Available online at www.isr-publications.com/jnsa
J. Nonlinear Sci. Appl., 10 (2017), 5763–5769
Research Article

ISSN: 2008-1898

Journal of Nonlinear Sciences and Applications

Journal Homepage: www.tjnsa.com - www.isr-publications.com/jnsa

Fixed point theorems in fuzzy cone metric spaces

Saif Ur Rehman, Hong-Xu Li*

Department of Mathematics, Sichuan University, Chengdu, Sichuan 610064, P. R. China.

Communicated by C. Zaharia

Abstract

In this paper we prove some fixed point theorems in fuzzy cone metric spaces under some fuzzy cone contractive type conditions. Our results generalize the "fuzzy cone Banach contraction theorem" given by [T. Öner, M. B. Kandemire, B. Tanay, J. Nonlinear Sci. Appl., **8** (2015), 610–616] recently.

Keywords: Fixed point, fuzzy cone metric space, contraction condition.
2010 MSC: 47H10, 54H25.

1. Introduction

Huang and Zhang [7] generalized the concept of metric space by replacing the real numbers with an ordered Banach space, and proved some fixed point results for nonlinear mappings satisfying some contraction conditions. After that lots of works were devoted to the problems on cone metric spaces (see, e.g., [1, 3, 8, 11–15, 21, 22] and references therein).

The fuzzy set theory was introduced by Zadeh [23]. In [10] Kramosil and Michalek introduced the fuzzy metric spaces which perform the probabilistic metric spaces approach to the fuzzy setting. Later on, George and Veeramani in [4] gave a stronger form of metric fuzziness. Some fixed point results for set-valued mappings on fuzzy metric spaces can be found in [5, 6, 9, 19].

In 2015, Oner et al. [17] introduced the notation of fuzzy cone metric space which generalized the notation of fuzzy metric space by George and Veeramani. They also presented some structural properties of fuzzy cone metric spaces and proved a fixed point theorem under a fuzzy cone contraction condition. Some fixed point theorems and common fixed point theorems concerning fuzzy cone metric spaces were obtained in [2, 18], and some more properties for fuzzy cone metric spaces can be found in [15, 16].

The purpose of this paper is to give further study of fixed point theory in fuzzy cone metric spaces. Some fixed point theorems are proved under some fuzzy cone contractive type conditions. Our results generalize the "fuzzy cone Banach contraction theorem" given in [17] (see Remark 3.5 and Example 3.6).

2. Preliminaries

Definition 2.1 ([20]). A binary operation $*: [0,1] \times [0,1] \to [0,1]$ is called a continuous t-norm if it satisfies the following conditions:

*Corresponding author
Email addresses: saif.urrehman27@yahoo.com (Saif Ur Rehman), hoxuli@scu.edu.cn (Hong-Xu Li)

doi:10.22436/jnsa.010.11.14

Received 2017-02-24

(1) $*$ is commutative and associative;
(2) $*$ is continuous;
(3) $1 * a = a$ for all $a \in [0, 1]$;
(4) $a * b \leqslant c * d$, whenever $a \leqslant c$ and $b \leqslant d$, for each $a, b, c, d \in [0, 1]$.

The basic three continuous t-norms are (see [20]):

(i) the minimum t-norm defined by $a * b = \min\{a, b\}$;
(ii) the product t-norm defined by $a * b = ab$;
(iii) the Lukasiewicz t-norm defined by $a * b = \max\{a + b - 1, 0\}$.

Definition 2.2 ([4])**.** A three-tuple $(X, M, *)$ is said to be a fuzzy metric space if X is an arbitrary set, $*$ is a continuous t-norm, and M is a fuzzy set on $X^2 \times (0, \infty)$ satisfying the following conditions:

(fm1) $M(x, y, t) > 0$ and $M(x, y, t) = 1$ iff $x = y$;
(fm2) $M(x, y, t) = M(y, x, t)$;
(fm3) $M(x, y, t) * M(y, z, s) \leqslant M(x, z, t + s)$;
(fm4) $M(x, y, \cdot) : (0, \infty) \to [0, 1]$ is continuous

for all $x, y, z \in X$ and $t, s > 0$.

