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Extinction in a nonautonomous system of Volterra integrodifferential equations

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Abstract

A nonautonomous system of Volterra integrodifferential equations is studied in this paper. It is shown that if the coefficients are continuous, bounded above and below by positive constants and satisfy certain inequalities, then one of the components will be driven to extinction while the other one will stabilize at the certain positive solution of a nonlinear single species model.

Keywords: Extinction, nonautonomous, Volterra integrodifferential equation, global attractivity.
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1. Introduction

An important and ubiquitous problem in predator-prey theory and related topics in mathematical ecology concerns the long-term coexistence of species. In the past few years, permanence and extinction of different types of ecosystems have been studied wildly both in theories and applications; see, for example, [9–11, 14].

Consider the nonautonomous Lotka-Volterra system of differential equations

$$x_i'(t) = x_i(t)\left[b_i(t) - \sum_{j=1}^{n} a_{ij}(t)x_j(t)\right], \quad i = 1, 2, \cdots, n, \quad n \geqslant 2, \tag{1.1}$$

where $x_i(t)$ is population density of the i-th species at time t; $a_{ij}(t)$ and $b_i(t)$, $i, j = 1, 2, \cdots, n$, are continuous bounded functions defined on R.

Given a function $f(t)$ which is defined on R, set

$$f^u = \sup\{f(t) | t \in R\}, \qquad f^l = \inf\{f(t) | t \in R\}.$$

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Assume that

$$a_{ij}^l > 0, \quad a_{ij}^u < +\infty, \quad i, j = 1, 2, \cdots, n, \tag{1.2}$$

$$b_i^l > 0, \quad b_i^u < +\infty, \quad i = 1, 2, \cdots, n, \tag{1.3}$$

i.e., the coefficients of system (1.1) are bounded above and below by strictly positive reals.

Montes de Oca and Zeeman [13] studied system (1.1), under which the functions $a_{ij}(t)$ and $b_i(t)$ were assumed to satisfy conditions (1.2) and (1.3). It was shown that if for each $k > 1$, there exists $i_k < k$ such that for any $j \leqslant k$ the inequality

$$\frac{b_k^u}{a_{kj}^l} < \frac{b_{i_k}^l}{a_{i_k j}^u}$$

holds, then every solution $(x_1(t), x_2(t), \cdots, x_n(t))^T$ of system (1.1) with $x_i(t_0) > 0$, $i = 1, 2, \cdots, n$, for some $t_0 \in R$ has the property

$$\lim_{t \to +\infty} (x_1(t) - u_1^*(t)) = 0, \quad \lim_{t \to +\infty} x_j(t) = 0, \quad j = 2, 3, \cdots, n,$$

where $u_1^*(t)$ is the unique solution of the logistic differential equation

$$u'(t) = u(t)[b_1(t) - a_{11}(t)],$$

which is bounded above and below by strictly positive reals for all $t \in R$.

Montes de Oca and Zeeman [12], Zeeman [18], Ahmad [1, 2], Teng [15] and Zhao et al. [19] have also studied the extinction of species in system (1.1), especially in [19], Zhao et al. obtained the same results as [1, 12, 13, 18] did under the weaker assumption that for each $k > 1$, there exists $i_k < k$ such that for any $j \leqslant k$ the inequality

$$\sup_{t \in [t_0, +\infty)} \frac{b_k(t)}{b_{i_k}(t)} < \inf_{t \in [t_0, +\infty)} \frac{a_{kj}(t)}{a_{i_k j}(t)}$$

holds for some $t_0 \in R$.

For when the growth rates have averages, and the interaction coefficients are constants, Ahmad and Lazer [3–5] have given sufficient conditions involving the averages of the growth rates for one species to be extinct in system (1.1). The work of Tineo [16] has complemented that in [3] concerning the extinction of one species or persistence of the rest of the species.

However, in the real world, the growth rate of a natural species population will often not respond immediately to changes in its own population or that of an interacting species, but rather will do so after a time lag. Research has shown that time delays have a great destabilizing influence on species populations. In [17], Wang studied the permanence of an autonomous two species competitive system with infinite delays, but the extinction has not been considered. Besides, system in the nature world, the nonautonomous case is more realistic. The main goal of this paper is to study the extinction of a competitive system with infinite delays which is modified by [17, system (14)].

