Available online at www.isr-publications.com/jnsa
J. Nonlinear Sci. Appl., 11 (2018), 49–72
Research Article

ISSN : 2008-1898

Journal of Nonlinear Sciences and Applications

Journal Homepage: www.isr-publications.com/jnsa

On some rational systems of difference equations

Check for updates

M. M. El-Dessoky[a,b,*], Abdul Khaliq[c], Asim Asiri[a]

[a]Mathematics Department, Faculty of Science, King AbdulAziz University, P. O. Box 80203, Jeddah 21589, Saudi Arabia.
[b]Department of Mathematics, Faculty of Science, Mansoura University, Mansoura 35516, Egypt.
[c]Department of Mathematics, Riphah International University, Lahore, Pakistan.

Communicated by K. Q. Lan

Abstract

Our goal in this paper is to find the form of solutions for the following systems of rational difference equations:

$$x_{n+1} = \frac{x_{n-3}y_{n-4}}{y_n(\pm 1 \pm x_{n-3}y_{n-4})}, \quad y_{n+1} = \frac{y_{n-3}x_{n-4}}{x_n(\pm 1 \pm y_{n-3}x_{n-4})}, \quad n = 0, 1, \ldots,$$

where the initial conditions have non-zero real numbers.

Keywords: Form of solution, stability, rational difference equations, rational systems.

2010 MSC: 39A10.

1. Introduction

Difference equations emerge as a natural representation of discovered evolution phenomena because most analysis of time evolving variables are discrete. Furthermore, non-linear difference equations of order greater than one are of immense importance in applications. Also several results in the theory of difference equations have been obtained as discrete analogues and as numerical solutions of differential equations. This is notably true in the case of Lyapunov theory of stability, which models various diverse phenomena in biology, ecology, physiology, physics, engineering, and economics. So, recently there has been an increasing interest in the study of qualitative analysis of rational difference equations and rational systems of difference equations. Although difference equations looks simple in form, but it is quite difficult to understand thoroughly the behaviors of their solutions, see [1–30] and the references cited therein.

*Corresponding author
Email addresses: dessokym@mans.edu.eg (M. M. El-Dessoky), khaliqsyed@gmail.com (Abdul Khaliq), amkasiri@kau.edu.sa (Asim Asiri)

doi: 10.22436/jnsa.011.01.05

Received: 2017-08-18 Revised: 2017-09-19 Accepted: 2017-11-05

The periodicity of the positive solutions of the rational difference system

$$x_{n+1} = \frac{\alpha x_{n-1}}{b y_n x_{n-1} + c}, \qquad y_{n+1} = \frac{\alpha y_{n-1}}{\beta x_n y_{n-1} + \gamma}$$

has been studied by Stevic in [27]. Camouzis [3] studied the boundedness, global stability, periodicity character and gave the solution of some special cases of the difference equation

$$x_{n+1} = \frac{\alpha_1 + \gamma_1 y_n + \beta_1 x_n}{A_1 + C_1 y_n + B_1 x_n}, \qquad y_{n+1} = \frac{\alpha_2 + \gamma_2 y_n + \beta_2 x_n}{A_2 + C_2 y_n + B_2 x_n}.$$

Kurbanli et al. [21] has studied the following system of difference equation

$$x_{n+1} = \frac{x_{n-1}}{x_{n-1} y_n + 1}, \qquad y_{n+1} = \frac{y_{n-1}}{y_{n-1} x_n + 1}.$$

El-Dessoky [10] investigated the periodic nature and the form of the solutions of nonlinear recurrence relations systems of order three

$$x_{n+1} = \frac{y_{n-2} y_{n-1}}{x_n \left(\pm y_{n-1} y_{n-2} \pm 1\right)}, \; y_{n+1} = \frac{x_{n-2} x_{n-1}}{y_n \left(\pm x_{n-1} x_{n-2} \pm 1\right)}.$$

Din et al. [6] investigated the dynamics of a system of fourth-order rational difference equations

$$x_{n+1} = \frac{\alpha x_{n-3}}{\beta + \gamma y_{n-3} y_{n-2} y_{n-1} y_n}, \qquad y_{n+1} = \frac{\alpha_1 y_{n-3}}{\beta_1 + \gamma_1 x_{n-3} x_{n-2} x_{n-1} x_n}.$$

The main objective of this paper is to investigate the forms and expressions of the solution of the difference equations systems

$$x_{n+1} = \frac{x_{n-3} y_{n-4}}{y_n(\pm 1 \pm x_{n-3} y_{n-4})}, \qquad y_{n+1} = \frac{y_{n-3} x_{n-4}}{x_n(\pm 1 \pm y_{n-3} x_{n-4})}, \qquad n = 0, 1, \ldots,$$

with initial conditions having non-zero real numbers and

$$[y_n(\pm 1 \pm x_{n-3} y_{n-4})] \neq 0, \;\; [x_n(\pm 1 \pm y_{n-3} x_{n-4})] \neq 0.$$

2. The first system: $x_{n+1} = x_{n-3} y_{n-4}/y_n(1 + x_{n-3} y_{n-4})$, $y_{n+1} = y_{n-3} x_{n-4}/x_n(1 + y_{n-3} x_{n-4})$

In this section, we study the existence of analytical forms of the solutions for the following system of difference equations:

$$x_{n+1} = \frac{x_{n-3} y_{n-4}}{y_n(1 + x_{n-3} y_{n-4})}, \qquad y_{n+1} = \frac{y_{n-3} x_{n-4}}{x_n(1 + y_{n-3} x_{n-4})}, \qquad n = 0,\ 1, \ldots, \tag{2.1}$$

with non-zero initial conditions $x_{-4};\ x_{-3};\ x_{-2};\ x_{-1}; x_0;\ y_{-4};\ y_{-3};\ y_{-2};\ y_{-1}$ and y_0 are real numbers.