Throughout this paper $\mathbb{E}$ denotes a real Banach space and θ denotes the zero of $\mathbb{E}$, and $\mathbb{N}$ denotes the set of natural numbers.

Definition 2.3 ([7])**.** A subset P of $\mathbb{E}$ is called a cone if

1) P is closed, nonempty, and $P \neq \{\theta\}$;
2) if $a, b \in [0, \infty)$ and $x, y \in P$, then $ax + by \in P$;
3) if both $x \in P$ and $-x \in P$, then $x = \theta$.

For a given cone $P \subset \mathbb{E}$, a partial ordering $\preceq$ on $\mathbb{E}$ via P is defined by $x \preceq y$ if and only if $y - x \in P$. $x \prec y$ stands for $x \preceq y$ and $x \neq y$, while $x \ll y$ stands for $y - x \in \text{int}(P)$. Throughout this paper, we assume that all cones has nonempty interior.

Definition 2.4 ([17])**.** A three-tuple $(X, M, *)$ is said to be a fuzzy cone metric space if P is a cone of $\mathbb{E}$, X is an arbitrary set, $*$ is a continuous t-norm, and M is a fuzzy set on $X^2 \times \text{int}(P)$ satisfying the following conditions:

(fcm1) $M(x, y, t) > 0$ and $M(x, y, t) = 1$ iff $x = y$;
(fcm2) $M(x, y, t) = M(y, x, t)$;
(fcm3) $M(x, y, t) * M(y, z, s) \leqslant M(x, z, t + s)$;
(fcm4) $M(x, y, .) : \text{int}(P) \to [0, 1]$ is continuous

for all $x, y, z \in X$ and $t, s \in \text{int}(P)$.

Remark 2.5. If we take $E = \mathbb{R}, P = [0, \infty)$ and $a * b = ab$, then every fuzzy metric space becomes a fuzzy cone metric space. For some more examples of fuzzy cone metric spaces we refer the readers to [17].

Definition 2.6 ([17])**.** Let $(X, M, *)$ be a fuzzy cone metric space, $x \in X$ and (x_n) be a sequence in X. Then

(i) (x_n) is said to converge to x if for $t \gg \theta$ and $r \in (0, 1)$ there exists $n_1 \in \mathbb{N}$ such that $M(x_n, x, t) > 1 - r$ for all $n \geqslant n_1$. We denote this by $\lim_{n \to \infty} x_n = x$ or $x_n \to x$ as $n \to \infty$;
(ii) (x_n) is said to be a Cauchy sequence if for $r \in (0, 1)$ and $t \gg \theta$ there exists $n_1 \in \mathbb{N}$ such that $M(x_m, x_n, t) > 1 - r$ for all $m, n \geqslant n_1$;
(iii) $(X, M, *)$ is said to be a complete cone metric space if every Cauchy sequence is convergent in X;

(iv) (x_n) is said to be fuzzy cone contractive if there exists $r \in (0,1)$ such that

$$\frac{1}{M(x_n, x_{n+1}, t)} - 1 \leqslant r\left(\frac{1}{M(x_{n-1}, x_n, t)} - 1\right)$$

for all $t \gg \theta$, $n \geqslant 1$.

Definition 2.7. *Let $(X, M, *)$ be a fuzzy cone metric space. The fuzzy cone metric M is triangular if*

$$\frac{1}{M(x,z,t)} - 1 \leqslant \left(\frac{1}{M(x,y,t)} - 1\right) + \left(\frac{1}{M(y,z,t)} - 1\right)$$

for all $x, y, z \in X$ and each $t \gg \theta$.

We recall some properties of fuzzy cone metric spaces from [17].