Motivated by the above, in the present paper, we shall study the following nonautonomous system of Volterra integrodifferential equations

$$\begin{aligned} x_1'(t) &= x_1(t)\left[r_1(t) - a_1(t)\int_0^{+\infty} k_1(s)x_1(t-s)ds - b_1(t)\int_0^{+\infty} k_2(s)x_2(t-s)ds\right], \\ x_2'(t) &= x_2(t)\left[r_2(t) - a_2(t)\int_0^{+\infty} k_1(s)x_1(t-s)ds - b_2(t)\int_0^{+\infty} k_2(s)x_2(t-s)ds\right], \end{aligned} \tag{1.4}$$

where $x_1(t), x_2(t)$ are population density of species x_1 and x_2 at time t, respectively; $r_i(t)$, $a_i(t)$, $b_i(t)$, $i = 1, 2$ are continuous, positive and bounded functions; $k_i : [0, +\infty) \to [0, +\infty)$, $i = 1, 2$ are piecewise continuous and integrable on $[0, +\infty)$ with $\int_0^{+\infty} k_i(s)ds = 1$.

The initial conditions of (1.4) are of the form

$$x_i(s) = \varphi_i(s) > 0, \quad s \in (-\infty, 0], \quad \varphi_i(0) > 0, \tag{1.5}$$

where φ_i, $i = 1, 2$ are bounded and continuous functions on $(-\infty, 0]$.

2. Basic results

In this section, we shall develop some preliminary results, which will be used to prove the main result.

Lemma 2.1 ([6]). *If $a > 0, b > 0$, and $x' \geqslant x(b - ax)$, when $t \geqslant 0$ and $x(0) > 0$, then*

$$\liminf_{t\to+\infty} x(t) \geqslant \frac{b}{a}.$$

If $a > 0, b > 0$, and $x' \leqslant x(b - ax)$, when $t \geqslant 0$ and $x(0) > 0$, then

$$\limsup_{t\to+\infty} x(t) \leqslant \frac{b}{a}.$$

Lemma 2.2. *Assume that $x(t) = (x_1(t), x_2(t))^T$ is any positive solution of system (1.4) with initial condition (1.5), then there exists a positive constant M_0 such that*

$$\limsup_{t\to+\infty} x_i(t) \leqslant M_0, \quad i = 1, 2,$$

i.e., any positive solution of system (1.4) are ultimately bounded above by a positive constant.

Proof. The positivity of the solution $x(t) = (x_1(t), x_2(t))^T$ of (1.4) and (1.5) for $t > 0$ is immediate from the form of (1.4) and the assumptions on the initial values.

From the first equation of system (1.4) and by the positivity of $x_1(t)$ and (1.4), we have

$$\begin{aligned} x_1'(t) &\leqslant x_1(t)\left[r_1^u - a_1^l \int_0^{+\infty} k_1(s)x_1(t-s)ds\right] & (2.1)\\ &\leqslant r_1^u x_1(t), \quad t > 0. & (2.2)\end{aligned}$$

For $t - s > 0$, it follows from (2.2) that

$$\frac{dx_1(t)}{x_1(t)} \leqslant r_1^u x_1(t). \tag{2.3}$$

Integrating (2.3) on $[t - s, t]$, we derive

$$x_1(t) \leqslant x_1(t-s)e^{r_1^u s},$$

which leads to

$$x_1(t-s) \geqslant x_1(t)e^{-r_1^u s}. \tag{2.4}$$

It follows from (2.1) and (2.4) that

$$x_1'(t) \leqslant x_1(t)\left[r_1^u - a_1^l\left(\int_0^{+\infty} k_1(s)e^{-r_1^u s}ds\right)x_1(t)\right].$$

By Lemma 2.1, we have

$$\limsup_{t\to+\infty} x_1(t) \leqslant \frac{r_1^u}{a_1^l \int_0^{+\infty} k_1(s)e^{-r_1^u s}ds} := M_1.$$

Similarly to the above analysis, from the second equation of system (1.4), we have

$$\limsup_{t\to+\infty} x_2(t) \leqslant \frac{r_2^u}{a_2^l \int_0^{+\infty} k_2(s)e^{-r_2^u s}ds} := M_2.$$

Set $M_0 = \max\{M_1, M_2\}$, then the conclusion of Lemma 2.2 follows. This completes the proof. □

