d_1(gy_n, gy_{n+1}) + d_1(gz_n, gz_{n+1})]$$
$$+ k_3[d_1(gx_{n+1}, gx_{n+2}) + d_1(gy_{n+1}, gy_{n+2}) + d_1(gz_{n+1}, gz_{n+2})].$$

It follows from (2.2) that

$$\begin{aligned} d_1(gx_{n+1}, gx_{n+2}) &+ d_1(gy_{n+1}, gy_{n+2}) + d_1(gz_{n+1}, gz_{n+2}) \\ &\leqslant \left(\frac{k_1+k_2}{1-k_3}\right)[d_1(gx_n, gx_{n+1}) + d_1(gy_n, gy_{n+1}) + d_1(gz_n, gz_{n+1})]. \end{aligned} \tag{2.3}$$

Taking $k = \frac{k_1+k_2}{1-k_3}$, by the condition $0 \leqslant k_1 + k_2 + k_3 < 1$, then we have $0 \leqslant k < 1$. (2.3) implies that

$$\begin{aligned} d_1(gx_{n+1}, gx_{n+2}) &+ d_1(gy_{n+1}, gy_{n+2}) + d_1(gz_{n+1}, gz_{n+2}) \\ &\leqslant k[d_1(gx_n, gx_{n+1}) + d_1(gy_n, gy_{n+1}) + d_1(gz_n, gz_{n+1})]. \end{aligned} \tag{2.4}$$

By taking $\delta_n = d_1(gx_n, gx_{n+1}) + d_1(gy_n, gy_{n+1}) + d_1(gz_n, gz_{n+1})$. Repeating the above inequality (2.4) $n+1$ times, we obtain,

$$\delta_{n+1} \leqslant k\delta_n \leqslant k^2\delta_{n-1} \leqslant \cdots \leqslant k^{n+1}\delta_0. \tag{2.5}$$

As $(x, y, z) = (x_n, y_n, z_n)$ and $(u, v, w) = (x_{n+2}, y_{n+2}, z_{n+2})$ in (2.1), also with (2.5), we get

$$\begin{aligned} d_1(gx_{n+1}, gx_{n+3}) &+ d_1(gy_{n+1}, gy_{n+3})) + d_1(gz_{n+1}, gz_{n+3}) \\ &= d_1(F(x_n, y_n, z_n), F(x_{n+2}, y_{n+2}, z_{n+2})) + d_1(F(y_n, z_n, x_n), F(y_{n+2}, z_{n+2}, x_{n+2})) \\ &\quad + d_1(F(z_n, x_n, y_n), F(z_{n+2}, x_{n+2}, y_{n+2})) \\ &\leqslant k_1[d_2(gx_n, gx_{n+2}) + d_2(gy_n, gy_{n+2}) + d_2(gz_n, gz_{n+2})] \\ &\quad + k_2[d_2(gx_n, F(x_n, y_n, z_n)) + d_2(gy_n, F(y_n, z_n, x_n)) + d_2(gz_n, F(z_n, x_n, y_n))] \\ &\quad + k_3[d_2(gx_{n+2}, F(x_{n+2}, y_{n+2}, z_{n+2})) \\ &\quad + d_2(gy_{n+2}, F(y_{n+2}, z_{n+2}, x_{n+2})) + d_2(gz_{n+2}, F(z_{n+2}, x_{n+2}, y_{n+2}))] \\ &= k_1[d_2(gx_n, gx_{n+2}) + d_2(gy_n, gy_{n+2}) + d_2(gz_n, gz_{n+2})] \\ &\quad + k_2[d_2(gx_n, gx_{n+1}) + d_2(gy_n, gy_{n+1}) + d_2(gz_n, gz_{n+1})] \\ &\quad + k_3[d_2(gx_{n+2}, gx_{n+3}) + d_2(gy_{n+2}, gy_{n+3}) + d_2(gz_{n+2}, gz_{n+3})] \\ &\leqslant k_1[d_2(gx_n, gx_{n+2}) + d_2(gy_n, gy_{n+2}) + d_2(gz_n, gz_{n+2})] \\ &\quad + k_2[d_2(gx_n, gx_{n+1}) + d_2(gy_n, gy_{n+1}) + d_2(gz_n, gz_{n+1})] \\ &\quad + k_3k^2[d_2(gx_n, gx_{n+1}) + d_2(gy_n, gy_{n+1}) + d_2(gz_n, gz_{n+1})] \\ &= k_1[d_2(gx_n, gx_{n+2}) + d_2(gy_n, gy_{n+2}) + d_2(gz_n, gz_{n+2})] \\ &\quad + (k_2 + k_3k^2)[d_2(gx_n, gx_{n+1}) + d_2(gy_n, gy_{n+1}) + d_2(gz_n, gz_{n+1})] \\ &\leqslant k_1[d_1(gx_n, gx_{n+2}) + d_1(gy_n, gy_{n+2}) + d_1(gz_n, gz_{n+2})] \\ &\quad + (k_2 + k_3k^2)[d_1(gx_n, gx_{n+1}) + d_1(gy_n, gy_{n+1}) + d_1(gz_n, gz_{n+1})]. \end{aligned} \tag{2.6}$$

By taking

$$\delta_n^* = d_1(gx_n, gx_{n+2}) + d_1(gy_n, gy_{n+2}) + d_1(gz_n, gz_{n+2}),$$

from $k = \frac{k_1+k_2}{1-k_3} \in [0, 1)$ we have

$$k_1 + k_2 + k_3k^2 \leqslant k_1 + k_2 + k_3k = k_1 + k_2 + k_3\frac{k_1+k_2}{1-k_3} = \frac{k_1+k_2}{1-k_3} = k.$$

Consequently, by the use of (2.5) and (2.6), we have

$$\delta_{n+1}^* \leqslant k_1\delta_n^* + (k_2 + k_3k^2)\delta_n \leqslant (k_1 + k_2 + k_3k^2)\max\{\delta_n, \delta_n^*\} \leqslant k\max\{\delta_n, \delta_n^*\}. \tag{2.7}$$

It follows from (2.5) and (2.7) that

$$\begin{aligned}\delta^*_{n+1} &\leqslant k\max\{\delta_n,\delta^*_n\}\leqslant k\max\{k\delta_{n-1},k\max\{\delta_{n-1},\delta^*_{n-1}\}\}=k^2\max\{\delta_{n-1},\delta^*_{n-1}\}\\ &\leqslant k^3\max\{\delta_{n-2},\delta^*_{n-2}\}\leqslant\cdots\leqslant k^{n+1}\max\{\delta_0,\delta^*_0\}.\end{aligned}\tag{2.8}$$

Next, we show that $\{gx_n\}$, $\{gy_n\}$ and $\{gz_n\}$ are Cauchy sequences in $g(X)$. For this, we consider $d_1(x_n, x_{n+p})$ in two cases.

Case 1. p is an odd number, assume that $p = 2m + 1$, then using (RbM3) we obtain

$$\begin{aligned}d_1(gx_n,gx_{n+p}) &= d_1(gx_n,gx_{n+2m+1})\\
&\leqslant s[d_1(gx_n,gx_{n+1})+d_1(gx_{n+1},gx_{n+2})+d_1(gx_{n+2},gx_{n+2m+1})]\\
&\leqslant s[d_1(gx_n,gx_{n+1})+d_1(gx_{n+1},gx_{n+2})]\\
&\quad+s^2[d_1(gx_{n+2},gx_{n+3})+d_1(gx_{n+3},gx_{n+4})+d_1(gx_{n+4},gx_{n+2m+1})]\\
&\leqslant s[d_1(gx_n,gx_{n+1})+d_1(gx_{n+1},gx_{n+2})]+s^2[d_1(gx_{n+2},gx_{n+3})+d_1(gx_{n+3},gx_{n+4})]\\
&\quad+s^3[d_1(gx_{n+4},gx_{n+5})+d_1(gx_{n+5},gx_{n+6})+d_1(gx_{n+6},gx_{n+2m+1})]\\
&\leqslant\cdots\\
&\leqslant s[d_1(gx_n,gx_{n+1})+d_1(gx_{n+1},gx_{n+2})]+s^2[d_1(gx_{n+2},gx_{n+3})+d_1(gx_{n+3},gx_{n+4})]\\
&\quad+s^3[d_1(gx_{n+4},gx_{n+5})+d_1(gx_{n+5},gx_{n+6})]+\cdots\\
&\quad+s^m[d_1(gx_{n+2m-2},gx_{n+2m-1})+d_1(gx_{n+2m-1},gx_{n+2m})+d_1(gx_{n+2m},gx_{n+2m+1})].\end{aligned}$$