Theorem 2.1. *Suppose that $\{x_n,\ y_n\}$ is a solution for the system (2.1), then for $n = 0, 1, 2, \ldots$, we obtain all solutions of system (2.1) are given by the following expression.*

$$x_{4n-3} = \left(\frac{bf^n}{l^n}\right) \prod_{i=0}^{n-1} \frac{(1 + ihb)(1 + ild)}{(1 + (i+1)bf)(1 + idh)}, \qquad x_{4n-2} = \left(\frac{ce^n}{a^n}\right) \prod_{i=0}^{n-1} \frac{(1 + (i+1)ag)(1 + ikc)}{(1 + (i+1)cg)(1 + iek)},$$

$$x_{4n-1} = \left(\frac{df^n}{l^n}\right) \prod_{i=0}^{n-1} \frac{(1 + (i+1)hb)(1 + ild)}{(1 + (i+1)bf)(1 + (i+1)dh)}, \qquad x_{4n} = \left(\frac{e^{n+1}}{a^n}\right) \prod_{i=0}^{n-1} \frac{(1 + (i+1)ag)(1 + (i+1)kc)}{(1 + (i+1)cg)(1 + (i+1)ek)},$$

and

$$y_{4n-3} = \left(\frac{ga^n}{e^n}\right) \prod_{i=0}^{n-1} \frac{(1 + icg)(1 + iek)}{(1 + (i+1)ag)(1 + ikc)}, \qquad y_{4n-2} = \left(\frac{l^n h}{f^n}\right) \prod_{i=0}^{n-1} \frac{(1 + (i+1)bf)(1 + idh)}{(1 + (i+1)hb)(1 + ild)},$$

$$y_{4n-1}=\frac{(ka)^n}{e^n}\prod_{i=0}^{n-1}\frac{(1+(i+1)cg)(1+iek)}{(1+(i+1)ag)(1+(i+1)kc)},\qquad y_{4n}=\left(\frac{l^{n+1}}{f^n}\right)\prod_{i=0}^{n-1}\frac{(1+(i+1)bf)(1+(i+1)dh)}{(1+(i+1)hb)(1+(i+1)ld)},$$

where $x_{-4}=a,\ x_{-3}=b,\ x_{-2}=c,\ x_{-1}=d,\ x_0=e,\ y_{-4}=f,\ y_{-3}=g,\ y_{-2}=h,\ y_{-1}=k,$ *and* $y_0=l$ *with the initial value must be non-zero and* $ag,\ kc,\ hb,\ ld,\ cg,\ ek,\ bf,\ dh\notin\left\{-\frac{1}{n},\ n=1,\ 2,\dots\right\}$.

Proof. For $n=0$ the result holds. Now suppose that $n>0$ and that our assumption holds for $n-1$. That is,

$$x_{4n-7}=\left(\frac{bf^{n-1}}{l^{n-1}}\right)\prod_{i=0}^{n-2}\frac{(1+ihb)(1+ild)}{(1+(i+1)bf)(1+idh)},\qquad x_{4n-6}=\left(\frac{ce^{n-1}}{a^{n-1}}\right)\prod_{i=0}^{n-2}\frac{(1+(i+1)ag)(1+ikc)}{(1+(i+1)cg)(1+iek)},$$
$$x_{4n-5}=\left(\frac{df^{n-1}}{l^{n-1}}\right)\prod_{i=0}^{n-2}\frac{(1+(i+1)hb)(1+ild)}{(1+(i+1)bf)(1+(i+1)dh)},\quad x_{4n-4}=\left(\frac{e^{n}}{a^{n-1}}\right)\prod_{i=0}^{n-2}\frac{(1+(i+1)ag)(1+(i+1)kc)}{(1+(i+1)cg)(1+(i+1)ek)},$$

and

$$y_{4n-7}=\left(\frac{ga^{n-1}}{e^{n-1}}\right)\prod_{i=0}^{n-2}\frac{(1+icg)(1+iek)}{(1+(i+1)ag)(1+ikc)},\qquad y_{4n-6}=\left(\frac{l^{n-1}h}{f^{n-1}}\right)\prod_{i=0}^{n-2}\frac{(1+(i+1)bf)(1+idh)}{(1+(i+1)hb)(1+ild)},$$
$$y_{4n-5}=\left(\frac{ka^{n-1}}{e^{n-1}}\right)\prod_{i=0}^{n-2}\frac{(1+(i+1)cg)(1+iek)}{(1+(i+1)ag)(1+(i+1)kc)},\quad y_{4n-4}=\left(\frac{l^{n}}{f^{n-1}}\right)\prod_{i=0}^{n-2}\frac{(1+(i+1)bf)(1+(i+1)dh)}{(1+(i+1)hb)(1+(i+1)ld)}.$$