Lemma 2.8 ([17])**.** *Let $(X, M, *)$ be a fuzzy cone metric space. The following statements hold.*

(i) *Let $x \in X$ and (x_n) be a sequence in X. (x_n) converges to x if and only if $M(x_n, x, t) \to 1$ as $n \to \infty$ for each $t \gg \theta$.*

(ii) *For $t \gg \theta$, the open ball $B(x, r, t)$ with center x and radius $r \in (0,1)$ is defined by $B(x,r,t) = \{y \in X : M(x,y,t) > 1 - r\}$. Let*

$$\tau_{fc} = \{A \subset X : x \in A \text{ if and only if there exist } r \in (0,1) \text{ and } t \gg \theta \text{ such that } B(x,r,t) \subset A\},$$

then (X, τ_{fc}) is a Hausdorff topology space.

Remark 2.9. It is easy to see that the convergence of a sequence in a fuzzy cone metric space is in the sense of the Hausdorff topology τ_{fc} given in Lemma 2.8 (ii). So the limit of a convergent sequence in a fuzzy cone metric space is unique.

Definition 2.10 ([17])**.** Let $(X, M, *)$ be a fuzzy cone metric space and $T : X \to X$. Then T is said to be fuzzy cone contractive if there exists $a \in (0,1)$ such that

$$\frac{1}{M(Tx, Ty, t)} - 1 \leqslant a\left(\frac{1}{M(x,y,t)} - 1\right) \tag{2.1}$$

for each $x, y \in X$ and $t \gg \theta$. a is called the contraction constant of T.

The following "fuzzy cone Banach contraction theorem" is obtained in [17].

Theorem 2.11. *Let $(X, M, *)$ be a complete fuzzy cone metric space in which fuzzy cone contractive sequences are Cauchy and $T : X \to X$ be a fuzzy cone contractive mapping. Then T has a unique fixed point.*

As a further study of the fuzzy cone metric space $(X, M, *)$, we prove some fixed point theorems for a mapping $T : X \to X$ under the following fuzzy cone contractive type conditions:

$$\begin{aligned}\frac{1}{M(Tx, Ty, t)} - 1 \leqslant\, & a\left(\frac{1}{M(x,y,t)} - 1\right) + b\left(\frac{1}{M(x,Tx,t)} - 1\right) \\ & + c\left(\frac{1}{M(y,Ty,t)} - 1\right) + d\left(\frac{1}{M(y,Tx,t)} - 1\right),\end{aligned} \tag{2.2}$$

and symmetrically

$$\begin{aligned}\frac{1}{M(Tx, Ty, t)} - 1 \leqslant\, & a\left(\frac{1}{M(x,y,t)} - 1\right) + b\left(\frac{1}{M(x,Tx,t)} - 1\right) \\ & + c\left(\frac{1}{M(y,Ty,t)} - 1\right) + d\left(\frac{1}{M(x,Ty,t)} - 1\right),\end{aligned} \tag{2.3}$$

where $t \gg \theta$, $a, b, c, d \in [0, +\infty)$. We note that (2.2) or (2.3) is the same as (2.1) if $a \in (0, 1)$ and $b = c = d = 0$. On the other hand, T may not be a fuzzy cone contractive mapping if it satisfies (2.2) or (2.3). This can be seen in the example at the end of this paper. Therefore our results generalize properly Theorem 2.11 (see Remark 3.5 and Example 3.6).

We only study the cases with condition (2.2) since the cases with condition (2.3) can be done symmetrically.

3. Main result

We are now in a position to give our first main result.

