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Split equality problem for κ-asymptotically strictly pseudo-nonspreading mapping in Hilbert space

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Abstract

In this paper, we consider the split equality problem (SEP) in Hilbert space. We propose and investigate a new iterative algorithm for solving split equality problem for κ-asymptotically strictly pseudo-nonspreading mapping. Finally, a numerical example is given to illustrate the feasibility of the proposed algorithm.

Keywords: General split equality problem, κ-asymptotically strictly pseudo-nonspreading mapping.
2010 MSC: 47J25, 47H09, 65J25.

1. Introduction

In the present paper, we are concerned with the split equality problem (SEP). SEP was proposed by Moudafi in [8]. Let H_1, H_2 and H_3 be three real Hilbert spaces. Let $T_i : H_1 \to H_1$ be a κ-asymptotically strictly pseudo-nonspreading mapping. Denote by $\text{Fix}(T_i)$ the set of fixed points of T_i $(i = 1, 2, \cdots, m)$. Set $C = \bigcap_{i=1}^m \text{Fix}(T_i)$. Let $A : H_1 \to H_3$ and $B : H_2 \to H_3$ be two bounded linear operators. Let Q be a nonempty closed convex subset of H_2.

The so-called SEP can mathematically be formulated as

$$\text{finding } x \in C,\ y \in Q \text{ such that } Ax = By. \tag{1.1}$$

We use Γ to denote the solution set of SEP, that is

$$\Gamma = \{(x, y) \in H_1 \times H_2,\ Ax = By,\ x \in C,\ y \in Q\}.$$

When $B = I$ (the identity mapping on Hilbert space H), problem (1.1) is equivalent to the well-known split feasibility problem (SFP).

As we have known, the SEP has received much attention due to its application in various disciplines such as medical image reconstruction and radiation therapy treatment planning [3, 4].

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To solve the SEP, Moudafi [7, 8] put forward an alternating CQ-algorithm and the relaxed alternating CQ-algorithm. In this paper, inspired by Chang [9], we propose and investigate a new iterative algorithm for solving split equality problem for κ-asymptotically strictly pseudo-nonspreading mapping and show the convergence of the presented algorithm. At last we give a numerical example for SEP in R^2.

2. Preliminaries

We recall some definitions, notations and conclusions which will be used in proving our main result. Let H be a real Hilbert space with inner product $\langle\cdot,\cdot\rangle$ and the norm $\|\cdot\|$. We write $x_n \to x$ (respectively, $x_n \rightharpoonup x$), the strong (respectively weak) convergence of the sequence $\{x_n\}$ to x.

Let E be a Banach space. A mapping T with domain $D(T)$ in E is said to be demi-closed, if for any sequence $x_n \subset E$, $x_n \rightharpoonup x^* \in D(T)$ and $\| x_n - Tx_n \| \to 0$, then $Tx^* = x^*$.

A Banach space E is said to have the Opial property, if for any sequence $\{x_n\}$ with $x_n \rightharpoonup x^*$, we have

$$\liminf_{n\to\infty} \|x_n - x^*\| < \liminf_{n\to\infty} \|x_n - y\|$$

for all $y \in E$ with $y \neq x^*$.

Remark 2.1. It is known that each Hilbert space possesses the Opial property.

Definition 2.2. Let H be a Hilbert space and K be a nonempty closed convex subset of H. We denote by $\mathrm{Fix}(T)$ the fixed points set of T. T is said to be

(i) Nonexpasive, if $\|Tx - Ty\| \leqslant \|x - y\|$ for all $x, y \in K$.

(ii) κ-strictly pseudo-nonspreading [2], if there exists $\kappa \in [0, 1)$ such that

$$\|Tx - Ty\|^2 \leqslant \|x - y\|^2 + \kappa\|x - Tx - (y - Ty)\|^2 + 2\langle x - Tx, y - Ty\rangle$$

for all $x, y \in D(T)$.

(iii) κ-asymptotically strictly pseudo-contraction [5], if there exists a constant $\kappa \in [0, 1)$ and a sequence $k_n \geqslant 1$ and $\lim_{n\to\infty} k_n = 1$ such that

$$\|T^n x - T^n y\|^2 \leqslant k_n\|x - y\|^2 + \kappa\|x - T^n x - (y - T^n y)\|^2$$

for all $x, y \in D(T)$.