Now, it follows from system (2.1) that

$$\begin{aligned}
&x_{4n-3}\\
&=\frac{x_{4n-7}y_{4n-8}}{y_{4n-4}(1+x_{4n-7}y_{4n-8})}\\
&=\frac{\left(\frac{bf^{n-1}}{l^{n-1}}\prod_{i=0}^{n-2}\frac{(1+ihb)(1+ild)}{(1+(i+1)bf)(1+idh)}\right)\left(\frac{l^{n-1}}{f^{n-2}}\prod_{i=0}^{n-3}\frac{(1+(i+1)bf)(1+(i+1)dh)}{(1+(i+1)hb)(1+(i+1)ld)}\right)}{\left(\frac{l^{n}}{f^{n-1}}\prod_{i=0}^{n-2}\frac{(1+(i+1)bf)(1+(i+1)dh)}{(1+(i+1)hb)(1+(i+1)ld)}\right)\left(1+\left(\frac{bf^{n-1}}{l^{n-1}}\prod_{i=0}^{n-2}\frac{(1+ihb)(1+ild)}{(1+(i+1)bf)(1+idh)}\right)\left(\frac{l^{n-1}}{f^{n-2}}\prod_{i=0}^{n-3}\frac{(1+(i+1)bf)(1+(i+1)dh)}{(1+(i+1)hb)(1+(i+1)ld)}\right)\right)}\\
&=\frac{\left(\frac{bf}{1+(n-1)bf}\right)}{\left(\frac{l^{n}}{f^{n-1}}\prod_{i=0}^{n-2}\frac{(1+(i+1)bf)(1+(i+1)dh)}{(1+(i+1)hb)(1+(i+1)ld)}\right)\left(1+\frac{bf}{1+(n-1)bf}\right)}\\
&=\frac{f^{n-1}\left(\frac{bf}{1+(n-1)bf}\right)}{l^{n}\left(1+\frac{bf}{1+(n-1)bf}\right)}\left(\prod_{i=0}^{n-2}\frac{(1+(i+1)hb)(1+(i+1)ld)}{(1+(i+1)bf)(1+(i+1)dh)}\right)\\
&=\left(\frac{bf^{n}}{l^{n}(1+(n-1)bf+bf)}\right)\prod_{i=0}^{n-2}\frac{(1+(i+1)hb)(1+(i+1)ld)}{(1+(i+1)bf)(1+(i+1)dh)}\\
&=\frac{bf^{n}}{l^{n}(1+nbf)}\prod_{i=0}^{n-2}\frac{(1+(i+1)hb)(1+(i+1)ld)}{(1+(i+1)bf)(1+(i+1)dh)}=\frac{bf^{n}}{l^{n}}\prod_{i=0}^{n-1}\frac{(1+ihb)(1+ild)}{(1+(i+1)bf)(1+idh)}.
\end{aligned}$$

And similarly

$$\begin{aligned}
&y_{4n-3}\\
&=\frac{y_{4n-7}x_{4n-8}}{x_{4n-4}(1+y_{4n-7}x_{4n-8})}\\
&=\frac{\left(\frac{ga^{n-1}}{e^{n-1}}\prod_{i=0}^{n-2}\frac{(1+icg)(1+iek)}{(1+(i+1)ag)(1+ikc)}\right)\left(\frac{e^{n-1}}{a^{n-2}}\prod_{i=0}^{n-3}\frac{(1+(i+1)ag)(1+(i+1)kc)}{(1+(i+1)cg)(1+(i+1)ek)}\right)}{\left(\frac{e^{n}}{a^{n-1}}\prod_{i=0}^{n-2}\frac{(1+(i+1)ag)(1+(i+1)kc)}{(1+(i+1)cg)(1+(i+1)ek)}\right)\left(1+\left(\frac{ga^{n-1}}{e^{n-1}}\prod_{i=0}^{n-2}\frac{(1+icg)(1+iek)}{(1+(i+1)ag)(1+ikc)}\right)\left(\frac{e^{n-1}}{a^{n-2}}\prod_{i=0}^{n-3}\frac{(1+(i+1)ag)(1+(i+1)kc)}{(1+(i+1)cg)(1+(i+1)ek)}\right)\right)}\\
&=\frac{\left(\frac{ag}{1+(n-1)ag}\right)}{\left(\frac{e^{n}}{a^{n-1}}\prod_{i=0}^{n-2}\frac{(1+(i+1)ag)(1+(i+1)kc)}{(1+(i+1)cg)(1+(i+1)ek)}\right)\left(1+\frac{ag}{1+(n-1)ag}\right)}
\end{aligned}$$

$$= \frac{a^{n-1}\left(\frac{ag}{1+(n-1)ag}\right)}{e^n\left(1+\frac{ag}{1+(n-1)ag}\right)}\left(\prod_{i=0}^{n-2}\frac{(1+(i+1)cg)(1+(i+1)ek)}{(1+(i+1)ag)(1+(i+1)kc)}\right)$$

$$= \left(\frac{a^n g}{e^n(1+nag)}\right)\prod_{i=0}^{n-2}\frac{(1+(i+1)cg)(1+(i+1)ek)}{(1+(i+1)ag)(1+(i+1)kc)} = \left(\frac{a^n g}{e^n}\right)\prod_{i=0}^{n-1}\frac{(1+icg)(1+iek)}{(1+(i+1)ag)(1+ikc)}.$$

Again, from system (2.1) we get that,

$$x_{4n} = \frac{x_{4n-4}y_{4n-5}}{y_{4n-1}(1+x_{4n-4}y_{4n-5})}$$

$$= \frac{\left(\frac{e^n}{a^{n-1}}\prod_{i=0}^{n-2}\frac{(1+(i+1)ag)(1+(i+1)kc)}{(1+(i+1)cg)(1+(i+1)ek)}\right)\left(\frac{ka^{n-1}}{e^{n-1}}\prod_{i=0}^{n-2}\frac{(1+(i+1)cg)(1+iek)}{(1+(i+1)ag)(1+(i+1)kc)}\right)}{\left(\frac{ka^n}{e^n}\prod_{i=0}^{n-1}\frac{(1+(i+1)cg)(1+iek)}{(1+(i+1)ag)(1+(i+1)kc)}\right)\left(1+\left(\frac{e^n}{a^{n-1}}\prod_{i=0}^{n-2}\frac{(1+(i+1)ag)(1+(i+1)kc)}{(1+(i+1)cg)(1+(i+1)ek)}\right)\left(\frac{ka^{n-1}}{e^{n-1}}\prod_{i=0}^{n-2}\frac{(1+(i+1)cg)(1+iek)}{(1+(i+1)ag)(1+(i+1)kc)}\right)\right)}$$

