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Convergence analysis of a Halpern-like iterative algorithm in Hilbert spaces

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Abstract

In this paper, a Halpern-like iterative algorithm is investigated for finding a solution of a split feasibility problem and a solution to a nonexpansive operator equation. Strong convergence theorems are established in the framework of infinite dimensional Hilbert spaces.

Keywords: Convergence analysis, Hilbert space, monotone mapping, split feasibility problem.
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1. Introduction and preliminaries

Let C and Q be nonempty closed and convex subsets of real Hilbert spaces H_1 and H_2, respectively, and let Proj_C and Proj_Q be the metric projections from H_1 and H_2 to C and Q, respectively, and $A : H_1 \to H_2$ be a bounded linear operator.

Recall that the split feasibility problem is formulated as to find a point $q \in H_1$ such that:

$$q \in C \ \text{ and } \ Aq \in Q. \tag{1.1}$$

It is easy to see that $q \in H_1$ solves equation (1.1) if and only if it solves the following fixed point equation

$$q = \mathrm{Proj}_C(I - \gamma A^*(I - P_Q)A)q, \ \ x \in C,$$

where A^* is the adjoint of A.

In 1994, Censor and Elfving [6] introduced the split feasibility problem in finite dimensional Hilbert spaces for modeling inverse problems which arise from phase retrievals and in medical image reconstruction. It has been found that the split feasibility problem can also be used in various disciplines such as image restoration, computer tomograph and radiation therapy treatment planning [4, 5, 7].

Recently, Byrne [4] considered the split feasibility problem in an infinite dimensional Hilbert space. In many disciplines, including image restoration, computer tomograph, control theory, and quantum

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physics, problems arise in infinite dimension spaces. Therefore, it is important to consider the split feasibility problem in the framework of infinite dimensional spaces. On the other hand, the split feasibility problem covers convex feasibility problems that it is to find a common element in the intersection of a family of closed and convex subsets of Hilbert spaces. Recently, many authors have studied the split feasibility problem via fixed point methods; see [8, 11, 15, 17–19] and the references therein.

Let H be a Hilbert space. Recall that a mapping $T : H \to H$ is said to be monotone iff

$$\langle Tx - Ty, x - y\rangle \geqslant 0, \quad \forall x, y \in H.$$

$T : H \to H$ is said to be inverse-strongly monotone iff there exists a constant $\nu > 0$ such that

$$\langle Tx - Ty, x - y\rangle \geqslant \nu\|Tx - Ty\|^2, \quad \forall x, y \in H.$$

We also say that T is ν-inverse-strongly monotone. It is obvious that if F is ν-inverse-strongly monotone, then it is $\frac{1}{\nu}$-Lipschitz continuous and monotone.

Recall that a mapping $T : H \to H$ is said to be nonexpansive iff

$$\|Tx - Ty\| \leqslant \|x - y\|, \quad \forall x, y \in H.$$

From Browder [3], we see that every nonexpansive mapping on bounded closed and convex subsets of Hilbert spaces has a nonempty fixed point set; see [3] and the references therein. Halpern-like iterative algorithms have recently investigated to study fixed points of nonexpansive mappings and zero points of monotone operators; see [1, 2, 9, 10, 13, 14, 16] and the references therein. The advantage of Halpern-like iterative algorithms is that strong convergence can be guaranteed without any compact assumptions.

Let D be a nonempty closed and convex subset of real Hilbert space H.

Recall that the metric projection $P_D^H : H \to D$ from H onto D of H is defined as follows: for each point $x \in H$, there exists a unique point $P_D^H x \in D$ with the property:

$$\|x - P_D^H x\| \leqslant \|x - y\|.$$

Thus for any $x \in H$, $\tilde{x} = P_D^H x$ iff $\tilde{x} \in D$ and $\|x - \tilde{x}\| = \inf\{\|x - y\| : y \in D\}$. We also have the following facts

$$\|x - y\|^2 - \|(I - P_D^H)x - (I - P_D^H)y\|^2 \geqslant \|P_D^H x - P_D^H y\|^2, \quad \forall x, y \in H,$$
$$\langle (I - P_D^H)x - (I - P_D^H)y, x - y\rangle \geqslant \|(I - P_D^H)x - (I - P_D^H)y\|^2, \quad \forall x, y \in H,$$

and

$$\langle P_D^H x - P_D^H y, x - y\rangle \geqslant \|P_D^H x - P_D^H y\|^2, \quad \forall x, y \in H.$$