Consider the following logistic equations

$$x'(t) = x(t)\left[r_1(t) - a_1(t)\int_0^{+\infty} k_1(s)x(t-s)ds\right], \tag{2.5}$$

and

$$x'(t) = x(t)[r_1(t) - a_1(t)x(t)]. \tag{2.6}$$

Lemma 2.3 ([8])**.** *Assume that $r_1(t)$, $a_1(t)$ are strictly positive, bounded, and continuous on* R*. Furthermore, suppose that $\int_0^{+\infty} s^2k_1(s)ds < +\infty$ and $(a_1^u)^2M_1K < a^l$, where $K = \int_0^{+\infty} sk_1(s)ds < +\infty$. Let $x_1(t)$, $x_2(t)$ be any two positive solutions of* (2.5)*, then $\lim\limits_{t\to+\infty}[x_1(t) - x_2(t)] = 0$.*

Lemma 2.4 ([1, 7])**.** *Assume that $r_1(t)$, $a_1(t)$ are strictly positive, bounded, and continuous on* R*. The logistic equation* (2.6) *has a unique solution $x^*(t)$ such that $\delta_1 \leqslant x^*(t) \leqslant \delta_2$, where δ_1, δ_2 are numbers satisfying $0 < \delta_1 < \frac{r_1^l}{a_1^u}$ and $\frac{r_1^u}{a_1^l} < \delta_2$. If $x(t)$ is a solution of equation* (2.6) *with $x(0) > 0$, then $\lim\limits_{t\to+\infty}[x(t) - x^*(t)] = 0$.*

3. Main results

In this section, we study the extinction of all species but x_1 of system (1.4).

Theorem 3.1. *Assume that the inequality*

$$\limsup_{t\to+\infty} \frac{r_2(t)}{r_1(t)} \leqslant \liminf_{t\to+\infty}\left\{\frac{a_2(t)}{a_1(t)}, \frac{b_2(t)}{b_1(t)}\right\} \tag{3.1}$$

holds, then the species x_2 will be driven to extinction, that is, for any positive solution $(x_1(t), x_2(t))^T$ of system (1.4)*, $x_2(t) \to 0$ exponentially as $t \to +\infty$, and $\int_0^{+\infty} x_2(t)dt < +\infty$.*

Proof. Let $x(t) = (x_1(t), x_2(t))^T$ be a solution of system (1.4) with $x_i(0) > 0$, $i = 1, 2$. First we show that $x_2(t) \to 0$ exponentially as $t \to +\infty$.

From (1.4) we have

$$\begin{aligned}
\frac{x_1'(t)}{x_1(t)} &= r_1(t) - a_1(t)\int_0^{+\infty} k_1(s)x_1(t-s)ds - b_1(t)\int_0^{+\infty} k_2(s)x_2(t-s)ds,\\
\frac{x_2'(t)}{x_2(t)} &= r_2(t) - a_2(t)\int_0^{+\infty} k_1(s)x_1(t-s)ds - b_2(t)\int_0^{+\infty} k_2(s)x_2(t-s)ds.
\end{aligned} \tag{3.2}$$

By (3.1), we can choose $\alpha, \beta, \varepsilon > 0$ such that

$$\limsup_{t\to+\infty} \frac{r_2(t)}{r_1(t)} < \frac{\alpha}{\beta} - \varepsilon < \frac{\alpha}{\beta} < \liminf_{t\to+\infty}\left\{\frac{a_2(t)}{a_1(t)}, \frac{b_2(t)}{b_1(t)}\right\}.$$

And so, there exists a $T_1 > 0$ such that

$$r_2(t)\beta - r_1(t)\alpha < -\varepsilon\beta r_1(t) < -\varepsilon\beta r_1^l < 0, \tag{3.3}$$

$$\alpha a_1(t) - \beta a_2(t) < 0, \tag{3.4}$$

$$\alpha b_1(t) - \beta b_2(t) < 0 \tag{3.5}$$

for all $t > T_1$.