$$= \frac{\left(\frac{ek}{1+(n-1)ek}\right)}{\left(\frac{ka^n}{e^n}\prod_{i=0}^{n-1}\frac{(1+(i+1)cg)(1+iek)}{(1+(i+1)ag)(1+(i+1)kc)}\right)\left(1+\frac{ek}{1+(n-1)ek}\right)}$$

$$= \frac{e^n\left(\frac{ek}{1+(n-1)ek}\right)}{ka^n\left(1+\frac{ek}{1+(n-1)ek}\right)}\left(\prod_{i=0}^{n-1}\frac{(1+(i+1)ag)(1+(i+1)kc)}{(1+(i+1)cg)(1+iek)}\right)$$

$$= \left(\frac{e^{n+1}}{a^n(1+(n-1)ek+ek)}\right)\prod_{i=0}^{n-1}\frac{(1+(i+1)ag)(1+(i+1)kc)}{(1+(i+1)cg)(1+iek)}$$

$$= \left(\frac{e^{n+1}}{a^n(1+nek)}\right)\prod_{i=0}^{n-1}\frac{(1+(i+1)ag)(1+(i+1)kc)}{(1+(i+1)cg)(1+iek)} = \frac{e^{n+1}}{a^n}\prod_{i=0}^{n-1}\frac{(1+(i+1)ag)(1+(i+1)kc)}{(1+(i+1)cg)(1+(i+1)ek)},$$

and

$$y_{4n} = \frac{y_{4n-4}x_{4n-5}}{x_{4n-1}(1+y_{4n-4}x_{4n-5})}$$

$$= \frac{\left(\frac{l^n}{f^{n-1}}\prod_{i=0}^{n-2}\frac{(1+(i+1)bf)(1+(i+1)dh)}{(1+(i+1)hb)(1+(i+1)ld)}\right)\left(\frac{df^{n-1}}{l^{n-1}}\prod_{i=0}^{n-2}\frac{(1+(i+1)hb)(1+ild)}{(1+(i+1)bf)(1+(i+1)dh)}\right)}{\left(\frac{df^n}{l^n}\prod_{i=0}^{n-1}\frac{(1+(i+1)hb)(1+ild)}{(1+(i+1)bf)(1+(i+1)dh)}\right)\left(1+\left(\frac{l^n}{f^{n-1}}\prod_{i=0}^{n-2}\frac{(1+(i+1)bf)(1+(i+1)dh)}{(1+(i+1)hb)(1+(i+1)ld)}\right)\left(\frac{df^{n-1}}{l^{n-1}}\prod_{i=0}^{n-2}\frac{(1+(i+1)hb)(1+ild)}{(1+(i+1)bf)(1+(i+1)dh)}\right)\right)}$$

$$= \frac{\left(\frac{ld}{1+(n-1)ld}\right)}{\left(\frac{df^n}{l^n}\prod_{i=0}^{n-1}\frac{(1+(i+1)hb)(1+ild)}{(1+(i+1)bf)(1+(i+1)dh)}\right)\left(1+\frac{ld}{1+(n-1)ld}\right)} = \frac{l^n\left(\frac{ld}{1+(n-1)ld}\right)}{df^n\left(1+\frac{ld}{1+(n-1)ld}\right)}\prod_{i=0}^{n-1}\frac{(1+(i+1)bf)(1+(i+1)dh)}{(1+(i+1)hb)(1+ild)}$$

$$= \left(\frac{l^{n+1}d}{df^n(1+nld)}\right)\prod_{i=0}^{n-1}\frac{(1+(i+1)bf)(1+(i+1)dh)}{(1+(i+1)hb)(1+ild)} = \left(\frac{l^{n+1}}{f^n}\right)\prod_{i=0}^{n-1}\frac{(1+(i+1)bf)(1+(i+1)dh)}{(1+(i+1)hb)(1+(i+1)ld)}.$$

Also, we can prove the other relations. This completes the proof. □

Example 2.2. In order to verify the results of this section, we deal with some numerical example for the difference system (2.1) with the initial conditions $x_{-4} = 4$, $x_{-3} = 3$, $x_{-2} = 9$, $x_{-1} = 4$, $x_0 = 3$, $y_{-4} = 1.2$, $y_{-3} = 9$, $y_{-2} = 1.5$, $y_{-1} = 5$, and $y_0 = 9$ (see Figure 1).

3. The second system: $x_{n+1} = x_{n-3}y_{n-4}/y_n(1+x_{n-3}y_{n-4})$, $y_{n+1} = y_{n-3}x_{n-4}/x_n(1-y_{n-3}x_{n-4})$

In this section, we study the existence of analytical forms of the solutions for the following system of difference equations:

$$x_{n+1} = \frac{x_{n-3}y_{n-4}}{y_n(1+x_{n-3}y_{n-4})}, \qquad y_{n+1} = \frac{y_{n-3}x_{n-4}}{x_n(1-y_{n-3}x_{n-4})}, \qquad n = 0,\ 1, \ldots, \tag{3.1}$$

having non-zero initial conditions x_{-4}, x_{-3}, x_{-2}, x_{-1}, x_0, y_{-4}, y_{-3}, y_{-2}, y_{-1}, and y_0, where $x_{-3}y_{-4} \neq -1$ and $x_{-2}y_{-2} \neq 1$.