(iv) κ-asymptotically strictly pseudo-nonspreading [9], if there exists a constant $\kappa \in [0, 1)$ and a sequence $k_n \geqslant 1$ and $\lim_{n\to\infty} k_n = 1$, such that

$$\|T^n x - T^n y\|^2 \leqslant k_n\|x - y\|^2 + \kappa\|x - T^n x - (y - T^n y)\|^2 + 2\langle x - T^n x, y - T^n y\rangle$$

for all $x, y \in D(T)$.

Remark 2.3. κ-asymptotically strictly pseudo-nonspreading is much more general than κ-strictly pseudo-nonspreading and κ-asymptotically strictly pseudo-contraction.

For every point $x \in H$, there exists a unique point $P_K x \in K$, such that

$$\| x - P_K x \| \leqslant \| x - y \|, \ \forall y \in K,$$

where P_K is called the metric projection of H onto K. We know that P_K is a nonexpansive mapping of H onto K.

Lemma 2.4 ([9]). *Let K be a nonempty closed convex subset of a real Hilbert space H, and let $T : K \to K$ be a continuous κ-asymptotically strictly pseudo-nonspreading. If $\mathrm{Fix}(T)$ is nonempty, then it is a closed and convex subset.*

Lemma 2.5 ([1]). *Let* $\{a_n\}_{n=0}^{\infty}$, $\{b_n\}_{n=0}^{\infty}$ *be sequences of nonnegative numbers satisfying* $a_{n+1} \leqslant a_n + b_n$, *for all* $n \geqslant 0$. *If* $\Sigma_{n=0}^{\infty} b_n < \infty$, *then* $\lim_{n\to\infty} a_n$ *exists.*

Lemma 2.6 ([6]). *We use* Γ *to denote the solution set of SEP, that is*

$$\Gamma = \{(x, y) \in H_1 \times H_2,\ Ax = By,\ x \in C,\ y \in Q\},$$

and assume consistency of SEP, so that Γ *is nonempty closed convex.*

Let $S = C \times Q$, C and Q be two nonempty closed convex subsets of real Hilbert spaces H_1 and H_2, define

$$G = [A \quad -B],$$

$G : H \to H_3$, then

$$G^*G = \begin{bmatrix} A^*A & -A^*B \\ -B^*A & B^*B \end{bmatrix}.$$

The original problem can be reformulated as

$$\text{finding} \quad w = \begin{bmatrix} x \\ y \end{bmatrix} \quad \text{with} \quad Gw = 0.$$

Then, $w = \begin{bmatrix} x \\ y \end{bmatrix}$ solves the SEP if and only if w solves the fixed point equation

$$P_S(w - \gamma G^*Gw) = w.$$

Lemma 2.7 ([10]). *Let* $J = I - \gamma G^*G$, *where* $0 < \gamma < 2/\rho(G^*G)$, $\rho(G^*G)$ *being the spectral radius of the self-adjoint operator* G^*G *on* H. *Then we have the following property:*

$$\mathrm{Fix}(J) = \{(x, y) \in H,\ Ax = By\},$$
$$\mathrm{Fix}(P_S J) = \mathrm{Fix}(P_S) \bigcap \mathrm{Fix}(J) = \Gamma.$$

3. Main result

Theorem 3.1. *Let* H_1 *and* H_2 *be two real Hilbert spaces. Let* $T_i : H_1 \to H_1$ *be a* κ*-asymptotically strictly pseudo-nonspreading mapping. Let* $C = \bigcap_{i=1}^{m} \mathrm{Fix}(T_i)$ *and* Q *be any nonempty closed convex set of* H_2. *Let* $S = C \times Q$ *and* P_S *be the metric projection of* $H = H_1 \times H_2$ *to S. Let* $\{w_n\}$ *be generated by*

$$\begin{cases} w_1 \in H, \\ u_n = (I - \gamma G^*G)w_n, \\ w_{n+1} = P_S[(1-\alpha_n)u_n + \alpha_n T_{[n]}^n u_n], \end{cases} \tag{3.1}$$

where $[n] = n$ *mod* m, $0 < \gamma < \lambda = 2/\rho(G^*G)$, *and* $\rho(G^*G)$ *being the spectral radius of the self-adjoint operator* G^*G, $\{\alpha_n\}$ *is a sequence in* $(0, 1-\kappa]$, *and* $\sum_{n=0}^{\infty} \alpha_n < \infty$ *and we remember* T_i *as* $T_i \oplus I$. *If* Γ *is nonempty, then the sequence* w_n *converges weakly to a point* $w \in \Gamma$.