Lemma 1.1 ([12])**.** *Let $\{a_n\}$ be a sequence of nonnegative real numbers such that*

$$a_{n+1} \leqslant (1 - t_n)a_n + b_n, \quad \forall n \geqslant 0,$$

where $\{t_n\} \subset (0, 1)$ and $\{b_n\}$ is a sequence of real numbers. Assume that

$$\sum_{n=0}^{\infty} t_n = \infty \text{ and } \limsup_{n\to\infty} \frac{b_n}{t_n} \leqslant 0.$$

Then $\lim_{n\to\infty} a_n = 0$.

Lemma 1.2 ([3])**.** *Let H be a Hilbert space and let S be a nonexpansive mapping on H. Then $I - S$ is demiclosed at the origin. That is, $\{x_n\}$ converges weakly to p and $\{x_n - Sx_n\}$ converges strongly to 0. Then p is a fixed point of S.*

2. Main results

Theorem 2.1. *Let C be a nonempty closed and convex subset of Hilbert space H_1 and let Q be a nonempty closed and convex subset of Hilbert space H_2. Let Proj_C be the metric projection from Hilbert space H_1 onto C and let Proj_Q be the metric projection from Hilbert space H_2 onto Q. Let $S : C \to C$ be a nonexpansive mapping with fixed* *points. Let* $A : H_1 \to H_2$ *be a bounded linear operator such that split feasibility problem* (1.1) *is consistent. Let* $\{x_n\}$ *be a sequence generated in the following iterative algorithm:* $x_1 \in C$ *is the initial and*

$$\begin{cases} \lambda_n = \mathrm{Proj}_C\Big((1-\beta_n)(x_n - \gamma_n A^*(I-\mathrm{Proj}_Q)Ax_n) + \beta_n Sx_n\Big), \\ x_{n+1} = \alpha_n u + (1-\alpha_n)\lambda_n, \quad n \geqslant 1, \end{cases}$$

where u *is a fixed element in* C, $\{\alpha_n\}$, $\{\beta_n\}$ *are two sequences in* $(0,1)$ *with* $\lim_{n\to\infty}\alpha_n = 0$, $\sum_{n=1}^{\infty}\alpha_n = \infty$, $\sum_{n=1}^{\infty}|\alpha_n - \alpha_{n+1}| < \infty$, $0 < \liminf_{n\to\infty}\beta_n \leqslant \limsup_{n\to\infty}\beta_n < 1$, $\sum_{n=1}^{\infty}|\beta_n - \beta_{n+1}| < \infty$, *and* $\{\gamma_n\}$ *is a sequence with* $0 < \liminf_{n\to\infty}\gamma_n \leqslant \limsup_{n\to\infty}\gamma_n < \frac{2}{\|A\|^2}$ *and* $\sum_{n=1}^{\infty}|\gamma_n - \gamma_{n+1}| < \infty$. *If* $\mathrm{Sol(SFP)} \cap \mathrm{Fix}(S) \neq \emptyset$, *then* $\{x_n\}$ *converges strongly to a point* $\bar{u}$ *in* $\mathrm{Fix}(S) \cap \mathrm{Sol(SFP)}$, *where* $\bar{u} = \mathrm{Proj}_{\mathrm{Sol(SFP)}\cap\mathrm{Fix}(S)}u$.