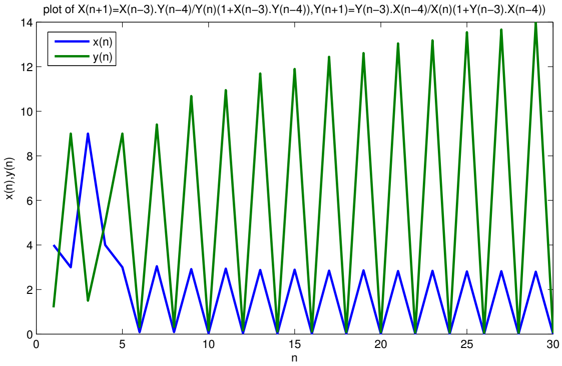

Figure 1

Theorem 3.1. *Suppose that* $\{x_n, y_n\}$ *is a solution for the system* (3.1), *then for* $n = 0, 1, \ldots,$ *one obtains*

$$x_{4n-3} = \left(\frac{bf^n}{l^n}\right)\prod_{i=0}^{n-1}\frac{(1-ihb)(1-ild)}{(1+(i+1)bf)(1+idh)}, \qquad x_{4n-2} = \left(\frac{ce^n}{a^n}\right)\prod_{i=0}^{n-1}\frac{(1-(i+1)ag)(1-ikc)}{(1+(i+1)cg)(1+iek)},$$

$$x_{4n-1} = \left(\frac{df^n}{l^n}\right)\prod_{i=0}^{n-1}\frac{(1-(i+1)hb)(1-ild)}{(1+(i+1)bf)(1+(i+1)dh)}, \qquad x_{4n} = \left(\frac{e^{n+1}}{a^n}\right)\prod_{i=0}^{n-1}\frac{(1-(i+1)ag)(1-(i+1)kc)}{(1+(i+1)cg)(1+(i+1)ek)},$$

and

$$y_{4n-3} = \left(\frac{ga^n}{e^n}\right)\prod_{i=0}^{n-1}\frac{(1+icg)(1+iek)}{(1-(i+1)ag)(1-ikc)}, \qquad y_{4n-2} = \left(\frac{l^n h}{f^n}\right)\prod_{i=0}^{n-1}\frac{(1+(i+1)bf)(1+idh)}{(1-(i+1)hb)(1-ild)},$$

$$y_{4n-1} = \left(\frac{ka^n}{e^n}\right)\prod_{i=0}^{n-1}\frac{(1+(i+1)cg)(1+iek)}{(1-(i+1)ag)(1-(i+1)kc)}, \qquad y_{4n} = \left(\frac{l^{n+1}}{f^n}\right)\prod_{i=0}^{n-1}\frac{(1+(i+1)bf)(1+(i+1)dh)}{(1-(i+1)hb)(1-(i+1)ld)},$$

where $x_{-4} = a,\ x_{-3} = b,\ x_{-2} = c,\ x_{-1} = d,\ x_0 = e,\ y_{-4} = f,\ y_{-3} = g,\ y_{-2} = h,\ y_{-1} = k$, and $y_0 = l$ with the initial value must be non-zero and $ag,\ kc,\ hb,\ ld \notin \left\{\frac{1}{n},\ n = 1,\ 2, \ldots\right\}$ and $cg,\ ek,\ bf,\ dh \notin \left\{-\frac{1}{n},\ n = 1,\ 2, \ldots\right\}$.

Proof. For $n = 0$ the result holds. Now suppose that $n > 0$ and that our assumption holds for $n-1$. That is,

$$x_{4n-7} = \left(\frac{bf^{n-1}}{l^{n-1}}\right)\prod_{i=0}^{n-2}\frac{(1-ihb)(1-ild)}{(1+(i+1)bf)(1+idh)},$$

$$x_{4n-6} = \left(\frac{ce^{n-1}}{a^{n-1}}\right)\prod_{i=0}^{n-2}\frac{(1-(i+1)ag)(1-ikc)}{(1+(i+1)cg)(1+iek)},$$

$$x_{4n-5} = \left(\frac{df^{n-1}}{l^{n-1}}\right)\prod_{i=0}^{n-2}\frac{(1-(i+1)hb)(1-ild)}{(1+(i+1)bf)(1+(i+1)dh)},$$

$$x_{4n-4} = \left(\frac{e^{n}}{a^{n-1}}\right)\prod_{i=0}^{n-2}\frac{(1-(i+1)ag)(1-(i+1)kc)}{(1+(i+1)cg)(1+(i+1)ek)},$$

and

$$y_{4n-7} = \left(\frac{ga^{n-1}}{e^{n-1}}\right)\prod_{i=0}^{n-2}\frac{(1+icg)(1+iek)}{(1-(i+1)ag)(1-ikc)},$$

$$y_{4n-6} = \left(\frac{l^{n-1}h}{f^{n-1}}\right)\prod_{i=0}^{n-2}\frac{(1+(i+1)bf)(1+idh)}{(1-(i+1)hb)(1-ild)},$$
$$y_{4n-5} = \left(\frac{ka^{n-1}}{e^{n-1}}\right)\prod_{i=0}^{n-2}\frac{(1+(i+1)cg)(1+iek)}{(1-(i+1)ag)(1-(i+1)kc)},$$
$$y_{4n-4} = \left(\frac{l^{n}}{f^{n-1}}\right)\prod_{i=0}^{n-2}\frac{(1+(i+1)bf)(1+(i+1)dh)}{(1-(i+1)hb)(1-(i+1)ld)}.$$