Proof. The proof is divided into five steps.

(1) We first prove the $\lim_{n\to\infty} \| w_n - p \|$ exists, for any $p \in \Gamma$.
Since $p \in \Gamma$, we have $p \in \bigcap_{i=1}^{m} \mathrm{Fix}(T_i)$ and $p \in \mathrm{Fix}(P_S J)$. It follows from (3.1) that

$$\begin{aligned} \|w_{n+1} - p\|^2 &= \|P_S[(1-\alpha_n)u_n + \alpha_n T_{[n]}^n u_n] - P_S p\|^2 \\ &\leqslant \|u_n - p + \alpha_n(T_{[n]}^n u_n - u_n)\|^2 \\ &= \|u_n - p\|^2 + 2\alpha_n\langle u_n - p, T_{[n]}^n u_n - u_n\rangle + \alpha_n^2\|T_{[n]}^n u_n - u_n\|^2. \end{aligned} \tag{3.2}$$

Because T_i is a κ-asymptotically strictly pseudo-nonspreading for any $v \in H$, we have

$$\|T^n_{[n]}u_n - T^n_{[n]}v\|^2 \leqslant k_n\|u_n - v\|^2 + 2\langle T^n_{[n]}u_n - u_n, v - T^n_{[n]}v\rangle + \kappa\|T^n_{[n]}u_n - u_n - (v - T^n_{[n]}v)\|^2.$$

Taking $v = p$, we have

$$\|T^n_{[n]}u_n - p\|^2 \leqslant k_n\|u_n - p\|^2 + \kappa\|T^n_{[n]}u_n - u_n\|^2.$$

Observe that

$$\begin{aligned}\|T^n_{[n]}u_n - p\|^2 &= \|T^n_{[n]}u_n - u_n + u_n - p\|^2\\ &= \|T^n_{[n]}u_n - u_n\|^2 + 2\langle T^n_{[n]}u_n - u_n, u_n - p\rangle + \|u_n - p\|^2\\ &\leqslant k_n\|u_n - p\|^2 + \kappa\|T^n_{[n]}u_n - u_n\|^2.\end{aligned}$$

Simplify the above inequality, we have

$$2\alpha_n\langle T^n_{[n]}u_n - u_n, u_n - p\rangle \leqslant \alpha_n(\kappa - 1)\|T^n_{[n]}u_n - u_n\|^2 + \alpha_n(k_n - 1)\|u_n - p\|^2. \tag{3.3}$$

It follows from (3.2) and (3.3) that

$$\begin{aligned}\|w_{n+1} - p\|^2 &\leqslant (\alpha_n(k_n - 1) + 1)\|u_n - p\|^2 + \alpha_n^2\|T^n_{[n]}u_n - u_n\|^2 + \alpha_n(k - 1)\|T^n_{[n]}u_n - u_n\|^2\\ &= (\alpha_n(k_n - 1) + 1)\|u_n - p\|^2 - \alpha_n(1 - \kappa - \alpha_n)\|T^n_{[n]}u_n - u_n\|^2.\end{aligned} \tag{3.4}$$

On the other hand,

$$\begin{aligned}\|u_n - p\|^2 &= \|(I - \gamma G^*G)w_n - p\|^2\\ &= \|w_n - p - \gamma G^*Gw_n\|^2\\ &= \|w_n - p\|^2 - 2\gamma\langle w_n - p, G^*Gw_n\rangle + \gamma^2\|G^*Gw_n\|^2\\ &= \|w_n - p\|^2 - 2\gamma\langle Gw_n - Gp, Gw_n\rangle + \gamma^2\langle G^*Gw_n, G^*Gw_n\rangle\\ &= \|w_n - p\|^2 - 2\gamma\langle Gw_n - Gp, Gw_n\rangle + \gamma^2\langle Gw_n, GG^*Gw_n\rangle\\ &\leqslant \|w_n - p\|^2 - 2\gamma\|Gw_n\|^2 + (2\gamma^2/\lambda)\|Gw_n\|^2\\ &= \|w_n - p\|^2 - \gamma(2 - 2\gamma/\lambda)\|Gw_n\|^2.\end{aligned} \tag{3.5}$$