That is

$$\begin{aligned}d_1(gx_n,gx_{n+p}) &= d_1(gx_n,gx_{n+2m+1})\\
&\leqslant s[d_1(gx_n,gx_{n+1})+d_1(gx_{n+1},gx_{n+2})]+s^2[d_1(gx_{n+2},gx_{n+3})+d_1(gx_{n+3},gx_{n+4})]\\
&\quad+s^3[d_1(gx_{n+4},gx_{n+5})+d_1(gx_{n+5},gx_{n+6})]+\cdots\\
&\quad+s^m[d_1(gx_{n+2m-2},gx_{n+2m-1})+d_1(gx_{n+2m-1},gx_{n+2m})]+s^m d_1(gx_{n+2m},gx_{n+2m+1}).\end{aligned}\tag{2.9}$$

We can similarly prove the following result

$$\begin{aligned}d_1(gy_n,gy_{n+p}) &= d_1(gy_n,gy_{n+2m+1})\\
&\leqslant s[d_1(gy_n,gy_{n+1})+d_1(gy_{n+1},gy_{n+2})]+s^2[d_1(gy_{n+2},gy_{n+3})+d_1(gy_{n+3},gy_{n+4})]\\
&\quad+s^3[d_1(gy_{n+4},gy_{n+5})+d_1(gy_{n+5},gy_{n+6})]+\cdots\\
&\quad+s^m[d_1(gy_{n+2m-2},gy_{n+2m-1})+d_1(gy_{n+2m-1},gy_{n+2m})]+s^m d_1(gy_{n+2m},gy_{n+2m+1}),\end{aligned}\tag{2.10}$$

and

$$\begin{aligned}d_1(gz_n,gz_{n+p}) &= d_1(gz_n,gz_{n+2m+1})\\
&\leqslant s[d_1(gz_n,gz_{n+1})+d_1(gz_{n+1},gz_{n+2})]+s^2[d_1(gz_{n+2},gz_{n+3})+d_1(gz_{n+3},gz_{n+4})]\\
&\quad+s^3[d_1(gz_{n+4},gz_{n+5})+d_1(gz_{n+5},gz_{n+6})]+\cdots\\
&\quad+s^m[d_1(gz_{n+2m-2},gz_{n+2m-1})+d_1(gz_{n+2m-1},gz_{n+2m})]+s^m d_1(gz_{n+2m},gz_{n+2m+1}).\end{aligned}\tag{2.11}$$

Combining (2.5), (2.9), (2.10) and (2.11), we have

$$\begin{aligned}d_1(gx_n,gx_{n+p})&+d_1(gy_n,gy_{n+p})+d_1(gz_n,gz_{n+p})\\
&= d_1(gx_n,gx_{n+2m+1})+d_1(gy_n,gy_{n+2m+1})+d_1(gz_n,gz_{n+2m+1})\\
&\leqslant s(\delta_n+\delta_{n+1})+s^2(\delta_{n+2}+\delta_{n+3})+\cdots+s^m(\delta_{n+2m-2}+\delta_{n+2m-1})+s^m\delta_{n+2m}\\
&\leqslant s\left(k^n+k^{n+1}\right)\delta_0+s^2\left(k^{n+2}+k^{n+3}\right)\delta_0+\cdots+s^m\left(k^{n+2m-2}+k^{n+2m-1}\right)\delta_0+s^m k^{n+2m}\delta_0\\
&= sk^n(1+k)\left[1+sk^2+\left(sk^2\right)^2+\cdots+\left(sk^2\right)^{m-1}\right]\delta_0+s^m k^{n+2m}\delta_0\end{aligned}$$

$$= \begin{cases} \left[sk^n(1+k)\cdot m + s^m k^{n+2m}\right]\delta_0, & sk^2=1, \\ \left(sk^n(1+k)\cdot \frac{1-(sk^2)^m}{1-sk^2} + s^m k^{n+2m}\right)\delta_0, & sk^2\neq 1, \end{cases} \tag{2.12}$$

$$\leqslant \begin{cases} \left[sk^n(1+k)\cdot m + s^m k^{n+2m}\right]\delta_0, & sk^2=1, \\ \left(\frac{sk^n(1+k)}{1-sk^2} + s^m k^{n+2m}\right)\delta_0, & sk^2\neq 1. \end{cases}$$

Case 2. p is an even number, assume that $p=2m$, then using (RbM3) we obtain

$$\begin{aligned} d_1(gx_n, gx_{n+p}) &= d_1(gx_n, gx_{n+2m}) \\ &\leqslant s[d_1(gx_n, gx_{n+1}) + d_1(gx_{n+1}, gx_{n+2}) + d_1(gx_{n+2}, gx_{n+2m})] \\ &\leqslant s[d_1(gx_n, gx_{n+1}) + d_1(gx_{n+1}, gx_{n+2})] \\ &\quad + s^2[d_1(gx_{n+2}, gx_{n+3}) + d_1(gx_{n+3}, gx_{n+4}) + d_1(gx_{n+4}, gx_{n+2m})] \\ &\leqslant \cdots \\ &\leqslant s[d_1(gx_n, gx_{n+1}) + d_1(gx_{n+1}, gx_{n+2})] + s^2[d_1(gx_{n+2}, gx_{n+3}) + d_1(gx_{n+3}, gx_{n+4})] \\ &\quad + \cdots + s^{m-1}[d_1(gx_{n+2m-4}, gx_{n+2m-3}) + d_1(gx_{n+2m-3}, gx_{n+2m-2})] \\ &\quad + s^{m-1} d_1(gx_{n+2m-2}, gx_{n+2m}). \end{aligned} \tag{2.13}$$

By similar arguments as above,

$$\begin{aligned} d_1(gy_n, gy_{n+p}) &= d_1(gy_n, gy_{n+2m}) \\ &\leqslant s[d_1(gy_n, gy_{n+1}) + d_1(gy_{n+1}, gy_{n+2})] + s^2[d_1(gy_{n+2}, gy_{n+3}) + d_1(gy_{n+3}, gy_{n+4})] \\ &\quad + \cdots + s^{m-1}[d_1(gy_{n+2m-4}, gy_{n+2m-3}) + d_1(gy_{n+2m-3}, gx_{n+2m-2})] \\ &\quad + s^{m-1}[d_1(gy_{n+2m-2}, gy_{n+2m})], \end{aligned} \tag{2.14}$$

and

$$\begin{aligned} d_1(gz_n, gz_{n+p}) &= d_1(gz_n, gz_{n+2m}) \\ &\leqslant s[d_1(gz_n, gz_{n+1}) + d_1(gz_{n+1}, gz_{n+2})] + s^2[d_1(gz_{n+2}, gz_{n+3}) + d_1(gz_{n+3}, gz_{n+4})] \\ &\quad + \cdots + s^{m-1}[d_1(gz_{n+2m-4}, gz_{n+2m-3}) \\ &\quad + d_1(gz_{n+2m-3}, gx_{n+2m-2})] + s^{m-1}[d_1(gz_{n+2m-2}, gz_{n+2m})]. \end{aligned} \tag{2.15}$$