Let

$$V(t) = x_1^{-\alpha}(t)x_2^{\beta}(t). \tag{3.6}$$

From (3.2), (3.3), (3.4), (3.5), it follows that

$$\begin{aligned} V'(t) &= V(t)\Big[\beta r_2(t) - \alpha r_1(t) + (\alpha a_1(t) - \beta a_2(t))\int_0^{+\infty} k_1(s)x_1(t-s)ds \\ &\quad + (\alpha b_1(t) - \beta b_2(t))\int_0^{+\infty} k_2(s)x_2(t-s)ds\Big] \\ &\leqslant V(t)\Big[\beta r_2(t) - \alpha r_1(t)\Big] \\ &< -\varepsilon\beta r_1^l V(t). \end{aligned} \tag{3.7}$$

Integrating inequality (3.7) from T_1 to t ($\geqslant T_1$), then

$$V(t) < V(T_1)\exp\{-\varepsilon\beta r_1^l V(t-T_1)\}. \tag{3.8}$$

By Lemma 2.2 we know that there exists $\xi > 0$ such that

$$x_i(t) < \xi, \quad i = 1,2, \quad t > T_1.$$

Together with (3.6) and (3.8), then

$$x_2(t) < C\exp\{-\varepsilon r_1^l(t-T_1)\},$$

where

$$C = \xi^{\frac{\alpha}{\beta}}(x_1(T_1))^{-\frac{\alpha}{\beta}}x_2(T_1).$$

Therefore, we have $x_2(t) \to 0$ exponentially as $t \to +\infty$. This completes the proof. □

Lemma 3.2. *Under the assumption of Theorem 3.1, and*

$$\int_0^{+\infty} k_1(s)e^{-[r^l - a_1^u M_1]s}ds < +\infty. \tag{3.9}$$

Let $x(t) = (x_1(t), x_2(t))^T$ be any positive solution of system (1.4) with initial condition (1.5), then there exists a positive constant m_0 such that

$$\liminf_{t\to+\infty} x_1(t) \geqslant m_0,$$

where m_0 is a constant independent of any positive solution of system (1.4), i.e., the first species $x_1(t)$ of system (1.4) is permanent.

Proof. Let $x(t) = (x_1(t), x_2(t))^T$ be a solution of system (1.4) with initial condition (1.5). Let ε be an enough small positive constant. For this ε, it follows from Lemma 2.2 and Theorem 3.1 that there exists a $T_2 > T_1$ such that

$$x_1(t) < M_1 + \varepsilon, \quad x_2(t) < \varepsilon, \quad t > T_2. \tag{3.10}$$

From the first equation of system (1.4), for $t > T_2$,

$$\begin{aligned} x_1'(t) &\geqslant x_1(t)\Big[r_1^l - a_1^u\int_0^{+\infty} k_1(s)x_1(t-s)ds - b_1^u\int_0^{+\infty} k_2(s)x_2(t-s)ds\Big] \\ &= x_1(t)\Big[r_1^l - a_1^u\int_0^{t-T_2} k_1(s)x_1(t-s)ds \\ &\quad - a_1^u\int_{t-T_2}^{+\infty} k_1(s)x_1(t-s)ds - b_1^u\int_0^{+\infty} k_2(s)x_2(t-s)ds\Big], \end{aligned} \tag{3.11}$$

which together with (3.10) implies

$$x_1'(t) > x_1(t)\Big[r_1^l - b_1^u\varepsilon - a_1^u(M_1+\varepsilon)\int_0^{t-T_2} k_1(s)ds$$

$$- a_1^u \int_{t-T_2}^{+\infty} k_1(s)x_1(t-s)ds\Big].$$

Let $\eta(t)$ be defined by

$$\eta(t) = r_1^l - b_1^u\varepsilon - a_1^u(M_1+\varepsilon)\int_0^{t-T_2} k_1(s)ds$$
$$- a_1^u \int_{t-T_2}^{+\infty} k_1(s)x_1(t-s)ds.$$

Then, by the boundedness of $x_1(t)$ and the property of the kernel $k_1(s)$,

$$\lim_{t\to+\infty} \eta(t) = r_1^l - b_1^u\varepsilon - a_1^u(M_1+\varepsilon), \tag{3.12}$$

and also from (3.11),

$$x_1'(t) > \eta(t)x_1(t), \quad t > T_2,$$

which leads to

$$x_1(t-s) < x_1(t)e^{-\int_{t-s}^{t}\eta(r)dr}, \quad t-s > T_2. \tag{3.13}$$

It follows from (3.11) and (3.13) that

$$\begin{aligned} x_1'(t) > x_1(t)\Big[r_1^l - b_1^u\varepsilon - a_1^u\Big(\int_0^{t-T_2} k_1(s)e^{-\int_{t-s}^{t}\eta(r)dr}ds\Big)x_1(t) \\ - a_1^u \int_{t-T_2}^{+\infty} k_1(s)x_1(t-s)ds\Big]. \end{aligned} \tag{3.14}$$