Deducing from system (3.1) we get,

$$\begin{aligned}
x_{4n-3} &= \frac{x_{4n-7}y_{4n-8}}{y_{4n-4}(1+x_{4n-7}y_{4n-8})}\\
&= \frac{\left(\frac{bf^{n-1}}{l^{n-1}}\prod_{i=0}^{n-2}\frac{(1-ihb)(1-ild)}{(1+(i+1)bf)(1+idh)}\right)\left(\frac{l^{n-1}}{f^{n-2}}\prod_{i=0}^{n-3}\frac{(1+(i+1)bf)(1+(i+1)dh)}{(1-(i+1)hb)(1-(i+1)ld)}\right)}{\left(\frac{l^{n}}{f^{n-1}}\prod_{i=0}^{n-2}\frac{(1+(i+1)bf)(1+(i+1)dh)}{(1-(i+1)hb)(1-(i+1)ld)}\right)\left(1+\left(\frac{bf^{n-1}}{l^{n-1}}\prod_{i=0}^{n-2}\frac{(1-ihb)(1-ild)}{(1+(i+1)bf)(1+idh)}\right)\left(\frac{l^{n-1}}{f^{n-2}}\prod_{i=0}^{n-3}\frac{(1+(i+1)bf)(1+(i+1)dh)}{(1-(i+1)hb)(1-(i+1)ld)}\right)\right)}\\
&= \frac{\left(\frac{bf}{1+(n-1)bf}\right)}{\left(\frac{l^{n}}{f^{n-1}}\prod_{i=0}^{n-2}\frac{(1+(i+1)bf)(1+(i+1)dh)}{(1-(i+1)hb)(1-(i+1)ld)}\right)\left(1+\frac{bf}{1+(n-1)bf}\right)}\\
&= \frac{f^{n-1}\left(\frac{bf}{1+(n-1)bf}\right)}{l^{n}\left(1+\frac{bf}{1+(n-1)bf}\right)}\left(\prod_{i=0}^{n-2}\frac{(1-(i+1)hb)(1-(i+1)ld)}{(1+(i+1)bf)(1+(i+1)dh)}\right)\\
&= \left(\frac{bf^{n}}{l^{n}(1+(n-1)bf+bf)}\right)\prod_{i=0}^{n-2}\frac{(1-(i+1)hb)(1-(i+1)ld)}{(1+(i+1)bf)(1+(i+1)dh)}\\
&= \left(\frac{bf^{n}}{l^{n}(1+nbf)}\right)\prod_{i=0}^{n-2}\frac{(1-(i+1)hb)(1-(i+1)ld)}{(1+(i+1)bf)(1+(i+1)dh)} = \left(\frac{bf^{n}}{l^{n}}\right)\prod_{i=0}^{n-1}\frac{(1-ihb)(1-ild)}{(1+(i+1)bf)(1+idh)}.
\end{aligned}$$

Similarly,

$$\begin{aligned}
y_{4n-3} &= \frac{y_{4n-7}x_{4n-8}}{x_{4n-4}(1-y_{4n-7}x_{4n-8})}\\
&= \frac{\left(\frac{ga^{n-1}}{e^{n-1}}\prod_{i=0}^{n-2}\frac{(1+icg)(1+iek)}{(1-(i+1)ag)(1-ikc)}\right)\left(\frac{e^{n-1}}{a^{n-2}}\prod_{i=0}^{n-3}\frac{(1-(i+1)ag)(1-(i+1)kc)}{(1+(i+1)cg)(1+(i+1)ek)}\right)}{\left(\frac{e^{n}}{a^{n-1}}\prod_{i=0}^{n-2}\frac{(1-(i+1)ag)(1-(i+1)kc)}{(1+(i+1)cg)(1+(i+1)ek)}\right)\left(1-\left(\frac{ga^{n-1}}{e^{n-1}}\prod_{i=0}^{n-2}\frac{(1+icg)(1+iek)}{(1-(i+1)ag)(1-ikc)}\right)\left(\frac{e^{n-1}}{a^{n-2}}\prod_{i=0}^{n-3}\frac{(1-(i+1)ag)(1-(i+1)kc)}{(1+(i+1)cg)(1+(i+1)ek)}\right)\right)}\\
&= \frac{\left(\frac{ag}{1-(n-1)ag}\right)}{\frac{e^{n}}{a^{n-1}}\left(1-\frac{ag}{1+(n-1)ag}\right)\prod_{i=0}^{n-2}\frac{(1-(i+1)ag)(1-(i+1)kc)}{(1+(i+1)cg)(1+(i+1)ek)}}\\
&= \frac{a^{n-1}\left(\frac{ag}{1-(n-1)ag}\right)}{e^{n}\left(1-\frac{ag}{1-(n-1)ag}\right)}\left(\prod_{i=0}^{n-2}\frac{(1+(i+1)cg)(1+(i+1)ek)}{(1-(i+1)ag)(1-(i+1)kc)}\right)\\
&= \left(\frac{a^{n}g}{e^{n}(1-nag)}\right)\prod_{i=0}^{n-2}\frac{(1+(i+1)cg)(1+(i+1)ek)}{(1-(i+1)ag)(1-(i+1)kc)} = \left(\frac{a^{n}g}{e^{n}}\right)\prod_{i=0}^{n-1}\frac{(1+icg)(1+iek)}{(1-(i+1)ag)(1-ikc)}.
\end{aligned}$$