Combining (2.5), (2.8), (2.13), (2.14) and (2.15), we have

$$\begin{aligned} & d_1(gx_n, gx_{n+p}) + d_1(gy_n, gy_{n+p}) + d_1(gz_n, gz_{n+p}) \\ &= d_1(gx_n, gx_{n+2m}) + d_1(gy_n, gy_{n+2m}) + d_1(gz_n, gz_{n+2m}) \\ &\leqslant s(\delta_n + \delta_{n+1}) + s^2(\delta_{n+2} + \delta_{n+3}) + \cdots + s^{m-1}(\delta_{n+2m-4} + \delta_{n+2m-3}) + s^{m-1}\delta^*_{n+2m-2} \\ &\leqslant s\left(k^n + k^{n+1}\right)\delta_0 + s^2\left(k^{n+2} + k^{n+3}\right)\delta_0 \\ &\quad + \cdots + s^{m-1}\left(k^{n+2m-4} + k^{n+2m-3}\right)\delta_0 + s^{m-1}k^{n+2m-2}\max\{\delta_0, d_0^*\} \\ &= sk^n(1+k)\left[1 + sk^2 + \left(sk^2\right)^2 + \cdots + \left(sk^2\right)^{m-2}\right]\delta_0 + s^{m-1}k^{n+2m-2}\max\{\delta_0, \delta_0^*\} \\ &= \begin{cases} sk^n(1+k)(m-1)\delta_0 + s^{m-1}k^{n+2m-2}\max\{\delta_0, \delta_0^*\}, & sk^2 = 1, \\ sk^n(1+k)\cdot\frac{1-(sk^2)^{m-1}}{1-sk^2}\cdot\delta_0 + s^{m-1}k^{n+2m-2}\max\{d_0, d_0^*\}, & sk^2 \neq 1, \end{cases} \\ &\leqslant \begin{cases} sk^n(1+k)(m-1)\delta_0 + s^{m-1}k^{n+2m-2}\max\{\delta_0, \delta_0^*\}, & sk^2 = 1, \\ \frac{sk^n(1+k)}{1-sk^2}\cdot\delta_0 + s^{m-1}k^{n+2m-2}\max\{\delta_0, \delta_0^*\}, & sk^2 \neq 1. \end{cases} \end{aligned} \tag{2.16}$$

Since $k \in [0,1)$, so $k^n \to 0$ as $n \to \infty$. Taking limit as $n \to \infty$ in (2.12) and (2.16), we get

$$\lim_{n\to\infty} [d_1(gx_n, gx_{n+p}) + d_1(gy_n, gy_{n+p}) + d_1(gz_n, gz_{n+p})] = 0,$$

which implies that $\{gx_n\}$, $\{gy_n\}$ and $\{gz_n\}$ are Cauchy sequences in $g(X)$. Since $g(X)$ is complete, then there exist $x, y, z \in X$ such that

$$\lim_{n\to\infty} gx_n = gx, \quad \lim_{n\to\infty} gy_n = gy, \quad \text{and} \quad \lim_{n\to\infty} gz_n = gz.$$

It follows from (2.1) and (2.5) that

$$\begin{aligned}
d_1(gx_{n+1}, &F(x,y,z)) + d_1(gy_{n+1}, F(y,z,x)) + d_1(gz_{n+1}, F(z,x,y)) \\
&= d_1(F(x_n,y_n,z_n), F(x,y,z)) + d_1(F(y_n,z_n,x_n), F(y,z,x)) + d_1(F(z_n,x_n,y_n), F(z,x,y)) \\
&\leqslant k_1[d_2(gx_n, gx) + d_2(gy_n, gy) + d_2(gz_n, gz)] \\
&\quad + k_2[d_2(gx_n, F(x_n,y_n,z_n)) + d_2(gy_n, F(y_n,x_n,z_n)) + d_2(gz_n, F(z_n,x_n,y_n))] \\
&\quad + k_3[d_2(gx, F(x,y,z)) + d_2(gy, F(y,z,x)) + d_2(gz, F(z,x,y))] \\
&= k_1[d_2(gx_n, gx) + d_2(gy_n, gy) + d_2(gz_n, gz)] \\
&\quad + k_2[d_2(gx_n, gx_{n+1}) + d_2(gy_n, gy_{n+1}) + d_2(gz_n, gz_{n+1})] \\
&\quad + k_3[d_2(gx, F(x,y,z)) + d_2(gy, F(y,z,x)) + d_2(gz, F(z,x,y))] \\
&\leqslant k_1[d_1(gx_n, gx) + d_1(gy_n, gy) + d_1(gz_n, gz)] \\
&\quad + k_2[d_1(gx_n, gx_{n+1}) + d_1(gy_n, gy_{n+1}) + d_1(gz_n, gz_{n+1})] \\
&\quad + k_3[d_1(gx, F(x,y,z)) + d_1(gy, F(y,z,x)) + d_1(gz, F(z,x,y))] \\
&= k_1[d_1(gx_n, gx) + d_1(gy_n, gy) + d_1(gz_n, gz)] + k_2\delta_n \\
&\quad + k_3[d_1(gx, F(x,y,z)) + d_1(gy, F(y,z,x)) + d_1(gz, F(z,x,y))] \\
&\leqslant k_1[d_1(gx_n, gx) + d_1(gy_n, gy) + d_1(gz_n, gz)] + k_2k^n\delta_0 \\
&\quad + k_3[d_1(gx, F(x,y,z)) + d_1(gy, F(y,z,x)) + d_1(gz, F(z,x,y))].
\end{aligned} \tag{2.17}$$

Applying (RbM3), (2.17) and (2.5) we have

$$\begin{aligned}
d_1(gx, F(x,y,z)) &+ d_1(gy, F(y,z,x)) + d_1(gz, F(z,x,y)) \\
&\leqslant s[d_1(gx, gx_n) + d_1(gx_n, gx_{n+1}) + d_1(gx_{n+1}, F(x,y,z))] \\
&\quad + s[d_1(gy, gy_n) + d_1(gy_n, gy_{n+1}) + d_1(gy_{n+1}, F(y,z,x))] \\
&\quad + s[d_1(gz, gz_n) + d_1(gz_n, gz_{n+1}) + d_1(gz_{n+1}, F(z,x,y))] \\
&= s[d_1(gx, gx_n) + d_1(gy, gy_n) + d_1(gz, gz_n)] + s\delta_n \\
&\quad + s[d_1(gx_{n+1}, F(x,y,z)) + d_1(gy_{n+1}, F(y,z,x)) + d_1(gz_{n+1}, F(z,x,y))] \\
&\leqslant s(1+k_1)[d_1(gx_n, gx) + d_1(gy_n, gy) + d_1(gz_n, gz)] + s(1+k_2)k^n\delta_0 \\
&\quad + sk_3[d_1(gx, F(x,y,z)) + d_1(gy, F(y,z,x)) + d_1(gz, F(z,x,y))].
\end{aligned} \tag{2.18}$$

By taking $n \to \infty$ in the above inequality (2.18), we have

$$\begin{aligned}
d_1(gx, F(x,y,z)) &+ d_1(gy, F(y,z,x)) + d_1(gz, F(z,x,y)) \\
&\leqslant sk_3[d_1(gx, F(x,y,z)) + d_1(gy, F(y,z,x)) + d_1(gz, F(z,x,y))].
\end{aligned} \tag{2.19}$$

By the condition $0 \leqslant sk_3 < 1$ and (2.19), we can easily obtain that

$$d_1(gx, F(x,y,z)) + d_1(gy, F(y,z,x)) + d_1(gz, F(z,x,y)) = 0,$$

which implies that

$$gx = F(x,y,z)), gy = F(y,z,x)), gz = F(z,x,y).$$

Therefore, we conclude that (x, y, z) is the tripled coincidence point of g and F.

Next, we show the uniqueness of the triple point of coincidence of g and F. Assume that (x^*, y^*, z^*) is another tripled coincidence point of mappings g and F. By (2.1), we derive

$$\begin{aligned} d_1(gx, gx^*)+&d_1(gy, gy^*) + d_1(gz, gz^*) \\ &= d_1(F(x, y, z), F(x^*, y^*, z^*)) + d_1(F(y, z, x), F(y^*, z^*, x^*)) + d_1(F(z, x, y), F(z^*, x^*, y^*)) \\ &\leqslant k_1[d_2(gx, gx^*) + d_2(gy, gy^*)) + d_2(gz, gz^*)] \\ &\quad + k_2[d_2(gx, F(x, y, z)) + d_2(gy, F(y, z, x)) + d_2(gz, F(z, x, y))] \\ &\quad + k_3[d_2(gx^*, F(x^*, y^*, z^*)) + d_2(gy^*, F(y^*, z^*, x^*)) + d_2(gz^*, F(z^*, x^*, y^*))] \\ &= k_1[d_2(gx, gx^*) + d_2(gy, gy^*) + d_2(gz, gz^*)] \\ &\leqslant k_1[d_1(gx, gx^*) + d_1(gy, gy^*) + d_1(gz, gz^*)]. \end{aligned} \tag{2.20}$$

By virtue of $0 \leqslant k_1 \leqslant k_1 + k_2 + k_3 < 1$ and (2.20), we deduce

$$d_1(gx, gx^*) + d_1(gy, gy^*) + d_1(gz, gz^*) = 0.$$

This implies that $gx = gx^*$, $gy = gy^*$ and $gz = gz^*$. So that the triple point of coincidence of g and F is unique.