Proof. First, we show that sequence $\{x_n\}$ is bounded. Define a mapping $T : H_1 \to H_1$ by

$$Tx = A^*(I-\mathrm{Proj}_Q)Ax, \quad \forall x \in H_1.$$

Using the properties of metric projection Proj_Q, we find that

$$\begin{aligned} \langle Tx - Ty, x-y\rangle &= \langle A^*(I-\mathrm{Proj}_Q)Ax - A^*(I-\mathrm{Proj}_Q)Ay, x-y\rangle \\ &= \langle (I-\mathrm{Proj}_Q)Ax - (I-\mathrm{Proj}_Q)Ay, Ax-Ay\rangle \\ &\geqslant \|(I-\mathrm{Proj}_Q)Ax - (I-\mathrm{Proj}_Q)Ay\|^2 \\ &\geqslant \frac{1}{\|A\|^2}\|A^*(I-\mathrm{Proj}_Q)Ax - A^*(I-\mathrm{Proj}_Q)Ay\|^2 \\ &= \frac{1}{\|A\|^2}\|Tx-Ty\|^2. \end{aligned} \tag{2.1}$$

This shows that T is $\frac{1}{\|A\|^2}$-inverse-strongly monotone. From the restriction imposed on $\{\gamma_n\}$, we may, without loss of generality, assume that $0 < \gamma \leqslant \gamma_n \leqslant \bar{\gamma} < \frac{2}{\|A\|^2}$, where γ and $\bar{\gamma}$ are real constants. Since T is $\frac{1}{\|A\|^2}$-inverse-strongly monotone, we find that

$$\begin{aligned} \|(I-\gamma_n T)x - (I-\gamma_n T)y\|^2 &= \|(x-y) - \gamma_n(Tx-Ty)\|^2 \\ &= \gamma_n^2\|Tx-Ty\|^2 + \|x-y\|^2 - 2\gamma_n\langle x-y, Tx-Ty\rangle \\ &\leqslant \gamma_n^2\|Tx-Ty\|^2 + \|x-y\|^2 - \frac{2\gamma_n}{\|A\|^2}\|Tx-Ty\|^2 \\ &= \|x-y\|^2 - \gamma_n(\frac{2}{\|A\|^2} - \gamma_n)\|Tx-Ty\|^2 \\ &\leqslant \|x-y\|^2. \end{aligned}$$

This shows that $(I-\gamma_n T)$ is a nonexpansive mapping. Letting $x \in A^{-1}(Q)$, we find from the definition of T that $x \in T^{-1}(0)$. This proves $A^{-1}(Q)$ is a subset of $T^{-1}(0)$. Letting $x \in T^{-1}(0)$, we have $Tx = 0$. Take a point $y \in \mathrm{Sol(SFP)} \cap \mathrm{Fix}(S)$. This implies $Ay = \mathrm{Proj}_Q Ay$. Hence, $Ty = 0$. Thanks to (2.1), one arrives at

$$0 = \langle Tx-Ty, x-y\rangle = \langle (I-\mathrm{Proj}_Q)Ax - (I-\mathrm{Proj}_Q)Ay, Ax-Ay\rangle \geqslant \|(I-\mathrm{Proj}_Q)Ax\|^2.$$

This proves $x \in A^{-1}(Q)$, that is, $T^{-1}(0)$ is a subset of $A^{-1}(Q)$. This completes the proof that

$$T^{-1}(0) = A^{-1}(Q).$$

It follows that

$$\mathrm{Fix}(S) \cap \mathrm{Sol(SFP)} = \mathrm{Fix}(S) \cap T^{-1}(0) \cap C.$$

On the other hand, we have

$$\begin{aligned}\|x_{n+1}-x^*\| &\leqslant (1-\alpha_n)\|\mathrm{Proj}_C\Big((1-\beta_n)(I-\gamma_n T)x_n+\beta_n Sx_n\Big)-\mathrm{Proj}_C x^*\|+\alpha_n\|u-x^*\|\\ &\leqslant (1-\alpha_n)\|\Big((1-\beta_n)(I-\gamma_n T)x_n+\beta_n Sx_n\Big)-x^*\|+\alpha_n\|u-x^*\|\\ &\leqslant (1-\alpha_n)(1-\beta_n)\|(I-\gamma_n T)x_n-x^*\|+(1-\alpha_n)\beta_n\|Sx_n-x^*\|+\alpha_n\|u-x^*\|\\ &\leqslant (1-\alpha_n)\|x_n-x^*\|+\alpha_n\|u-x^*\|\\ &\leqslant \max\{\|x_n-x^*\|,\|u-x^*\|\}.\end{aligned}$$