Again extracting from system (3.1), we get

$$\begin{aligned}
x_{4n} &= \frac{x_{4n-4}y_{4n-5}}{y_{4n-1}(1+x_{4n-4}y_{4n-5})}\\
&= \frac{\left(\frac{e^{n}}{a^{n-1}}\prod_{i=0}^{n-2}\frac{(1-(i+1)ag)(1-(i+1)kc)}{(1+(i+1)cg)(1+(i+1)ek)}\right)\left(\frac{ka^{n-1}}{e^{n-1}}\prod_{i=0}^{n-2}\frac{(1+(i+1)cg)(1+iek)}{(1-(i+1)ag)(1-(i+1)kc)}\right)}{\left(\frac{ka^{n}}{e^{n}}\prod_{i=0}^{n-1}\frac{(1+(i+1)cg)(1+iek)}{(1-(i+1)ag)(1-(i+1)kc)}\right)\left(1+\left(\frac{e^{n}}{a^{n-1}}\prod_{i=0}^{n-2}\frac{(1-(i+1)ag)(1-(i+1)kc)}{(1+(i+1)cg)(1+(i+1)ek)}\right)\left(\frac{ka^{n-1}}{e^{n-1}}\prod_{i=0}^{n-2}\frac{(1+(i+1)cg)(1+iek)}{(1-(i+1)ag)(1-(i+1)kc)}\right)\right)}\\
&= \frac{\left(\frac{ek}{1+(n-1)ek}\right)}{\frac{ka^{n}}{e^{n}}\prod_{i=0}^{n-1}\frac{(1+(i+1)cg)(1+iek)}{(1-(i+1)ag)(1-(i+1)kc)}\left(1+\frac{ek}{1+(n-1)ek}\right)}
\end{aligned}$$

$$= \frac{e^n\left(\frac{ek}{1+(n-1)ek}\right)}{ka^n\left(1+\frac{ek}{1+(n-1)ek}\right)}\left(\prod_{i=0}^{n-1}\frac{(1-(i+1)ag)(1-(i+1)kc)}{(1+(i+1)cg)(1+iek)}\right)$$
$$= \left(\frac{e^{n+1}}{a^n(1+(n-1)ek+ek)}\right)\prod_{i=0}^{n-1}\frac{(1-(i+1)ag)(1-(i+1)kc)}{(1+(i+1)cg)(1+iek)}$$
$$= \left(\frac{e^{n+1}}{a^n(1+nek)}\right)\prod_{i=0}^{n-1}\frac{(1-(i+1)ag)(1-(i+1)kc)}{(1+(i+1)cg)(1+iek)} = \left(\frac{e^{n+1}}{a^n}\right)\prod_{i=0}^{n-1}\frac{(1-(i+1)ag)(1-(i+1)kc)}{(1+(i+1)cg)(1+(i+1)ek)},$$

and similarly,

$$y_{4n}$$
$$= \frac{y_{4n-4}x_{4n-5}}{x_{4n-1}(1-y_{4n-4}x_{4n-5})}$$
$$= \frac{\left(\frac{l^n}{f^{n-1}}\prod_{i=0}^{n-2}\frac{(1+(i+1)bf)(1+(i+1)dh)}{(1-(i+1)hb)(1-(i+1)ld)}\right)\left(\frac{df^{n-1}}{l^{n-1}}\prod_{i=0}^{n-2}\frac{(1-(i+1)hb)(1-ild)}{(1+(i+1)bf)(1+(i+1)dh)}\right)}{\left(\frac{df^n}{l^n}\prod_{i=0}^{n-1}\frac{(1-(i+1)hb)(1-ild)}{(1+(i+1)bf)(1+(i+1)dh)}\right)\left(1+\left(\frac{l^n}{f^{n-1}}\prod_{i=0}^{n-2}\frac{(1+(i+1)bf)(1+(i+1)dh)}{(1-(i+1)hb)(1-(i+1)ld)}\right)\left(\frac{df^{n-1}}{l^{n-1}}\prod_{i=0}^{n-2}\frac{(1-(i+1)hb)(1-ild)}{(1+(i+1)bf)(1+(i+1)dh)}\right)\right)}$$
$$= \frac{\left(\frac{ld}{1-(n-1)ld}\right)}{\left(\frac{df^n}{l^n}\prod_{i=0}^{n-1}\frac{(1-(i+1)hb)(1-ild)}{(1+(i+1)bf)(1+(i+1)dh)}\right)\left(1-\frac{ld}{1-(n-1)ld}\right)} = \frac{l^n\left(\frac{ld}{1-(n-1)ld}\right)}{df^n\left(1-\frac{ld}{1-(n-1)ld}\right)}\prod_{i=0}^{n-1}\frac{(1+(i+1)bf)(1+(i+1)dh)}{(1-(i+1)hb)(1-ild)}$$
$$= \left(\frac{l^{n+1}d}{df^n(1-nld)}\right)\prod_{i=0}^{n-1}\frac{(1+(i+1)bf)(1+(i+1)dh)}{(1-(i+1)hb)(1-ild)} = \left(\frac{l^{n+1}}{f^n}\right)\prod_{i=0}^{n-1}\frac{(1+(i+1)bf)(1+(i+1)dh)}{(1-(i+1)hb)(1-(i+1)ld)}.$$

Also, we can prove the other relations. Thus the proof is completed. □

Example 3.2. In order to verify the results of this section, we deal with some numerical example for the difference system (3.1) with the initial conditions $x_{-4} = 5$, $x_{-3} = 2$, $x_{-2} = 4$, $x_{-1} = 8$, $x_0 = 3$, $y_{-4} = 2$, $y_{-3} = 3$, $y_{-2} = 5$, $y_{-1} = 8$, and $y_0 = 7$ (see Figure 2).