Next, we show that $gx = gy = gz$. In fact, it follows from (2.1) that

$$\begin{aligned} d_1(gx, gy)+&d_1(gy, gz) + d_1(gz, gx) \\ &= d_1(F(x, y, z), F(y, z, x)) + d_1(F(y, z, x), F(z, x, y)) + d_1(F(z, x, y), F(x, y, z)) \\ &\leqslant k_1[d_2(gx, gy) + d_2(gy, gz) + d_2(gz, gx)] \\ &\quad + k_2[d_2(gx, F(x, y, z)) + d_2(gy, F(y, z, x)) + d_2(gz, F(z, x, y))] \\ &\quad + k_3[d_2(gy, F(y, z, x)) + d_2(gz, F(z, x, y)) + d_2(gx, F(x, y, z))] \\ &= k_1[d_2(gx, gy) + d_2(gy, gz) + d_2(gz, gx)] \\ &\leqslant k_1[d_1(gx, gy) + d_1(gy, gz) + d_1(gz, gx)]. \end{aligned} \tag{2.21}$$

By making use of $0 \leqslant k_1 \leqslant k_1 + k_2 + k_3 < 1$ and (2.21), we deduce

$$d_1(gx, gy) + d_1(gy, gz) + d_1(gz, gx) = 0.$$

This means that $gx = gy = gz$.

Finally, if g and F are w-compatible, then we have $g(F(x, y, z)) = F(gx, gy, gz)$. Therefore, by taking $u = gx$, we have $u = gx = F(x, y, z) = gy = F(y, z, x) = gz = F(z, x, y)$, hence we have

$$gu = ggx = g(F(x, y, z)) = F(gx, gy, gz) = F(u, u, u).$$

Thus, (gu, gu, gu) is a coupled point of coincidence of g and F, and by its uniqueness, we get $gu = gx$. Thus, we obtain $u = gu = F(u, u, u)$. Therefore, (u, u, u) is the unique common tripled fixed point of g and F. This completes the proof of Theorem 2.1. □

In Theorem 2.1, if we take $d_1(x, y) = d_2(x, y) = d(x, y)$ for all $x, y \in X$, then we get the following corollary.

Corollary 2.2. *Let (X, d) be a rectangular b-metrics space with coefficient $s \geqslant 1$, and $F : X \times X \times X \to X$ and $g : X \to X$ be two mappings. Suppose that there exist k_1, k_2 and k_3 in $[0, 1)$ with $0 \leqslant k_1 + k_2 + k_3 < 1$ and $0 \leqslant sk_3 < 1$ such that the condition*

$$\begin{aligned} d(F(x, y, z), F(u, v, w))+&d(F(y, z, x), F(v, w, u)) + d(F(z, x, y), F(w, u, v)) \\ &\leqslant k_1[d(gx, gu) + d(gy, gv) + d(gz, gw)] \end{aligned}$$

$$+ k_2[d(gx, F(x, y, z)) + d(gy, F(y, z, x)) + d(gz, F(z, x, y))]$$
$$+ k_3[d(gu, F(u, v, w)) + d(gv, F(v, w, u)) + d(gw, F(w, u, v))],$$

holds for all $(x, y, z), (u, v, w) \in X \times X \times X$.

If $F(X \times X \times X) \subseteq g(X)$ *and* $g(X)$ *is complete, then* g *and* F *have a tripled coincidence point* $(x, y, z) \in X \times X \times X$, *satisfying that* $gx = F(x, y, z) = gy = F(y, z, x) = gz = F(z, x, y)$.

Moreover, if g *and* F *are* w*-compatible, then* g *and* F *have a unique common tripled fixed point of the form* (u, u, u), *which satisfies that* $u = gu = F(u, u, u)$.

Corollary 2.3. *Let* d_1 *and* d_2 *be two rectangular* b*-metrics on* X *such that* $d_2(x, y) \leqslant d_1(x, y)$ *for all* $x, y \in X$, (X, d_1) *with coefficient* $s \geqslant 1$, *and* $F : X \times X \times X \to X$ *and* $g : X \to X$ *be two mappings. Suppose that there exist* $a_i \in [0, 1)$ $(i = 1, 2, 3, \cdots, 9)$ *with* $0 \leqslant a_1 + a_2 + a_3 + \cdots + a_9 < 1$ *and* $0 \leqslant s(a_7 + a_8 + a_9) < 1$ *such that the condition*

$$\begin{aligned} d_1(F(x, y, z), F(u, v, w)) \leqslant\ & a_1 d_2(gx, gu) + a_2 d_2(gy, gv) + a_3 d_2(gz, gw) \\ & + a_4 d_2(gx, F(x, y, z)) + a_5 d_2(gy, F(y, z, x)) + a_6 d_2(gz, F(z, x, y)) \\ & + a_7 d_2(gu, F(u, v, w)) + a_8 d_2(gv, F(v, w, u)) + a_9 d_2(gw, F(w, u, v)), \end{aligned} \tag{2.22}$$

holds for all $(x, y, z), (u, v, w) \in X \times X \times X$.

If $F(X \times X \times X) \subseteq g(X)$ *and* $g(X)$ *is* d_1*-complete, then* g *and* F *have a tripled coincidence point* $(x, y, z) \in X \times X \times X$, *which satisfies that* $gx = F(x, y, z) = gy = F(y, z, x) = gz = F(z, x, y)$.

Moreover, if g *and* F *are* w*-compatible, then* g *and* F *have a unique common tripled fixed point of the form* (u, u, u), *which satisfies that* $u = gu = F(u, u, u)$.

Proof. Given $(x, y), (u, v), (z, w) \in X \times X$. It follows from (2.22) that

$$\begin{aligned} d_1(F(x, y, z), F(u, v, w)) \leqslant\ & a_1 d_2(gx, gu) + a_2 d_2(gy, gv) + a_3 d_2(gz, gw) \\ & + a_4 d_2(gx, F(x, y, z)) + a_5 d_2(gy, F(y, z, x)) + a_6 d_2(gz, F(z, x, y)) \\ & + a_7 d_2(gu, F(u, v, w)) + a_8 d_2(gv, F(v, w, u)) + a_9 d_2(gw, F(w, u, v)), \end{aligned} \tag{2.23}$$

$$\begin{aligned} d_1(F(y, z, x), F(v, w, u)) \leqslant\ & a_1 d_2(gy, gv) + a_2 d_2(gz, gw) + a_3 d_2(gx, gu) \\ & + a_4 d_2(gy, F(y, z, x)) + a_5 d_2(gz, F(z, x, y)) + a_6 d_2(gx, F(x, y, z)) \\ & + a_7 d_2(gv, F(v, w, u)) + a_8 d_2(gw, F(w, u, v)) + a_9 d_2(gu, F(u, v, w)), \end{aligned} \tag{2.24}$$

and

$$\begin{aligned} d_1(F(z, x, y), F(w, u, v)) \leqslant\ & a_1 d_2(gz, gw) + a_2 d_2(gx, gu) + a_3 d_2(gy, gv) \\ & + a_4 d_2(gz, F(z, x, y)) + a_5 d_2(gx, F(x, y, z)) + a_6 d_2(gy, F(y, z, x)) \\ & + a_7 d_2(gw, F(w, u, v)) + a_8 d_2(gu, F(u, v, w)) + a_9 d_2(gv, F(v, w, u)). \end{aligned} \tag{2.25}$$

Combining (2.23), (2.24) and (2.25), we have

$$\begin{aligned} d_1(F(x, y, z), F(u, v, w)) &+ d_1(F(y, z, x), F(v, w, u)) + d_1(F(z, x, y), F(w, u, v)) \\ \leqslant\ & (a_1 + a_2 + a_3)[d_2(gx, gu) + d_2(gy, gv) + d_2(gz, gw)] \\ & + (a_4 + a_5 + a_6)[d_2(gx, F(x, y, z)) + d_2(gy, F(y, z, x)) + d_2(gz, F(z, x, y))] \\ & + (a_7 + a_8 + a_9)[d_2(gu, F(u, v, w)) + d_2(gv, F(v, w, u)) + d_2(gw, F(w, u, v))]. \end{aligned}$$

Therefore, the result follows from Theorem 2.1. □

Remark 2.4. If we take $d_1(x, y) = d_2(x, y) = d(x, y)$ for all $x, y \in X$, where d is a rectangular b-metrics on X, then Corollary 2.3 is reduced to a new result.