It follows from (3.4) and (3.5) that

$$\begin{aligned}\|w_{n+1} - p\|^2 &\leqslant (\alpha_n(k_n - 1) + 1)\|w_n - p\|^2 - \gamma(\alpha_n(k_n - 1) + 1)(2 - 2\gamma/\lambda)\|Gw_n\|^2\\ &\quad - \alpha_n(1 - \kappa - \alpha_n)\|T^n_{[n]}u_n - u_n\|^2\\ &\leqslant (\alpha_n(k_n - 1) + 1)\|w_n - p\|^2.\end{aligned} \tag{3.6}$$

Put $x_n = \|w_n - p\|^2$ and $\beta_n = \alpha_n(k_n - 1)$, then (3.6) is equivalent to

$$x_{n+1} \leqslant (\beta_n + 1)x_n = x_n + \beta_n x_n. \tag{3.7}$$

By (3.7), we know

$$\begin{aligned}x_{n+1} &\leqslant (1 + \beta_n)x_n\\ &\leqslant (1 + \beta_n)(1 + \beta_{n-1})\cdots(1 + \beta_0)x_0\\ &= e^{\ln(1+\beta_n)+\ln(1+\beta_{n-1})+\cdots+\ln(1+\beta_0)}x_0\\ &\leqslant e^{\beta_n+\beta_{n-1}+\cdots+\beta_0}x_0\\ &\leqslant e^{\sum_{n=0}^{\infty}\beta_n}x_0\\ &< \infty,\end{aligned}$$

which implies that x_n is bounded. Since $\sum_{n=0}^{\infty}\alpha_n < \infty$, then $\sum_{n=0}^{\infty}\beta_n < \infty$. So $\sum_{n=0}^{\infty}\beta_n x_n < \infty$. By Lemma 2.5, we know $\lim_{n\to\infty} x_n$ exists. It also shows that the $\lim_{n\to\infty} \| w_n - p \|$ exists for any $p \in \Gamma$.

(2) We show that the $\lim_{n\to\infty} \| u_n - p \|$ exists for any $p \in \Gamma$.

By (3.6), we have

$$\gamma(\alpha_n(k_n-1)+1)(2-2\gamma/\lambda)\|Gw_n\|^2+\alpha_n(1-\kappa-\alpha_n)\|T^n_{[n]}u_n-u_n\|^2 \\ \leqslant (\alpha_n(k_n-1)+1)\|w_n-p\|^2-\|w_{n+1}-p\|^2,$$

which implies that

$$\lim_{n\to\infty}\|Gw_n\|=0, \tag{3.8}$$

and

$$\lim_{n\to\infty}\alpha_n\|T^n_{[n]}u_n-u_n\|=0. \tag{3.9}$$

On one hand, by (3.2), we know

$$\|w_{n+1}-p\|^2\leqslant\|u_n-p\|^2+2\alpha_n\|u_n-p\|\|T^n_{[n]}u_n-u_n\|+\alpha_n^2\|T^n_{[n]}u_n-u_n\|^2.$$