Noting that,

$$\lim_{t\to+\infty} a_1^u \int_{t-T_2}^{+\infty} k_1(s)x_1(t-s)ds = 0,$$

for the above $\varepsilon > 0$, there exists a $T_3 > T_2$ large enough such that

$$a_1^u \int_{t-T_2}^{+\infty} k_1(s)x_1(t-s)ds < \varepsilon, \tag{3.15}$$

also from (3.12), we have

$$r_1^l - b_1^u\varepsilon - a_1^u(M_1+\varepsilon) - \varepsilon < \eta(t) < r_1^l - b_1^u\varepsilon - a_1^u(M_1+\varepsilon) + \varepsilon, \quad t > T_3. \tag{3.16}$$

By (3.16), for $t > T_3$,

$$a_1^u \int_0^{t-T_2} k_1(s)e^{-\int_{t-s}^{t}\eta(r)dr}ds < a_1^u \int_0^{+\infty} k_1(s)e^{-[r_1^l - b_1^u\varepsilon - a_1^u(M_1+\varepsilon)-\varepsilon]s}ds.$$

From (3.14), (3.15), (3.16), we derive that, for $t > T_3$,

$$x_1'(t) > x_1(t)\Big[r_1^l - b_1^u\varepsilon - a_1^u\Big(\int_0^{+\infty} k_1(s)e^{-[r_1^l - b_1^u\varepsilon - a_1^u(M_1+\varepsilon)-\varepsilon]s}\Big)x_1(t) - \varepsilon\Big].$$

Let $\varepsilon \to 0$, then

$$x_1'(t) \geqslant x_1(t)\Big[r_1^l - a_1^u\Big(\int_0^{+\infty} k_1(s)e^{-[r_1^l - a_1^u M_1]s}\Big)x_1(t)\Big].$$

By Lemma 2.1 and (3.9),

$$\liminf_{t\to+\infty} x_1(t) \geqslant \frac{r_1^l}{a_1^u\int_0^{+\infty} k_1(s)e^{-[r_1^l - a_1^u M_1]s}ds} := m_0.$$

This completes the proof. □

Theorem 3.3. *Under the assumptions of Theorem 3.1, Lemmas 2.3 and 3.2, let* $x(t) = (x_1(t), x_2(t))^T$ *be any positive solution of system (1.4) with initial condition (1.5), then the species* x_2 *will be driven to extinction, that is,* $x_2(t) \to 0$ *as* $t \to +\infty$, *and* $x_1(t) \to x^*(t)$ *as* $t \to +\infty$, *where* $x^*(t)$ *is any positive solution of equation (2.5).*

Proof. Let $x(t) = (x_1(t), x_2(t))^T$ be a solution of system (1.4) with $x_i(0) > 0$, $i = 1, 2$. From Lemmas 2.2 and 3.2, $x_1(t)$ is bounded above and below by positive constants on $[0, +\infty)$. To finish the proof of Theorem 3.3, it is enough to show that $x_1(t) \to x^*(t)$ as $t \to +\infty$, where $x^*(t)$ is any positive solution of equation (2.5).

Since

$$x_1'(t) \leqslant x_1(t)\left[r_1(t) - a_1(t)\int_0^{+\infty} k_1(s)x_1(t-s)ds\right],$$

then $x_1(t) \leqslant x^*(t)$ for all $t \geqslant 0$, where $x^*(t)$ is any positive solution of equation (2.5) with $x(0) = x_1(0)$. Clearly, $x^*(t)$ is bounded above and below by positive constants on $[0, +\infty)$.