The following corollary can be obtained from Theorem 2.1 immediately.

Corollary 2.5. *Let d_1 and d_2 be two rectangular b-metrics on X such that $d_2(x,y) \leqslant d_1(x,y)$ for all $x,y \in X$, and $F : X \times X \times X \to X$ and $g : X \to X$ be two mappings. Suppose that there exists $k \in [0,1)$ such that the condition*

$$\begin{aligned} &d_1(F(x,y,z),F(u,v,w)) + d_1(F(y,z,x),F(v,w,u)) + d_1(F(z,x,y),F(w,u,v)) \\ &\qquad \leqslant k[d_2(gx,gu) + d_2(gy,gv) + d_2(gz,gw)], \end{aligned}$$

holds for all $(x,y,z),(u,v,w) \in X \times X \times X$.

If $F(X \times X \times X) \subseteq g(X)$ and $g(X)$ is d_1-complete, then g and F have a tripled coincidence point $(x,y,z) \in X \times X \times X$, which satisfies that $gx = F(x,y,z) = gy = F(y,z,x) = gz = F(z,x,y)$.

Moreover, if g and F are w-compatible, then g and F have a unique common tripled fixed point of the form (u,u,u), which satisfies that $u = gu = F(u,u,u)$.

Corollary 2.6. *Let d_1 and d_2 be two rectangular b-metrics on X such that $d_2(x,y) \leqslant d_1(x,y)$ for all $x,y \in X$, and $F : X \times X \times X \to X$ and $g : X \to X$ be two mappings. Suppose that there exists $k \in [0,1)$ such that the condition*

$$\begin{aligned} &d_1(F(x,y,z),F(u,v,w)) + d_1(F(y,z,x),F(v,w,u)) + d_1(F(z,x,y),F(w,u,v)) \\ &\qquad \leqslant k[d_2(gx,F(x,y,z)) + d_2(gy,F(y,z,x)) + d_2(gz,F(z,x,y))], \end{aligned}$$

holds for all $(x,y,z),(u,v,w) \in X \times X \times X$.

If $F(X \times X \times X) \subseteq g(X)$ and $g(X)$ is d_1-complete., then g and F have a tripled coincidence point $(x,y,z) \in X \times X \times X$, which satisfies that $gx = F(x,y,z) = gy = F(y,z,x) = gz = F(z,x,y)$.

Moreover, if g and F are w-compatible, then g and F have a unique common tripled fixed point of the form (u,u,u), which satisfies that $u = gu = F(u,u,u)$.

Corollary 2.7. *Let d_1 and d_2 be two rectangular b-metrics on X such that $d_2(x,y) \leqslant d_1(x,y)$ for all $x,y \in X$, (X,d_1) with coefficient $s \geqslant 1$, and $F : X \times X \times X \to X$ and $g : X \to X$ be two mappings. Suppose that there exists $k \in [0,1)$ with $0 \leqslant sk < 1$ such that the condition*

$$\begin{aligned} &d_1(F(x,y,z),F(u,v,w)) + d_1(F(y,z,x),F(v,w,u)) + d_1(F(z,x,y),F(w,u,v)) \\ &\qquad \leqslant k[d_2(gu,F(u,v,w)) + d_2(gv,F(v,w,u)) + d_2(gw,F(w,u,v))], \end{aligned}$$

holds for all $(x,y,z),(u,v,w) \in X \times X \times X$.

If $F(X \times X \times X) \subseteq g(X)$ and $g(X)$ is d_1-complete, then g and F have a tripled coincidence point $(x,y,z) \in X \times X \times X$, which satisfies that $gx = F(x,y,z) = gy = F(y,z,x) = gz = F(z,x,y)$.

Moreover, if g and F are w-compatible, then g and F have a unique common tripled fixed point of the form (u,u,u), which satisfies that $u = gu = F(u,u,u)$.

Let $g = I_X$ (the identity mapping) in Theorem 2.1 and Corollaries 2.2, 2.3, 2.5–2.7. Then we have the following results.

Corollary 2.8. *Let d_1 and d_2 be two rectangular b-metrics on X such that $d_2(x,y) \leqslant d_1(x,y)$ for all $x,y \in X$, (X,d_1) with coefficient $s \geqslant 1$ and $F : X \times X \times X \to X$ be a mapping. Suppose that there exist k_1, k_2 and k_3 in $[0,1)$ with $0 \leqslant k_1 + k_2 + k_3 < 1$ and $0 \leqslant sk_3 < 1$ such that the condition*

$$\begin{aligned} &d_1(F(x,y,z),F(u,v,w)) + d_1(F(y,z,x),F(v,w,u)) + d_1(F(z,x,y),F(w,u,v)) \\ &\qquad \leqslant k_1[d_2(x,u) + d_2(y,v) + d_2(z,w)] + k_2[d_2(x,F(x,y,z)) + d_2(y,F(y,z,x)) \\ &\qquad\quad + d_2(z,F(z,x,y))] + k_3[d_2(u,F(u,v,w)) + d_2(v,F(v,w,u)) + d_2(w,F(w,u,v))], \end{aligned}$$

holds for all $(x,y,z),(u,v,w) \in X \times X \times X$.

If X is d_1-complete, then F has a unique tripled fixed point of the form (u,u,u), which satisfies that $u = F(u,u,u)$.

Corollary 2.9. *Let* (X, d) *be a complete rectangular* b*-metrics space with coefficient* $s \geqslant 1$*, and* $F : X \times X \times X \to X$ *be a mapping. Suppose that there exist* k_1, k_2 *and* k_3 *in* $[0, 1)$ *with* $0 \leqslant k_1 + k_2 + k_3 < 1$ *and* $0 \leqslant sk_3 < 1$ *such that the condition*

$$\begin{aligned} d(F(x, y, z),F(u, v, w)) &+ d(F(y, z, x), F(v, w, u)) + d(F(z, x, y), F(w, u, v)) \\ &\leqslant k_1[d(x, u) + d(y, v) + d(z, w)] + k_2[d(x, F(x, y, z)) + d(y, F(y, z, x)) + d(z, F(z, x, y))] \\ &\quad + k_3[d(u, F(u, v, w)) + d(v, F(v, w, u)) + d(w, F(w, u, v))], \end{aligned}$$

holds for all $(x, y, z), (u, v, w) \in X \times X \times X$.

Then F *has a unique tripled fixed point of the form* (u, u, u)*, which satisfies that* $u = gu = F(u, u, u)$.

Corollary 2.10. *Let* d_1 *and* d_2 *be two rectangular* b*-metrics on* X *such that* $d_2(x, y) \leqslant d_1(x, y)$ *for all* $x, y \in X$, (X, d_1) *with coefficient* $s \geqslant 1$ *and* $F : X \times X \times X \to X$ *be a mapping. Suppose that there exist* $a_i \in [0, 1)$ $(i = 1, 2, 3, \cdots, 9)$ *with* $0 \leqslant a_1 + a_2 + a_3 + \cdots + a_9 < 1$ *and* $0 \leqslant s(a_7 + a_8 + a_9) < 1$ *such that the condition*

$$\begin{aligned} d_1(F(x, y, z), F(u, v, w)) \leqslant{} & a_1 d_2(x, u) + a_2 d_2(y, v) + a_3 d_2(z, w) \\ & + a_4 d_2(x, F(x, y, z)) + a_5 d_2(y, F(y, z, x)) + a_6 d_2(z, F(z, x, y)) \\ & + a_7 d_2(u, F(u, v, w)) + a_8 d_2(v, F(v, w, u)) + a_9 d_2(w, F(w, u, v)), \end{aligned}$$

holds for all $(x, y, z), (u, v, w) \in X \times X \times X$.