According to (3.9), we obtain

$$\|w_{n+1}-p\|\leqslant\|u_n-p\|. \tag{3.10}$$

On the other hand, by (3.5) and (3.8) we have

$$\|u_n-p\|\leqslant\|w_n-p\|. \tag{3.11}$$

From (3.10) and (3.11), we get

$$\lim_{n\to\infty}\|w_n-p\|=\lim_{n\to\infty}\|u_n-p\|.$$

(3) We prove that

$$\lim_{n\to\infty}\|w_{n+1}-w_n\|=0,$$

and

$$\lim_{n\to\infty}\|u_{n+1}-u_n\|=0.$$

In fact, it follows from (3.1) that

$$\begin{aligned}\|w_{n+1}-w_n\|^2&=\|P_S[(1-\alpha_n)u_n+\alpha_nT^n_{[n]}u_n]-w_n\|^2\\&=\|P_S[(1-\alpha_n)u_n+\alpha_nT^n_{[n]}u_n]-P_Sw_n\|^2\\&\leqslant\|\alpha_n(T^n_{[n]}u_n-u_n)+(u_n-w_n)\|^2\\&\leqslant 2\alpha_n^2\|T^n_{[n]}u_n-u_n\|^2+2\|u_n-w_n\|^2\\&=2\alpha_n^2\|T^n_{[n]}u_n-u_n\|^2+2\gamma^2\|G^*Gw_n\|^2\\&=2\alpha_n^2\|T^n_{[n]}u_n-u_n\|^2+2\gamma^2\langle G^*Gw_n,G^*Gw_n\rangle\\&=2\alpha_n^2\|T^n_{[n]}u_n-u_n\|^2+2\gamma^2\langle Gw_n,GG^*Gw_n\rangle\\&\leqslant 2\alpha_n^2\|T^n_{[n]}u_n-u_n\|^2+(4\gamma^2/\lambda)\|Gw_n\|^2\\&=2(\alpha_n\|T^n_{[n]}u_n-u_n\|)^2+(4\gamma^2/\lambda)\|Gw_n\|^2.\end{aligned}$$

This together with (3.8) and (3.9) imply that

$$\lim_{n\to\infty}\|w_{n+1}-w_n\|=0. \tag{3.12}$$

Similarly, it follows from (3.1) and (3.12) that

$$\begin{aligned}\|u_{n+1}-u_n\|^2&=\|(I-\gamma G^*G)w_{n+1}-(I-\gamma G^*G)w_n\|^2\\&\leqslant 2\|w_{n+1}-w_n\|^2+2\|\gamma G^*G(w_{n+1}-w_n)\|^2\\&\leqslant 2\|w_{n+1}-w_n\|^2+(8\gamma^2/\lambda^2)\|w_{n+1}-w_n\|^2\\&=2(1+4\gamma^2/\lambda^2)\|w_{n+1}-w_n\|^2\\&\to 0\ \ (\text{as}\ \ n\to\infty).\end{aligned}$$

(4) In step (1), we have known $\{w_n\}$ is a bounded sequence. Let w^* be a weak cluster point of $\{w_n\}$, there exists a sequence $\{w_{n_k}\}$ such that $\{w_{n_k}\}$ converges weakly to w^*, then $Gw^* = 0$. It follows that $w^* \in \mathrm{Fix}(I - \gamma G^* G)$. On the other hand, when $n \geqslant 2$, $w_n = P_S w_n$, as w_{n_k} converges weakly to w^*, we obtain that $P_S w^* = w^*$, that is to say $w^* \in \mathrm{Fix}(P_S)$.

Therefore $w^* \in \mathrm{Fix}(P_S) \bigcap \mathrm{Fix}(I - \gamma G^* G)$.

(5) Next we prove that $w_n \rightharpoonup w^*$.

By contradiction, without loss of generality, we assume that there exists another subsequence $w_{n_l} \subset w_n$.

Case 1. If w_{n_l} is a convergent sequence, then $w_{n_l} \rightharpoonup w^{**}$, with $w^* \neq w^{**}$. Otherwise, Case 2, the sequence w_{n_l} has at least two subsequences not convergent to the same point. We assume $w_{n_{l_k}} \rightharpoonup w_*$, with $w^* \neq w_*$. Consequently, by the existence of $\lim_{n\to\infty} \| w_n - p \|$ and the Opial property, in Case 1, we have

$$\begin{aligned}
\liminf_{n_i\to\infty} \|w_{n_i} - w^*\| &< \liminf_{n_i\to\infty} \|w_{n_i} - w^{**}\| \\
&= \liminf_{n\to\infty} \|w_n - w^{**}\| \\
&= \liminf_{n_j\to\infty} \|w_{n_j} - w^{**}\| \\
&< \liminf_{n_j\to\infty} \|w_{n_j} - w^*\| \\
&= \liminf_{n\to\infty} \|w_n - w^*\| \\
&= \liminf_{n_i\to\infty} \|w_{n_i} - w^*\|.
\end{aligned}$$

This is a contradiction, then $w^* = w^{**}$. In Case 2, the same method as above, we have $w^* = w_*$. Therefore $w_n \rightharpoonup w^*$. This completes the proof. □

4. Numerical example for SEP in R^2

In this section, we consider split equality problem for some special cases of κ-asymptotically strictly pseudo-contraction mapping in Definition 2.2, $H_1 = H_2 = H_3 = R^2$.