Theorem 3.1. *Assume that $(X, M, *)$ is a complete fuzzy cone metric space in which M is triangular and $T : X \to X$ satisfies (2.2) with $a + b + c < 1$. Then T has a fixed point in X. If, in addition, $a + d < 1$, the fixed point of T is unique.*

Proof. Fix $x_0 \in X$ and construct a sequence (x_n) by $x_{n+1} = Tx_n, n \geqslant 0$. Then by (2.2), for $t \gg \theta, n \geqslant 1$,

$$\begin{aligned}\frac{1}{M(x_n, x_{n+1}, t)} - 1 &= \frac{1}{M(Tx_{n-1}, Tx_n, t)} - 1\\ &\leqslant a\left(\frac{1}{M(x_{n-1}, x_n, t)} - 1\right) + b\left(\frac{1}{M(x_{n-1}, Tx_{n-1}, t)} - 1\right) + c\left(\frac{1}{M(x_n, Tx_n, t)} - 1\right)\\ &= a\left(\frac{1}{M(x_{n-1}, x_n, t)} - 1\right) + b\left(\frac{1}{M(x_{n-1}, x_n, t)} - 1\right) + c\left(\frac{1}{M(x_n, x_{n+1}, t)} - 1\right).\end{aligned}$$

Then

$$\frac{1}{M(x_n, x_{n+1}, t)} - 1 \leqslant \kappa\left(\frac{1}{M(x_{n-1}, x_n, t)} - 1\right),$$

where $\kappa = \dfrac{a+b}{1-c} < 1$ since $a + b + c < 1$. This implies

$$\frac{1}{M(x_n, x_{n+1}, t)} - 1 \leqslant \kappa\left(\frac{1}{M(x_{n-1}, x_n, t)} - 1\right) \leqslant \cdots \leqslant \kappa^n\left(\frac{1}{M(x_0, x_1, t)} - 1\right),$$

which means that (x_n) is a fuzzy cone contractive sequence, and we get

$$\lim_{n\to\infty} M(x_n, x_{n+1}, t) = 1 \quad \text{for } t \gg \theta. \tag{3.1}$$

Noticing that M is triangular, then for all $m > n \geqslant n_0$,

$$\begin{aligned}\frac{1}{M(x_n, x_m, t)} - 1 &\leqslant \left(\frac{1}{M(x_n, x_{n+1}, t)} - 1\right) + \left(\frac{1}{M(x_{n+1}, x_{n+2}, t)} - 1\right) + \cdots + \left(\frac{1}{M(x_{m-1}, x_m, t)} - 1\right)\\ &\leqslant (\kappa^n + \kappa^{n+1} + \cdots + \kappa^{m-1})\left(\frac{1}{M(x_0, x_1, t)} - 1\right)\\ &\leqslant \frac{\kappa^n}{1-\kappa}\left(\frac{1}{M(x_0, x_1, t)} - 1\right)\\ &\to 0 \quad \text{as } n \to \infty,\end{aligned}$$

which yields that (x_n) is a Cauchy sequence in X. Since $(X, M, *)$ is complete, there exists $\nu \in X$ such that

$$\lim_{n\to\infty} M(x_n, \nu, t) = 1 \quad \text{for } t \gg \theta. \tag{3.2}$$

Since M is triangular, we have

$$\frac{1}{M(v,Tv,t)}-1\leqslant\left(\frac{1}{M(v,x_{n+1},t)}-1\right)+\left(\frac{1}{M(x_{n+1},Tv,t)}-1\right)\quad\text{for } t\gg\theta. \tag{3.3}$$

By (2.2), (3.1), and (3.2), for $t\gg\theta$,

$$\begin{aligned}&\frac{1}{M(x_{n+1},Tv,t)}-1\\&=\frac{1}{M(Tx_n,Tv,t)}-1\\&\leqslant a\left(\frac{1}{M(x_n,v,t)}-1\right)+b\left(\frac{1}{M(x_n,Tx_n,t)}-1\right)+c\left(\frac{1}{M(v,Tv,t)}-1\right)+d\left(\frac{1}{M(v,Tx_n,t)}-1\right)\\&=a\left(\frac{1}{M(x_n,v,t)}-1\right)+b\left(\frac{1}{M(x_n,x_{n+1},t)}-1\right)+c\left(\frac{1}{M(v,Tv,t)}-1\right)+d\left(\frac{1}{M(v,x_{n+1},t)}-1\right)\\&\to c\left(\frac{1}{M(v,Tv,t)}-1\right)\quad\text{as } n\to\infty.\end{aligned}$$