This proves that

$$\|x_{n+1}-x^*\|\leqslant \max\{\|x_1-x^*\|,\|u-x^*\|\}.$$

This completes the proof that $\{x_n\}$ is a bounded sequence. Note that

$$\lambda_n=\mathrm{Proj}_C\Big((1-\beta_n)(I-\gamma_n T)x_n+\beta_n Sx_n\Big).$$

It follows that

$$\begin{aligned}\|\lambda_{n+1}-\lambda_n\| &\leqslant \|\Big((1-\beta_{n+1})(I-\gamma_{n+1}T)x_{n+1}+\beta_{n+1}Sx_{n+1}\Big)-\Big((1-\beta_n)(I-\gamma_n T)x_n+\beta_n Sx_n\Big)\|\\ &\leqslant (1-\beta_{n+1})\|\big((I-\gamma_{n+1}T)x_{n+1}-(I-\gamma_n T)x_n\big)\|\\ &\quad+\beta_{n+1}\|Sx_{n+1}-Sx_n\|+|\beta_{n+1}-\beta_n|\|(I-\gamma_n T)x_n-Sx_n\|\\ &\leqslant (1-\beta_{n+1})\big(\|x_{n+1}-x_n\|+|\gamma_n-\gamma_{n+1}|\|Tx_n\|\big)\\ &\quad+\beta_{n+1}\|x_{n+1}-x_n\|+|\beta_{n+1}-\beta_n|\|(I-\gamma_n T)x_n-Sx_n\|\\ &\leqslant \|x_{n+1}-x_n\|+|\gamma_n-\gamma_{n+1}|\|Tx_n\|+|\beta_{n+1}-\beta_n|\|(I-\gamma_n T)x_n-Sx_n\|.\end{aligned}$$

This implies that

$$\begin{aligned}\|x_{n+2}-x_{n+1}\| &\leqslant |\alpha_{n+1}-\alpha_n|\|u-\lambda_n\|+(1-\alpha_{n+1})\|\lambda_{n+1}-\lambda_n\|\\ &\leqslant \|u-\lambda_n\|+(1-\alpha_{n+1})\|x_{n+1}-x_n\|\\ &\quad+|\gamma_n-\gamma_{n+1}|\|Tx_n\|+|\beta_{n+1}-\beta_n|\|(I-\gamma_n T)x_n-Sx_n\|\\ &\leqslant (1-\alpha_{n+1})\|x_{n+1}-x_n\|+(|\gamma_n-\gamma_{n+1}|+|\beta_{n+1}-\beta_n|+|\alpha_{n+1}-\alpha_n|)M,\end{aligned}$$

where

$$M=\max\{\sup_{n\geqslant 1}\{\|u-\lambda_n\|\},\sup_{n\geqslant 1}\{\|Tx_n\|\},\sup_{n\geqslant 1}\{\|(I-\gamma_n T)x_n-Sx_n\|\}\}.$$

Using Lemma 1.1, we find that

$$\lim_{n\to\infty}\|x_{n+1}-x_n\|=0. \tag{2.2}$$

On the other hand, we have

$$\begin{aligned}\|x^*-(I-\gamma_n T)x_n\|^2 &= \|(x_n-x^*)-\gamma_n(Tx_n-Tx^*)\|^2\\ &\leqslant \gamma_n^2\|Tx_n-Tx^*\|^2+\|x_n-x^*\|^2-\frac{2\gamma_n}{\|A\|^2}\|Tx_n-Tx^*\|^2\\ &= \|x_n-x^*\|^2-(\frac{2\gamma_n}{\|A\|^2}-\gamma_n^2)\|Tx_n-Tx^*\|^2\\ &= \|x_n-x^*\|^2-(\frac{2\gamma_n}{\|A\|^2}-\gamma_n^2)\|Tx_n\|^2.\end{aligned} \tag{2.3}$$