$T_i,\ i = 1,2$ are two κ-asymptotically strictly pseudo-contraction mappings $(\kappa(i) = 0)$,

$$T_1 = \begin{bmatrix} \sqrt{2}/2 & \sqrt{2}/2 \\ -\sqrt{2}/2 & \sqrt{2}/2 \end{bmatrix}, \quad T_2 = \begin{bmatrix} \sqrt{3}/2 & 1/2 \\ -1/2 & \sqrt{3}/2 \end{bmatrix},$$

$$\|T_i x - T_i y \|^2 \leqslant \|x - y \|^2 + \kappa_i \|x - T_i^n x - (y - T_i^n y)\|^2,$$

for all $x, y \in D(T)$. Here $k_n \equiv 1,\ \kappa = 0,\ i = 1,2$.

$C = \bigcap_{i=1}^2 \mathrm{Fix}(T_i),\ Q = [-1,1;-1,1],\ S = C \times Q,\ H = H_1 \times H_2$, we put $T_i \oplus I,\ i = 1,2$ still marked as $T_i,\ i = 1,2$,

$$T_1 = \begin{bmatrix} \sqrt{2}/2 & \sqrt{2}/2 & 0 & 0 \\ -\sqrt{2}/2 & \sqrt{2}/2 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ & & 0 & 1 \end{bmatrix}, \quad T_2 = \begin{bmatrix} \sqrt{3}/2 & 1/2 & 0 & 0 \\ -1/2 & \sqrt{3}/2 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}.$$

Let

$$A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}, \quad B = \begin{bmatrix} 2 & 3 \\ 1 & 4 \end{bmatrix}.$$

Let $P_s\{x_1, x_2, y_1, y_2\} = \{0, 0, y_1, y_2\}$, $\{y_1, y_2\} \in Q$, $\{w_n\}$ be generated by

$$w_1 = \{0, 0, 0.5, 0.5\} \in H, \quad u_n = (I - \gamma G^* G) w_n w_{n+1} = P_S[(1-\alpha_n) u_n + \alpha_n T_{[n]}^n u_n].$$

Here $[n] = n \bmod 2$,

$$G = \begin{bmatrix} 10 & 14 & -5 & -15 \\ 14 & 20 & -8 & -22 \\ -5 & -8 & 5 & 10 \\ -15 & -22 & 10 & 25 \end{bmatrix}, \quad G^* = \begin{bmatrix} 10 & 14 & -5 & -15 \\ 14 & 20 & -8 & -22 \\ -5 & -8 & 5 & 10 \\ -15 & -22 & 10 & 25 \end{bmatrix},$$

$$G^*G = \begin{bmatrix} 546 & 790 & -337 & -883 \\ 790 & 1144 & -490 & -1280 \\ -337 & -490 & 214 & 551 \\ -883 & -1280 & 551 & 1434 \end{bmatrix}.$$

Let $0 < \gamma = 0.0001 < 2/\rho(G^*G) = 0.00060009$, $\alpha_i = \frac{1}{n^{100/99}}$ be the same as above assumptions. Here $\Gamma = \{0, 0, 0, 0\}$,

$$\lim_{n\to\infty} w_n = \{0, 0, 0, 0\} \in \Gamma.$$

Table 1: Initial point $\{0, 0, 0.5, 0.5\}$.

Initial point	w_{2n+1}	Iter	Time	error
$\{0, 0, 0.5, 0.5\}$	$\{0, 0, 0.005, -0.0019\}$	$2 * 10000 + 2$	0.554	0.0054
$\{0, 0, 0.5, 0.5\}$	$\{0, 0, 0.6844 * 10^{-3}, -0.2633 * 10^{-3}\}$	$2 * 15000 + 2$	0.818	$7.3333 * 10^{-4}$
$\{0, 0, 0.5, 0.5\}$	$\{0, 0, 0.9357 * 10^{-4}, -0.36 * 10^{-4}\}$	$2 * 20000 + 2$	1.080	$1.0026 * 10^{-4}$
$\{0, 0, 0.5, 0.5\}$	$\{0, 0, 0.1749 * 10^{-5}, -0.0673 * 10^{-5}\}$	$2 * 30000 + 2$	1.63	$1.8739 * 10^{-6}$
$\{0, 0, 0.5, 0.5\}$	$\{0, 0, 0.3269 * 10^{-7}, -0.1258 * 10^{-7}\}$	$2 * 40000 + 2$	2.181	$3.5025 * 10^{-8}$

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