Then

$$\limsup_{n\to\infty}\left(\frac{1}{M(x_{n+1},Tv,t)}-1\right)\leqslant c\left(\frac{1}{M(v,Tv,t)}-1\right)\quad\text{for } t\gg\theta.$$

This together with (3.2) and (3.3) implies

$$\frac{1}{M(v,Tv,t)}-1\leqslant c\left(\frac{1}{M(v,Tv,t)}-1\right)\quad\text{for } t\gg\theta.$$

Noticing that $c<1$ since $a+b+c<1$, then $M(v,Tv,t)=1$, that is, $Tv=v$.

Now we prove the uniqueness. Let $u\in X$ be any fixed point of T in X. Then by (2.2), for every $t\gg\theta$,

$$\begin{aligned}&\frac{1}{M(u,v,t)}-1\\&=\frac{1}{M(Tu,Tv,t)}-1\\&\leqslant a\left(\frac{1}{M(u,v,t)}-1\right)+b\left(\frac{1}{M(u,Tu,t)}-1\right)+c\left(\frac{1}{M(v,Tv,t)}-1\right)+d\left(\frac{1}{M(v,Tu,t)}-1\right)\\&=a\left(\frac{1}{M(u,v,t)}-1\right)+b\left(\frac{1}{M(u,u,t)}-1\right)+c\left(\frac{1}{M(v,v,t)}-1\right)+d\left(\frac{1}{M(v,u,t)}-1\right)\\&=(a+d)\left(\frac{1}{M(u,v,t)}-1\right).\end{aligned}$$

Since $a+d<1$, we get $M(u,v,t)=1$ and $u=v$. That is T has a unique fixed point. □

The following corollary follows from Theorem 3.1 directly.

Corollary 3.2. *Assume that $(X,M,*)$ is a complete fuzzy cone metric space in which M is triangular and $T:X\to X$ satisfies (2.2) with $a+b+c+d<1$. Then T has a unique fixed point in X.*

By the proof of Theorem 3.1, it is easy to see that the condition "M is triangular" implies that "fuzzy cone contractive sequences are Cauchy". The next theorem shows that, if $c=0$ in (2.2), the condition "M is triangular" in Theorem 3.1 can be weakened to "fuzzy cone contractive sequences are Cauchy". In this case, condition (2.2) becomes

$$\frac{1}{M(Tx,Ty,t)}-1\leqslant a\left(\frac{1}{M(x,y,t)}-1\right)+b\left(\frac{1}{M(x,Tx,t)}-1\right)+d\left(\frac{1}{M(y,Tx,t)}-1\right). \tag{3.4}$$

Theorem 3.3. *Assume that* $(X, M, *)$ *is a complete fuzzy cone metric space in which fuzzy cone contractive sequences are Cauchy and* $T : X \to X$ *satisfies* (3.4) *with* $a + b < 1$. *Then* T *has a fixed point in* X. *If, in addition,* $a + d < 1$, *the fixed point of* T *is unique.*

Proof. Fix $x_0 \in X$ and construct a sequence (x_n) by $x_{n+1} = Tx_n, n \geqslant 0$. Replacing condition (2.2) by (3.4), the same arguments of the proof of Theorem 3.1 follow that (x_n) is a fuzzy cone contractive sequence and (3.1) holds. Then (x_n) is a Cauchy sequence. Since $(X, M, *)$ is complete, (3.2) also holds for some $\nu \in X$. By (3.1), (3.2), and (3.4),

$$\begin{aligned}\frac{1}{M(x_{n+1}, T\nu, t)} - 1 = &\frac{1}{M(Tx_n, T\nu, t)} - 1\\ \leqslant &a\left(\frac{1}{M(x_n, \nu, t)} - 1\right) + b\left(\frac{1}{M(x_n, Tx_n, t)} - 1\right) + d\left(\frac{1}{M(\nu, Tx_n, t)} - 1\right)\\ = &a\left(\frac{1}{M(x_n, \nu, t)} - 1\right) + b\left(\frac{1}{M(x_n, x_{n+1}, t)} - 1\right) + d\left(\frac{1}{M(\nu, x_{n+1}, t)} - 1\right)\\ \to &0 \quad \text{as } n \to \infty\end{aligned}$$

for $t \gg \theta$. Then

$$\lim_{n\to\infty} M(x_{n+1}, T\nu, t) = 1 \quad \text{for } t \gg \theta. \tag{3.5}$$