This in turn implies from (2.3) that

$$\begin{aligned}\|\lambda_n-x^*\|^2 &\leqslant \|\big((1-\beta_n)(I-\gamma_n T)x_n+\beta_n Sx_n\big)-x^*\|\\ &\leqslant (1-\beta_n)\|x^*-(I-\gamma_n T)x_n\|+\beta_n\|Sx_n-x^*\|\\ &\leqslant \|x^*-x_n\|^2-(1-\beta_n)(\frac{2\gamma_n}{\|A\|^2}-\gamma_n^2)\|Tx_n\|^2.\end{aligned}$$

Hence, we have

$$\begin{aligned}\|x_{n+1}-x^*\|^2 &\leqslant \alpha_n\|u-x^*\|^2+(1-\alpha_n)\|\lambda_n-x^*\|^2\\ &\leqslant \alpha_n\|u-x^*\|^2+(1-\alpha_n)\|x^*-x_n\|^2-(1-\alpha_n)(1-\beta_n)(\frac{2\gamma_n}{\|A\|^2}-\gamma_n^2)\|Tx_n\|^2\\ &\leqslant \alpha_n\|u-x^*\|^2+\|x^*-x_n\|^2-(1-\alpha_n)(1-\beta_n)(\frac{2\gamma_n}{\|A\|^2}-\gamma_n^2)\|Tx_n\|^2.\end{aligned}$$

It follows that

$$\begin{aligned}(1-\alpha_n)(1-\beta_n)(\frac{2\gamma_n}{\|A\|^2}-\gamma_n^2)\|Tx_n\|^2 &\leqslant \alpha_n\|u-x^*\|^2+\|x_n-p\|^2-\|x_{n+1}-p\|^2\\ &\leqslant \alpha_n\|u-x^*\|^2+(\|x_n-p\|+\|x_{n+1}-p\|)\|x_{n+1}-x_n\|.\end{aligned}$$

From the restriction imposed on $\{\beta_n\}$, we may, without loss of generality, assume that $0<\beta\leqslant\beta_n\leqslant\bar{\beta}<\frac{2}{\|A\|^2}$, where β and $\bar{\beta}$ are real constants. We find from (2.2) that

$$\lim_{n\to\infty}\|Tx_n\|=0. \tag{2.4}$$

Set $\bar{u}=\mathrm{Proj}_{Sol(SFP)\cap Fix(S)}u$. Now, we are in a position to show that

$$\limsup_{n\to\infty}\langle u-\bar{u},\lambda_n-\bar{u}\rangle\leqslant 0. \tag{2.5}$$

To this end, we take a subsequence $\{x_{n_j}\}$ of $\{x_n\}$ such that

$$\limsup_{n\to\infty}\langle u-\bar{u},x_n-\bar{u}\rangle=\lim_{j\to\infty}\langle u-\bar{u},x_{n_j}-\bar{u}\rangle.$$

Without loss of generality, let us assume that $\{x_{n_j}\}$ converges weakly to a point in C. Next we denote the point by ω. Setting $W=I-T$, we see that W is nonexpansive. From the demiclosed principal of W, we find from (2.4) that

$$\omega\in Fix(W)=T^{-1}(0)=A^{-1}(Q).$$

On the other hand, we find that

$$\begin{aligned}\|x_n-Sx_n\| &\leqslant \|x_n-x_{n+1}\|+\alpha_n\|u-Sx_n\|+(1-\alpha_n)\|\lambda_n-Sx_n\|\\ &\leqslant \|x_n-x_{n+1}\|+\alpha_n\|u-Sx_n\|+(1-\alpha_n)(1-\beta_n)\|(x_n-T)x_n-Sx_n\|\\ &\leqslant \|x_n-x_{n+1}\|+\alpha_n\|u-Sx_n\|+(1-\beta_n)\|x_n-Sx_n\|+\|Tx_n\|.\end{aligned}$$

This finds that

$$\|x_n-Sx_n\|\leqslant\frac{1}{\beta_n}\|x_n-x_{n+1}\|+\frac{\alpha_n}{\beta_n}\|u-Sx_n\|+\frac{1}{\beta_n}\|Tx_n\|.$$

Using (2.2) and (2.4), we find that

$$\lim_{n\to\infty}\|x_n-Sx_n\|=0.$$

From the demiclosed principal of S, we find that $\omega\in Fix(S)$. This obtains that (2.5) holds.