If (X, d_1) *is complete, then* F *has a unique tripled fixed point of the form* (u, u, u)*, which satisfies that* $u = gu = F(u, u, u)$.

Corollary 2.11. *Let* d_1 *and* d_2 *be two rectangular* b*-metrics on* X *such that* $d_2(x, y) \leqslant d_1(x, y)$ *for all* $x, y \in X$*, and* $F : X \times X \times X \to X$ *be a mapping. Suppose that there exists* $k \in [0, 1)$ *such that the condition*

$$\begin{aligned} d_1(F(x,y, z), F(u, v, w)) &+ d_1(F(y, z, x), F(v, w, u)) + d_1(F(z, x, y), F(w, u, v)) \\ &\leqslant k[d_2(x, u) + d_2(y, v) + d_2(z, w)], \end{aligned}$$

holds for all $(x, y, z), (u, v, w) \in X \times X \times X$.

If (X, d_1) *is complete, then* F *has a unique tripled fixed point of the form* (u, u, u)*, which satisfies that* $u = gu = F(u, u, u)$.

Corollary 2.12. *Let* d_1 *and* d_2 *be two rectangular* b*-metrics on* X *such that* $d_2(x, y) \leqslant d_1(x, y)$ *for all* $x, y \in X$*, and* $F : X \times X \times X \to X$ *be a mapping. Suppose that there exists* $k \in [0, 1)$ *such that the condition*

$$\begin{aligned} d_1(F(x,y, z), F(u, v, w)) &+ d_1(F(y, z, x), F(v, w, u)) + d_1(F(z, x, y), F(w, u, v)) \\ &\leqslant k[d_2(x, F(x, y, z)) + d_2(y, F(y, z, x)) + d_2(z, F(z, x, y))], \end{aligned}$$

holds for all $(x, y, z), (u, v, w) \in X \times X \times X$.

If (X, d_1) *is complete, then* F *has a unique tripled fixed point of the form* (u, u, u)*, which satisfies that* $u = gu = F(u, u, u)$.

Corollary 2.13. *Let* d_1 *and* d_2 *be two rectangular* b*-metrics on* X *such that* $d_2(x, y) \leqslant d_1(x, y)$ *for all* $x, y \in X$, (X, d_1) *with coefficient* $s \geqslant 1$ *and* $F : X \times X \times X \to X$ *and* $g : X \to X$ *be two mappings. Suppose that there exists* $k \in [0, 1)$ *with* $0 \leqslant sk < 1$ *such that the condition*

$$\begin{aligned} d_1(F(x,y, z), F(u, v, w)) &+ d_1(F(y, z, x), F(v, w, u)) + d_1(F(z, x, y), F(w, u, v)) \\ &\leqslant k[d_2(u, F(u, v, w)) + d_2(v, F(v, w, u)) + d_2(w, F(w, u, v))], \end{aligned}$$

holds for all $(x, y, z), (u, v, w) \in X \times X \times X$.

If (X, d_1) *is complete, then* F *has a unique tripled fixed point of the form* (u, u, u)*, which satisfies that* $u = gu = F(u, u, u)$.

Remark 2.14. If we take coefficient $s = 1$ (for the space X) in Theorem 2.1 and Corollaries 2.2, 2.3, 2.5–2.13, then several new results can be obtain in two rectangular metric space.

3. Application to integral equations

Example 3.1. Let $X = \mathbb{R}$ and define $d : X \times X \to \mathbb{R}^+$ as $d(x, y) = |x - y|^k$, $x, y \in X$, where $k \geqslant 1$. Then (X, d) be a rectangular b-metric space with coefficient $s = 3^{k-1}$.

In fact, obviously conditions (RbM1) and (RbM2) in Definition 1.3 are satisfied. Now we show that condition (RbM3) holds for k. By using the following inequality

$$(a+b+c)^k \leqslant 3^{k-1}(a^k+b^k+c^k),\ \forall\ a,\ b,\ c \in \mathbb{R}^+ \ \text{ and } \ k \geqslant 1,$$

we can get

$$\begin{aligned} d(x,y) = |x-y|^k &= |(x-z)+(z-w)+(w-y)|^k \\ &\leqslant (|x-z|+|z-w|+|w-y|)^k \\ &\leqslant 3^{k-1}\left(|x-z|^k+|z-w|^k+|w-y|^k\right) \\ &= 3^{k-1}\left(d(x,z)+d(z,w)+d(w,y)\right), \end{aligned}$$

for all $x, y, z, w \in X$. This means that (RbM3) holds, hence (X, d) is a rectangular b-metric space.

Example 3.2. Let $X = \mathbb{R}$ and d_1, d_2 are two rectangular b-metrics in X such that

$$d_1(x,y) = (x-y)^2, \quad d_2(x,y) = \frac{(x-y)^2}{4},\ \ \forall\ x,y \in X.$$

Define $F : X \times X \times X \to X$ and $g : X \to X$ respectively by

$$F(x,y,z) = \frac{x-y+z}{3},\ \ gx = 4x,\ \ \forall\ x,y,z \in X.$$

It is easy to see that $g(X)$ is d_1-complete, and F and g are ω-compatible.

On the other hand, we have

$$\begin{aligned} d_1(F(x,y,z),F(u,v,w)) &= (F(x,y,z)-F(u,v,w))^2 \\ &= \left(\frac{x-y+z}{3}-\frac{u-v+w}{3}\right)^2 = \left(\frac{x-u}{3}+\frac{v-y}{3}+\frac{z-w}{3}\right)^2 \\ &\leqslant 3\left(\frac{(x-u)^2}{9}+\frac{(y-v)^2}{9}+\frac{(z-w)^2}{9}\right) \\ &= \frac{(x-u)^2}{3}+\frac{(y-v)^2}{3}+\frac{(z-w)^2}{3} \\ &= \frac{(4x-4u)^2}{48}+\frac{(4y-4v)^2}{48}+\frac{(4z-4w)^2}{48} \\ &= \frac{(gx-gu)^2}{48}+\frac{(gy-gv)^2}{48}+\frac{(gz-gw)^2}{48} \\ &= \frac{1}{12}[d_2(gx,gu)+d_2(gy,gv)+d_2(gz,gw)]. \end{aligned}$$

By similar arguments as above, we can show that

$$d_1(F(y,z,x),F(v,w,u)) \leqslant \frac{1}{12}[d_2(gy,gv)+d_2(gz,gw)+d_2(gx,gu)],$$

and

$$d_1(F(z,x,y),F(w,u,v)) \leqslant \frac{1}{12}[d_2(gz,gw)+d_2(gx,gu)+d_2(gy,gv)].$$

Combining the above three inequalities, we obtain

$$\begin{aligned} d_1(F(x,y,z),F(u,v,w))+&d_1(F(y,z,x),F(v,w,u)) + d_1(F(z,x,y),F(w,u,v)) \\ &\leqslant \frac{1}{4}[d_2(gx,gu) + d_2(gy,gv) + d_2(gz,gw)]. \end{aligned}$$

Then by Corollary 2.5, F and g have a unique common tripled fixed point, in fact $(0,0,0)$ is the unique common tripled fixed point of mappings of F and g.

Next, we assume that $X = C[a,b]$ is the set of all continuous functions. Define two rectangular b-metrics respectively by

$$d_1(x,y) = \max_{t\in[a,b]} |x(t)-y(t)|^k, \quad d_2(x,y) = \frac{\max\limits_{t\in[a,b]} |x(t)-y(t)|^k}{3}, \quad \forall x,y \in X, \quad (k \geqslant 1).$$

Then the coefficient of two rectangular b-metrics is $s = 3^{k-1}$. Consider the nonlinear integral equation set as follows

$$\begin{cases} x(r) = K(r) + \int_a^b G(r,t)[f(r,x(t)) + g(r,y(t)) + h(r,z(t))]dt, \\ y(r) = K(r) + \int_a^b G(r,t)[f(r,y(t)) + g(r,z(t)) + h(r,x(t))]dt, \\ z(r) = K(r) + \int_a^b G(r,t)[f(r,z(t)) + g(r,x(t)) + h(r,y(t))]dt. \end{cases} \tag{3.1}$$

Now, we will analyze (3.1) under the following conditions:

(i) $f,g,h : [a,b] \times X \to \mathbb{R}$ are three continuous functions.