By Definition 2.4 (fcm3),

$$M(\nu, x_{n+1}, t) * M(x_{n+1}, T\nu, t) \leqslant M(\nu, T\nu, 2t) \quad \text{for } t \gg \theta. \tag{3.6}$$

Now it follows from (3.2), (3.5), (3.6), and the definition of $*$ that

$$1 = 1 * 1 = \lim_{n\to\infty} (M(\nu, x_{n+1}, t) * M(x_{n+1}, T\nu, t)) \leqslant M(\nu, T\nu, 2t)$$

for $t \gg \theta$. Therefore, $M(\nu, T\nu, 2t) = 1$ for $t \gg \theta$, and $T\nu = \nu$. The proof for the uniqueness of fixed point of T is the same as that of Theorem 3.1. □

By Theorem 3.3, we can get the following corollary immediately.

Corollary 3.4. *Assume that* $(X, M, *)$ *is a complete fuzzy cone metric space in which fuzzy cone contractive sequences are Cauchy and* $T : X \to X$ *satisfies* (3.4) *with* $a + b + d < 1$. *Then* T *has a unique fixed point in* X.

Remark 3.5. Theorem 3.3 and Corollary 3.4 include [17, Theorem 3.3] (i.e., Theorem 2.11). In fact, if $a \in (0, 1)$ and $b = d = 0$, the three results are the same.

Let us close this work by an example, where an existence and uniqueness of fixed point result is gotten by applying Theorem 3.1 or 3.3. However, [17, Theorem 3.3] is not applicable since the mapping is not fuzzy cone contractive.

Example 3.6. Let $X = [0, \infty)$, $*$ be a continuous t-norm, and $M : X^2 \times (0, \infty) \to [0, 1]$ be defined as

$$M(x, y, t) = \frac{t}{t + |x - y|}$$

for $x, y \in X$ and $t > 0$. Then it is easy to verify that M is triangular and $(X, M, *)$ is a complete fuzzy cone metric space. Define a mapping $T : X \to X$ as

$$Tx = \begin{cases} \frac{5}{4}x + 3, & x \in [0, 1],\\ \frac{3}{4}x + \frac{7}{2}, & x \in [1, \infty). \end{cases}$$

T is not fuzzy cone contractive since

$$\frac{1}{M(Tx,Ty,t)}-1=\frac{5}{4}\left(\frac{1}{M(x,y,t)}-1\right)$$

for $x,y\in[0,1], t>0$. So Theorem 2.11 is not applicable. But it is easy to verify that all conditions of Theorems 3.1 and 3.3 are satisfied with $a=1/4, c=0, b=d=2/3$. Therefore T has a unique fixed point in $[0,\infty)$. In fact, the unique fixed point of T is 14.

Acknowledgment

The authors are grateful to the referees for bringing to their attention Refs. [2, 15, 16, 18] and other helpful suggestions.

This work is supported by National Natural Science Foundation (NNSF) of China (Grant No. 11471227, 11561077).