Define a function $V(t)$ on $[0, +\infty)$ as

$$V(t) = -\ln\left(\frac{x_1(t)}{x^*(t)}\right) + b_1(t)\int_0^{+\infty}\left(\int_{t-\theta}^{t} k_2(\theta)x_2(s)ds\right)d\theta.$$

Calculating the derivative of $V(t)$ along the solution $x_1(t)$ and $x^*(t)$, it follows that

$$\begin{aligned} V'(t) &= -\left(\frac{x_1'(t)}{x_1(t)} - \frac{x^{*\prime}(t)}{x^*(t)}\right) + b_1(t)x_2(t) + b_1(t)\int_0^{+\infty} k_2(\theta)x_2(t-\theta)d\theta \\ &= -a_1(t)\int_0^{+\infty} k_1(s)[x^*(t-s) - x_1(t-s)]ds + b_1(t)x_2(t) \\ &\leqslant -a_1^l\int_0^{+\infty} k_1(s)[x^*(t-s) - x_1(t-s)]ds + b_1^u x_2(t). \end{aligned}$$

The above equality implies that

$$\frac{d}{dt}\left(-V(t) - a_1^l\int_0^t\int_0^{+\infty} k_1(s)[x^*(\theta-s) - x_1(\theta-s)]dsd\theta + b_1^u\int_0^t x_2(s)ds\right) \geqslant 0,$$

then,

$$-V(t) - a_1^l\int_0^t\int_0^{+\infty} k_1(s)[x^*(\theta-s) - x_1(\theta-s)]dsd\theta + b_1^u\int_0^t x_2(s)ds \geqslant V(0),$$

that is,

$$a_1^l\int_0^t\int_0^{+\infty} k_1(s)[x^*(\theta-s) + x_1(\theta-s)]dsd\theta \leqslant V(0) - V(t) + b_1^u\int_0^t x_2(s)ds.$$

Since $V(t)$ is bounded and $\int_0^{+\infty} x_2(t)dt < +\infty$, then

$$0 < \int_0^{+\infty}\int_0^{+\infty} k_1(s)[x^*(\theta-s) - x_1(\theta-s)]dsd\theta < \zeta < +\infty,$$

where $\zeta = V(0) + b_1^u\int_0^{+\infty} x_2(s)ds$.

On the other hand, $x^*(t) - x_1(t)$ is a nonnegative, bounded and differential function such that $x^{*\prime}(t) - x_1'(t)$ is bounded on $[0, +\infty)$. Hence, by the Mean Valued Theorem, $x^{*\prime}(t) - x_1'(t)$ is uniformly continuous on $[0, +\infty)$. Therefore,

$$\lim_{t\to+\infty}(x^*(t) - x_1(t)) = 0.$$

This completes the proof. $\square$

Theorem 3.4. *Under the assumptions of Theorem 3.1, Lemmas 2.4 and 3.2, let* $x(t) = (x_1(t), x_2(t))^T$ *be any positive solution of system (1.4) with initial condition (1.5), then the species* x_2 *will be driven to extinction, that is,* $x_2(t) \to 0$ *as* $t \to +\infty$, *and* $x_1(t) \to x^*(t)$ *as* $t \to +\infty$, *where* $x^*(t)$ *is any positive solution of (2.6).*

Proof. The logistic equation (2.6)

$$x'(t) = x(t)[r_1(t) - a_1(t)x(t)],$$

can be written as

$$x'(t) = x(t)\left[r_1(t) - a_1(t)\int_0^{+\infty} k_1(s)x(t)ds\right].$$

The following proof is similar to that of Theorem 3.3, we omit it here. This completes the proof. □

4. Numerical example and simulations

In this section, we give an example to illustrate the feasibility of our results with the two kernels

$$k_1(s) = \gamma_1 e^{-\gamma_1 s}, \qquad k_2(s) = \gamma_2^2 s e^{-\gamma_2 s},$$

where γ_1, γ_2 are positive constants. The simulations are based on the technique of converting the scalar integrodifferential equations into a system of ordinary differential (difference) equations and then numerically solving them using Matlab and its built in graphical output routine.

Consider the following equations

$$\begin{aligned} x_1'(t) &= x_1(t)\left[(4 + \sin\frac{\pi}{2}t) - \int_0^{+\infty} \gamma_1 e^{-\gamma_1 s} x_1(t-s)ds \right. \\ &\quad \left. - (2 - \cos\pi t)\int_0^{+\infty} \gamma_2^2 s e^{-\gamma_2 s} x_2(t-s)ds\right], \\ x_2'(t) &= x_2(t)\left[\frac{1}{2} - (2 - \sin t)\int_0^{+\infty} \gamma_1 e^{-\gamma_1 s} x_1(t-s)ds \right. \\ &\quad \left. - \int_0^{+\infty} \gamma_2^2 s e^{-\gamma_2 s} x_2(t-s)ds\right], \end{aligned} \tag{4.1}$$

where

$$\begin{aligned} &r_1(t) = 4 + \sin\frac{\pi}{2}t; a_1(t) = 1; b_1(t) = 2 - \cos\pi t, \\ &r_2(t) = \frac{1}{2}; a_2(t) = 2 - \sin t; b_2(t) = 1. \end{aligned}$$