(ii) $K : [a,b] \to \mathbb{R}$ are continuous functions.

(iii) $G : [a,b] \times [a,b] \to \mathbb{R}^+$ is a continuous function.

(iv) There exist $L_i > 0$ $(i = 1,2,3)$ such that for all $x,y \in X$,

$$\begin{cases} |f(r,x(t)) - f(r,y(t))| \leqslant L_1|x-y|, \\ |g(r,x(t)) - g(r,y(t))| \leqslant L_2|x-y|, \\ |h(r,x(t)) - h(r,y(t))| \leqslant L_3|x-y|. \end{cases}$$

(v)

$$\max_{r\in[a,b]} \left(\int_a^b G(r,t)dt\right)^k < \frac{1}{3^{2k+1}L^k},$$

where $L = \max\{L_1, L_2, L_3\}$.

Theorem 3.3. *Under the conditions* (i)-(v), *the integral equation* (3.1) *has a unique common solution on* $[a,b]$.

Proof. Define $F : X^3 \to X$ and $g : X \to X$ respectively by

$$\begin{aligned} F(x,y,z)(r) &= K(r) + \int_a^b G(r,t)[f(r,x(t)) + g(r,y(t)) + h(r,z(t))]dt, \quad \forall x,y,z \in X, \quad r \in [a,b] \\ gx &= x, \quad \forall x \in X. \end{aligned}$$

By Example 3.1, (iv) and (v), we have

$$
\begin{aligned}
&|F(x,y,z)(r)-F(u,v,w)(r)|^k\\
&\quad=\left|\int_a^b G(r,t)[f(r,x(t))-f(r,u(t))]dt+\int_a^b G(r,t)[g(r,y(t))-g(r,v(t))]dt\right.\\
&\qquad\left.+\int_a^b G(r,t)[h(r,z(t))-h(r,w(t))]dt\right|^k\\
&\quad\leqslant 3^{k-1}\left[\left|\int_a^b G(r,t)[f(r,x(t))-f(r,u(t))]dt\right|^k+\left|\int_a^b G(r,t)[g(r,y(t))-g(r,v(t))]dt\right|^k\right.\\
&\qquad\left.+\left|\int_a^b G(r,t)[h(t,z(t))-h(t,w(t))]dt\right|^k\right]\\
&\quad\leqslant 3^{k-1}\left[L_1^k\left(\max_{t\in[a,b]}|x(t)-u(t)|\right)^k+L_2^k\left(\max_{t\in[a,b]}|y(t)-v(t)|\right)^k\right.\\
&\qquad\left.+L_3^k\left(\max_{t\in[a,b]}|z(t)-w(t)|\right)^k\right]\left(\int_a^b G(r,t)dt\right)^k\\
&\quad\leqslant 3^{k-1}L^k\left[\max_{t\in[a,b]}|x(t)-u(t)|^k+\max_{t\in[a,b]}|y(t)-v(t)|^k+\max_{t\in[a,b]}|z(t)-w(t)|^k\right]\left(\int_a^b G(r,t)dt\right)^k\\
&\quad=3^{k-1}L^k[d_1(x,u)+d_1(y,v)+d_1(z,w)]\left(\int_a^b G(r,t)dt\right)^k\\
&\quad\leqslant 3^{k-1}L^k[d_1(x,u)+d_1(y,v)+d_1(z,w)]\cdot\frac{1}{3^{2k+1}L^k}\\
&\quad\leqslant \frac{1}{3^{k+2}}[d_1(x,u)+d_1(y,v)+d_1(z,w)].
\end{aligned}
$$

It follows from the above inequality that

$$
\begin{aligned}
d_1(F(x,y,z),F(u,v,w))&=\max_{r\in[a,b]}|F(x,y,z)(r)-F(u,v,w)(r)|^k\\
&\leqslant\frac{1}{3^{k+2}}[d_1(x,u)+d_1(y,v)+d_1(z,w)].
\end{aligned}\tag{3.2}
$$

By similar arguments as above,

$$
d_1(F(y,z,x),F(v,w,u))\leqslant\frac{1}{3^{k+2}}[d_1(y,v)+d_1(z,w)+d_1(x,u)],\tag{3.3}
$$

$$
d_1(F(z,x,y),F(w,u,v))\leqslant\frac{1}{3^{k+2}}[d_1(z,w)+d_1(x,u)+d_1(y,v)].\tag{3.4}
$$

It follows from (3.2), (3.3) and (3.4) that

$$
\begin{aligned}
&d_1(F(x,y,z),F(u,v,w))+d_1(F(y,z,x),F(v,w,u))+d_1(F(z,x,y),F(w,u,v))\\
&\qquad\leqslant\frac{1}{3^{k+1}}[d_1(x,u)+d_1(y,v)+d_1(z,w)]=\frac{1}{3^k}[d_2(x,u)+d_2(y,v)+d_2(z,w)].
\end{aligned}
$$

By Corollary 2.5 we assert that there exists $\tau\in X$ such that $F(\tau,\tau,\tau)=g\tau=\tau$ which implies that τ is the unique solution of equation set (3.1). □

Example 3.4. We assume that $X = C[0,1]$ is the set of all continuous functions defined on $[0,1]$. Define (X, d_1) and (X, d_2) respectively by

$$d_1(x,y) = \max_{r\in[0,1]} |x(r)-y(r)|^2, \quad d_2(x,y) = \max_{r\in[0,1]} \frac{|x(r)-y(r)|^2}{3}, \quad \forall x,y \in X.$$

Then the coefficient of two rectangular b-metrics is $k = 2$. Consider the following nonlinear integral equation set:

$$\begin{cases} x(r) = 2^r + \int_0^1 \frac{\sin(r\cdot\pi)}{3\sqrt{3}+t}\left[\frac{e^{-rx(t)}}{3} + \frac{\sin(y(t)\cdot\pi)}{6\pi+r} + \frac{\cos(r)}{4}\cdot\frac{|z(t)|}{1+|z(t)|}\right]dt, \\ y(r) = 2^r + \int_0^1 \frac{\sin(r\cdot\pi)}{3\sqrt{3}+t}\left[\frac{e^{-ry(t)}}{3} + \frac{\sin(z(t)\cdot\pi)}{6\pi+r} + \frac{\cos(r)}{4}\cdot\frac{|x(t)|}{1+|x(t)|}\right]dt, \\ z(r) = 2^r + \int_0^1 \frac{\sin(r\cdot\pi)}{3\sqrt{3}+t}\left[\frac{e^{-rz(t)}}{3} + \frac{\sin(x(t)\cdot\pi)}{6\pi+r} + \frac{\cos(r)}{4}\cdot\frac{|y(t)|}{1+|y(t)|}\right]dt. \end{cases} \tag{3.5}$$

Define the function $K : [0,1] \to \mathbb{R}$, $G : [0,1]\times[0,1] \to \mathbb{R}^+$, $f, g, h : [0,1]\times X \to \mathbb{R}$ respectively by: $K(r) = 2^r$, $G(r,t) = \frac{\sin(r\cdot\pi)}{3\sqrt{3}+t}$, $f(r,x(t)) = \frac{e^{-rx(t)}}{3}$, $g(r,x(t)) = \frac{\sin(x(t)\cdot\pi)}{6\pi+r}$, $h(r,x(t)) = \frac{\cos(r)}{4}\cdot\frac{|x(t)|}{1+|x(t)|}$.