Note that

$$\begin{aligned}\|x_{n+1}-\bar{u}\|^2 &\leqslant \alpha_n^2\|u-\bar{u}\|^2+(1-\alpha_n)^2\|\lambda_n-\bar{u}\|^2+2\alpha_n(1-\alpha_n)\langle u-\bar{u},\lambda_n-\bar{u}\rangle\\ &\leqslant \alpha_n^2\|u-\bar{u}\|^2+(1-\alpha_n)^2((1-\beta_n)\|(I-\gamma_nT)x_n-\bar{u}\|+\beta_n\|Sx_n-\bar{u}\|)^2\\ &\quad+2\alpha_n(1-\alpha_n)\langle u-\bar{u},\lambda_n-\bar{u}\rangle\\ &\leqslant (1-\alpha_n)^2(1-\beta_n)^2\|x_n-\bar{u}\|^2+\alpha_n^2\|u-\bar{u}\|^2+2\alpha_n(1-\alpha_n)\langle u-\bar{u},\lambda_n-\bar{u}\rangle\\ &\leqslant (1-\alpha_n)\|x_n-\bar{u}\|^2+\alpha_n\xi_n,\end{aligned}$$

where

$$\xi_n=\alpha_n\|u-\bar{u}\|^2+2\langle u-\bar{u},\lambda_n-\bar{u}\rangle.$$

Using Lemma 1.1, we find that $x_n\to\bar{u}$ as $n\to\infty$. This completes the proof. □

If $S = I$, the identity mapping, then Theorem 2.1 is reduced to the following result.

Corollary 2.2. *Let C be a nonempty closed and convex subset of Hilbert space H_1 and let Q be a nonempty closed and convex subset of Hilbert space H_2. Let Proj_C be the metric projection from Hilbert space H_1 onto C and let Proj_Q be the metric projection from Hilbert space H_2 onto Q. Let $S : C \to C$ be a nonexpansive mapping with fixed points. Let $A : H_1 \to H_2$ be a bounded linear operator such that split feasibility problem (1.1) is consistent. Let $\{x_n\}$ be a sequence generated in the following iterative algorithm: $x_1 \in C$ is the initial and*

$$\begin{cases} \lambda_n = \mathrm{Proj}_C\Big(x_n - (1-\beta_n)\gamma_n A^*(I - \mathrm{Proj}_Q)Ax_n\Big), \\ x_{n+1} = \alpha_n u + (1-\alpha_n)\lambda_n, \quad n \geqslant 1, \end{cases}$$

where u is a fixed element in C, $\{\alpha_n\}$ and $\{\beta_n\}$ are two sequences in $(0,1)$ with $\lim_{n\to\infty}\alpha_n = 0$, $\sum_{n=1}^{\infty}\alpha_n = \infty$, $\sum_{n=1}^{\infty}|\alpha_n - \alpha_{n+1}| < \infty$, $0 < \liminf_{n\to\infty}\beta_n \leqslant \limsup_{n\to\infty}\beta_n < 1$, $\sum_{n=1}^{\infty}|\beta_n - \beta_{n+1}| < \infty$, and $\{\gamma_n\}$ is a sequence with $0 < \liminf_{n\to\infty}\gamma_n \leqslant \limsup_{n\to\infty}\gamma_n < \frac{2}{\|A\|^2}$ and $\sum_{n=1}^{\infty}|\gamma_n - \gamma_{n+1}| < \infty$. Then $\{x_n\}$ converges strongly to a point $\bar{u}$ in $\mathrm{Sol(SFP)}$, where $\bar{u} = \mathrm{Proj}_{\mathrm{Sol(SFP)}}u$.

Remark 2.3. Fixed point methods are investigated to split feasibility problem (1.1). A strong convergence theorem of common solutions are established in the framework of infinite dimensional Hilbert spaces without any compact assumption. It is of interest to improve the results presented in this article to the framework of Banach spaces.

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