By simple computation, one could see that

$$\limsup_{t\to+\infty} \frac{r_2(t)}{r_1(t)} = \frac{1}{6}, \ \liminf_{t\to+\infty} \left\{\frac{a_2(t)}{a_1(t)}, \frac{b_2(t)}{b_1(t)}\right\} = \frac{1}{3},$$

that is, the condition of Theorem 3.1 holds.

Let $\gamma_1 = 10$, $\gamma_2 = 5$, the conditions in Lemmas 2.3 and 3.2 also hold.

From Theorem 3.3, species x_2 will be driven to extinction while species x_1 is asymptotically to any positive solution of

$$y'(t) = y(t)\left[(4 + \sin\frac{\pi}{2}t) - \int_0^{+\infty} \gamma_1 e^{-\gamma_1 s} y(t-s)ds\right].$$

The integrodifferential equations (4.1) can be converted into a system of ordinary differential equations by the introduction of auxiliary variables u, v, w, where

$$u(t) = \int_0^{+\infty} \gamma_1 e^{-\gamma_1 s} x_1(t-s)ds = \gamma_1 \int_{-\infty}^{t} e^{-\gamma_1(t-s)} x_1(s)ds,$$

$$v(t) = \int_0^{+\infty} \gamma_2^2 s e^{-\gamma_2 s} x_2(t-s)ds = \gamma_2^2 \int_{-\infty}^{t} (t-s)e^{-\gamma_2(t-s)} x_2(s)ds,$$
$$w(t) = \int_0^{+\infty} \gamma_2 e^{-\gamma_2 s} x_2(t-s)ds = \gamma_2 \int_{-\infty}^{t} e^{-\gamma_2(t-s)} x_2(s)ds,$$

then (4.1) can be transformed into

$$\begin{aligned} x_1'(t) &= x_1(t)\left[(4+\sin\frac{\pi}{2}t) - u(t) - (2-\cos\pi t)v(t)\right],\\ x_2'(t) &= x_2(t)\left[\frac{1}{2} - (2-\sin t)u(t)\right],\\ u'(t) &= -\gamma_1[u(t) - x_1(t)],\\ v'(t) &= \gamma_2[w(t) - v(t)],\\ w'(t) &= \gamma_2[x_2(t) - w(t)]. \end{aligned}$$

The solutions of (4.1) corresponding to $\gamma_1 = 10$, $\gamma_2 = 5$ and three initial values are displayed in Figures 1 and 2.

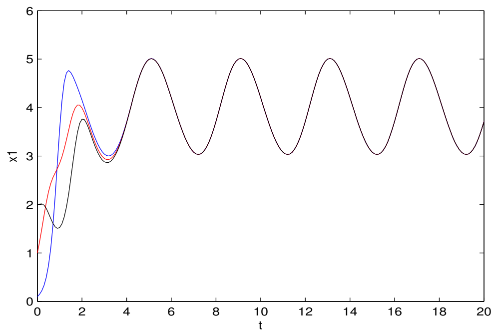

Figure 1: Dynamic behaviors of species x_1 in (4.1) with $\gamma_1 = 10, \gamma_2 = 5$ and the initial values $(x(0), y(0)) = (0.1, 0.1), (1, 1), (2, 2)$.

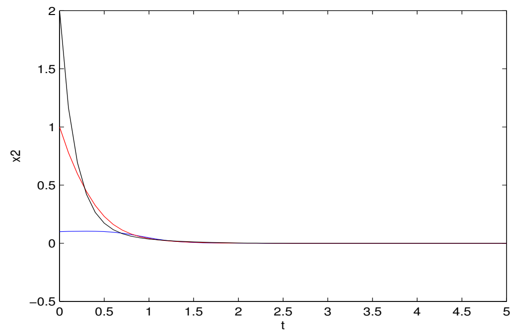

Figure 2: Dynamic behaviors of species x_2 in (4.1) with $\gamma_1 = 10, \gamma_2 = 5$ and the initial values $(x(0), y(0)) = (0.1, 0.1), (1, 1), (2, 2)$.

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