We can easily obtain that $K(t)$, $G(r,t)$, $f(r,x)$, $g(r,x)$, $h(r,x)$ are continuous functions. Also, for all $r \in [0,1]$ and $x, y \in X$, we have

$$\begin{aligned} |f(r,x)-f(r,y)| &= \left|\frac{e^{-rx}}{3} - \frac{e^{-ry}}{3}\right| = \left|\frac{-re^{-r\xi}}{3}(x-y)\right| \leqslant \frac{1}{3}|x-y|, \\ |g(r,x)-g(r,y)| &= \left|\frac{\sin(x\cdot\pi)}{6\pi+r} - \frac{\sin(y\cdot\pi)}{6\pi+r}\right| \leqslant \frac{|\sin(x\cdot\pi)-\sin(y\cdot\pi)|}{6\pi} \\ &= \frac{1}{6\pi}\left|\pi\cdot\cos(\eta\cdot\pi)\cdot(x-y)\right| \leqslant \frac{1}{6}\left|x-y\right|, \\ |h(r,x)-h(r,y)| &= \left|\frac{\cos(r)}{4}\cdot\frac{|x|}{1+|x|} - \frac{\cos(r)}{4}\cdot\frac{|y|}{1+|y|}\right| \leqslant \frac{1}{4}\left|\frac{|x|}{1+|x|} - \frac{|y|}{1+|y|}\right| \\ &\leqslant \frac{1}{4}\left|\frac{1+|x|-1}{1+|x|} - \frac{1+|y|-1}{1+|y|}\right| \leqslant \frac{1}{4}\left|\frac{1}{1+|y|} - \frac{1}{1+|x|}\right| \\ &= \frac{1}{4}\left|\frac{1}{(1+\zeta)^2}(|x|-|y|)\right| \leqslant \frac{1}{4}||x|-|y|| \leqslant \frac{1}{4}|x-y|. \end{aligned}$$

Here $r \in [0,1]$, $L_1 = \frac{1}{3}$, $L_2 = \frac{1}{6}$, $L_3 = \frac{1}{4}$, $L = \max\{L_1, L_2, L_3\} = \frac{1}{3}$, ξ, η exist between x and y, ζ exist between $|x|$ and $|y|$.

$$\max_{r\in[0,1]}\left(\int_0^1 G(r,t)dt\right)^2 = \max_{r\in[0,1]}\left(\int_0^1 \frac{\sin(r\cdot\pi)}{3\sqrt{3}+t}dt\right)^2 \leqslant \left(\frac{1}{3\sqrt{3}}\right)^2 = \frac{1}{3^3} = \frac{1}{3^{2k+1}L^k}.$$

Consequently, all the conditions of Theorem 3.3 are satisfied. Hence the integral equation set (3.5) has the unique solution in $C[0,1]$.

Acknowledgment

This work is supported by the National Natural Science Foundation of China (11071169, 11271105), the Natural Science Foundation of Zhejiang Province (Y6110287).

References

[1] T. Abdeljawad, D. Türkoğlu, *Locally convex valued rectangular metric spaces and the Kannan's fixed point theorem*, J. Comput. Anal. Appl., **14** (2012), 484–494. 1

[2] J. Ahmad, M. Arshad, C. Vetro, *On a theorem of Khan in a generalized metric space*, Int. J. Anal., **2013** (2013), 6 pages.

[3] M. Arshad, J. Ahmad, E. Karapınar, *Some common fixed point results in rectangular metric spaces*, Int. J. Anal., **2013** (2013), 7 pages. 1

[4] H. Aydi, M. Abbas, W. Sintunavarat, P. Kumam, *Tripled fixed point of W-compatible mappings in abstract metric spaces*, Fixed Point Theory Appl., **2012** (2012), 20 pages. 1.7, 1.8, 1.9
[5] H. Aydi, A. Felhi, S. Sahmim, *Common fixed points in rectangular* b*-metric spaces using* (E.A) *property*, J. Adv. Math. Stud., **8** (2015), 159–169. 1
[6] H. Aydi, E. Karapınar, H. Lakzian, *Fixed point results on a class of generalized metric spaces*, Math. Sci. (Springer), **2012** (2012), 6 pages. 1
[7] N. Bilgili, E. Karapınar, D. Turkoglu, *A note on common fixed points for* (ψ,α,β)*-weakly contractive mappings in generalized metric spaces*, Fixed Point Theory Appl., **2013** (2013), 6 pages. 1
[8] A. Branciari, *A fixed point theorem of Banach-Caccioppoli type on a class of generalized metric spaces*, Publ. Math. Debrecen, **57** (2000), 31–37. 1, 1.2
[9] N. Cakić, *Coincidence and common fixed point theorems for* (ψ,ϕ) *weakly contractive mappings in generalized metric spaces*, Filomat, **27** (2013), 1415–1423. 1
[10] S. Czerwik, *Contraction mappings in* b*-metric spaces*, Acta Math. Inform. Univ. Ostraviensis, **1** (1993), 5–11. 1, 1.1
[11] P. Das, L. K. Dey, *Fixed point of contractive mappings in generalized metric spaces*, Math. Slovaca, **59** (2009), 499–504. 1
[12] C. Di Bari, P. Vetro, *Common fixed points in generalized metric spaces*, Appl. Math. Comput., **218** (2012), 7322–7325. 1
[13] H.-S. Ding, M. Imdad, S. Radenović, J. Vujaković, *On some fixed point results in* b*-metric, rectangular and* b*-rectangular metric spaces*, Arab J. Math. Sci., **22** (2016), 151–164. 1
[14] H.-S. Ding, V. Ozturk, S. Radenović, *On some new fixed point results in* b*-rectangular metric spaces*, J. Nonlinear Sci. Appl., **8** (2015), 378–386. 1.4, 1
[15] İ. M. Erhan, E. Karapınar, T. Sekulić, *Fixed points of* (ψ,ϕ) *contractions on rectangular metric spaces*, Fixed Point Theory Appl., **2012** (2012), 12 pages. 1
[16] A. Flora, A. Bellour, A. Al-Bsoul, *Some results in fixed point theory concerning generalized metric spaces*, Mat. Vesnik, **61** (2009), 203–208. 1
[17] R. George, S. Radenović, K. P. Reshma, S. Shukla, *Rectangular b-metric space and contraction principles*, J. Nonlinear Sci. Appl., **8** (2015), 1005–1013. 1, 1.3, 1.4, 1.5
[18] R. George, R. Rajagopalan, *Common fixed point results for* $\psi-\phi$ *contractions in rectangular metric spaces*, Bull. Math. Anal. Appl., **5** (2013), 44–52. 1
[19] H. Işik, D. Türkoğlu, *Common fixed points for* (ψ,α,β)*-weakly contractive mappings in generalized metric spaces*, Fixed Point Theory Appl., **2013** (2013), 6 pages.
[20] W. A. Kirk, N. Shahzad, *Generalized metrics and Caristi's theorem*, Fixed Point Theory Appl., **2013** (2013), 9 pages.
[21] B. K. Lahiri, P. Das, *Fixed point of a Ljubomir Ćirić's quasi-contraction mapping in a generalized metric space*, Publ. Math. Debrecen, **61** (2002), 589–594.
[22] H. Lakzian, B. Samet, *Fixed points for* (ψ,ϕ)*-weakly contractive mappings in generalized metric spaces*, Appl. Math. Lett., **25** (2012), 902–906.
[23] V. La Rosa, P. Vetro, *Common fixed points for* α*-*ψ*-*ϕ*-contractions in generalized metric spaces*, Nonlinear Anal. Model. Control, **19** (2014), 43–54.
[24] S. K. Malhotra, S. Shukla, R. Sen, *Some fixed point theorems for ordered Reich type contractions in cone rectangular metric spaces*, Acta Math. Univ. Comenian. (N.S.), **82** (2013), 165–175.
[25] S. Moradi, D. Alimohammadi, *New extensions of Kannan fixed-point theorem on complete metric and generalized metric spaces*, Int. J. Math. Anal. (Ruse), **5** (2011), 2313–2320. 1
[26] J. R. Roshan, V. Parvaneh, Z. Kadelburg, N. Hussain, *New fixed point results in* b*-rectangular metric spaces*, Nonlinear Anal. Model. Control, **21** (2016), 614–634. 1, 1.3
[27] B. Samet, *A fixed point theorem in a generalized metric space for mappings satisfying a contractive condition of integral type*, Int. J. Math. Anal. (Ruse), **3** (2009), 1265–1271. 1
[28] B. Samet, *Discussion on "A fixed point theorem of Banach-Caccioppoli type on a class of generalized metric spaces" by A. Branciari [MR1771669]*, Publ. Math. Debrecen, **76** (2010), 493–494. 1
[29] B. Samet, C. Vetro, *Coupled fixed point,* F*-invariant set and fixed point of* N*-order*, Ann. Funct. Anal., **1** (2010), 46–56. 1.6
[30] S. Shukla, G-(F,τ)*-contractions in partial rectangular metric spaces endowed with a graph and fixed point theorems*, TWMS J. Appl. Eng. Math., **6** (2016), 342–353. 1