References

[1] M. Abbas, M. Ali Khan, S. Radenoviç, *Common coupled fixed point theorems in cone metric spaces for w-compatible mappings*, Appl. Math. Comput., **217** (2010), 195–202. 1

[2] A. M. Ali, G. R. Kanna, *Intuitionistic fuzzy cone metric spaces and fixed point theorems*, Internat. J. Math. Appl., **5** (2017), 25–36. 1, 3

[3] I. Altun, B. Damjanoviç, D. Djoriç, *Fixed point and common fixed point theorems on ordered cone metric spaces*, Appl. Math. Lett., **23** (2010), 310–316. 1

[4] A. George, P. Veeramani, *On some results in fuzzy metric spaces*, Fuzzy Sets and Systems, **64** (1994), 395–399. 1, 2.2

[5] M. Grabiec, *Fixed point in fuzzy metric spaces*, Fuzzy Sets and Systems, **27** (1988), 385–389. 1

[6] O. Hadžić, E. Pap, *Fixed point theorem for multivalued mappings in probabilistic metric spaces and an applications in fuzzy metric spaces*, Fuzzy Sets and Systems, **127** (2002), 333–344. 1

[7] L.-G. Huang, X. Zhang, *Cone metric spaces and fixed point theorems of contractive mappings*, J. Math. Anal. Appl., **332** (2007), 1468–1476. 1, 2.3

[8] S. Janković, Z. Kadelburg, S. Radenović, *On cone metric spaces: a survey*, Nonlinear Anal., **74** (2011), 2591–2601. 1

[9] F. Kiani, A. Amini-Haradi, *Fixed point and endpoint theorems for set-valued fuzzy contraction mapps in fuzzy metric spaces*, Fixed Point Theory Appl., **2011** (2011), 9 pages. 1

[10] I. Kramosil, J. Michálek, *Fuzzy metric and statistical metric spaces*, Kybernetika, **11** (1975), 336–344. 1

[11] A. Latif, N. Hussain, J. Ahmad, *Coincidence points for hybrid contractions in cone metric spaces*, J. Nonlinear Convex Anal., **17** (2016), 899–906. 1

[12] Z.-L. Li, S.-J. Jiang, *Quasi-contractions restricted with linear bounded mappings in cone metric spaces*, Fixed Point Theory Appl., **2014** (2014), 10 pages.

[13] N. Mehmood, A. Azam, L. D. R. Kočinac, *Multivalued fixed point results in cone metric spaces*, Topology Appl., **179** (2015), 156–170.

[14] B. H. Nguyen, D. T. Tran, *Fixed point theorems and the Ulam-Hyers stability in non-Archimedean cone metric spaces*, J. Math. Anal. Appl., **414** (2014), 10–20.

[15] T. Öner, *On some results in fuzzy cone metric spaces*, Int. J. Adv. Comput. Eng. Network., **4** (2016), 37–39. 1, 3

[16] T. Öner, *On the metrizability of fuzzy cone metric spaces*, Int. j. Eng. Technol. Manag. Appl. Sci., **2** (2016), 133–135. 1, 3

[17] T. Öner, M. B. Kandemire, B. Tanay, *Fuzzy cone meric spaces*, J. Nonlinear Sci. Appl., **8** (2015), 610–616. 1, 2.4, 2.5, 2.6, 2, 2.8, 2.10, 2, 3.5, 3

[18] N. Priyobarta, Y. Rohen, B. B. Upadhyay, *Some fixed point results in fuzzy cone metric spaces*, Int. J. Pure Appl. Math., **109** (2016), 573–582. 1, 3

[19] Z. Sadeghi, S. M. Vaezpour, C. Park, R. Saadati, C. Vetro, *Set-valued mappings in partially ordered fuzzy metric spaces*, J. Inequal. Appl., **2014** (2014), 17 pages. 1

[20] B. Schweizer, A. Sklar, *Statical metric spaces*, Pac. J. Math., **10** (1960), 314–334. 2.1, 2

[21] W. Shatanawi, E. Karapınar, H. Aydi, *Coupled coincidence points in partially ordered cone metric spaces with a c-distance*, J. Appl. Math., **2012** (2012), 15 pages. 1

[22] D. Turkoglu, M. Abuloha, *Cone metric spaces and fixed point theorems in diametrically contractive mappings*, Acta Math. Sin., **26** (2010), 489–496. 1

[23] L. A. Zadeh, *Fuzzy sets*, Information and Control, **8** (1965), 338–353. 1