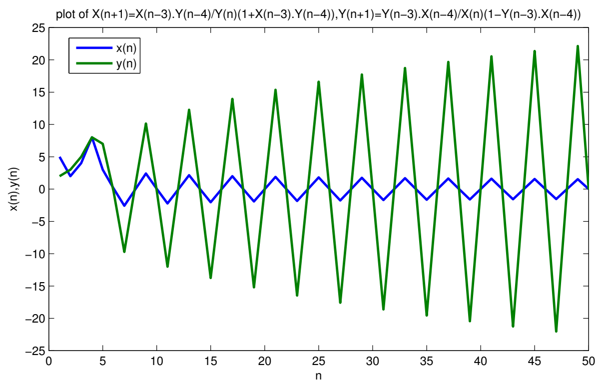

Figure 2

4. The third system: $x_{n+1} = x_{n-3}y_{n-4}/y_n(-1 + x_{n-3}y_{n-4})$, $y_{n+1} = y_{n-3}x_{n-4}/x_n(1 + y_{n-3}x_{n-4})$

In this section, we study the existence of analytical forms of the solutions for the following system of difference equations

$$x_{n+1} = \frac{x_{n-3}y_{n-4}}{y_n(-1+x_{n-3}y_{n-4})}, \qquad y_{n+1} = \frac{y_{n-3}x_{n-4}}{x_n(1+y_{n-3}x_{n-4})}, \qquad n = 0, 1, \dots, \tag{4.1}$$

with non-zero initial conditions x_{-4}, x_{-3}, x_{-2}, x_{-1}, x_0, y_{-4}, y_{-3}, y_{-2}, y_{-1}, and y_0 real numbers.

Theorem 4.1. *Suppose that* $\{x_n, y_n\}$ *is a solution for the system* (4.1) *then for* $n = 0, 1, 2, \ldots$, *one obtains*

$$x_{8n-4} = \frac{e^{2n}\prod_{i=0}^{n-1}(-1+(2i+1)ag)(-1+(2i+1)kc)(-1+2iag)(-1+2ikc)}{a^{2n-1}(1+cg)^n(1+ek)^n},$$

$$x_{8n-3} = \frac{bf^{2n}\prod_{i=0}^{n-1}(-1+(2i+1)hb)(-1+(2i+1)ld)(-1+2ihb)(-1+2ild)}{l^{2n}(1+bf)^n(1+dh)^n},$$

$$x_{8n-2} = \frac{ce^{2n}\prod_{i=0}^{n-1}(-1+(2i+1)ag)(-1+(2i+1)kc)(-1+(2i+2)ag)(-1+2ikc)}{a^{2n}(1+cg)^n(1+ek)^n},$$

$$x_{8n-1} = \frac{df^{2n}\prod_{i=0}^{n-1}(-1+(2i+1)hb)(-1+(2i+1)ld)(-1+(2i+2)hb)(-1+2ild)}{l^{2n}(1+bf)^n(1+dh)^n},$$

$$x_{8n} = \frac{e^{2n+1}\prod_{i=0}^{n-1}(-1+(2i+1)ag)(-1+(2i+1)kc)(-1+(2i+2)ag)(-1+(2i+2)kc)}{a^{2n}(1+cg)^n(1+ek)^n},$$

$$x_{8n+1} = \frac{bf^{2n+1}\prod_{i=0}^{n-1}(-1+(2i+1)hb)(-1+(2i+1)ld)(-1+(2i+2)hb)(-1+(2i+2)ld)}{l^{2n+1}(1+bf)^{n+1}(1+dh)^n},$$

$$x_{8n+2} = \frac{ce^{2n+1}(-1+ag)\prod_{i=0}^{n-1}(-1+(2i+2)ag)(-1+(2i+1)kc)(-1+(2i+3)ag)(-1+(2i+2)kc)}{a^{2n+1}(1+cg)^{n+1}(1+ek)^n},$$

$$x_{8n+3} = \frac{df^{2n+1}(-1+hb)\prod_{i=0}^{n-1}(-1+(2i+2)hb)(-1+(2i+1)ld)(-1+(2i+3)hb)(-1+(2i+2)ld)}{l^{2n+1}(1+bf)^{n+1}(1+dh)^{n+1}},$$

and

$$y_{8n-4} = \frac{l^{2n}(1+bf)^n(1+dh)^n}{f^{2n-1}\prod_{i=0}^{n-1}(-1+(2i+1)hb)(-1+(2i+1)ld)(-1+2ihb)(-1+2ild)},$$

$$y_{8n-3} = \frac{ga^{2n}(1+cg)^n(1+ek)^n}{e^{2n}\prod_{i=0}^{n-1}(-1+(2i+1)ag)(-1+(2i+1)kc)(-1+(2i+2)ag)(-1+2ikc)},$$

$$y_{8n-2} = \frac{hl^{2n}(1+bf)^n(1+dh)^n}{f^{2n}\prod_{i=0}^{n-1}(-1+(2i+1)hb)(-1+(2i+1)ld)(-1+(2i+2)hb)(-1+2ild)},$$

$$y_{8n-1} = \frac{ka^{2n}(1+cg)^n(1+ek)^n}{e^{2n}\prod_{i=0}^{n-1}(-1+(2i+1)ag)(-1+(2i+1)kc)(-1+(2i+2)ag)(-1+(2i+2)kc)},$$

$$y_{8n} = \frac{l^{2n+1}(1+bf)^n(1+dh)^n}{f^{2n}\prod_{i=0}^{n-1}(-1+(2i+1)hb)(-1+(2i+1)ld)(-1+(2i+2)hb)(-1+(2i+2)ld)},$$

$$y_{8n+1} = \frac{ga^{2n+1}(1+cg)^n(1+ek)^n}{e^{2n+1}(-1+ag)\prod_{i=0}^{n-1}(-1+(2i+2)ag)(-1+(2i+1)kc)(-1+(2i+3)ag)(-1+(2i+2)kc)},$$

$$y_{8n+2} = \frac{hl^{2n+1}(1+bf)^{n+1}(1+dh)^{n-1}}{f^{2n+1}(-1+hb)\prod_{i=0}^{n-1}(-1+(2i+2)hb)(-1+(2i+1)ld)(-1+(2i+3)hb)(-1+(2i+2)ld)},$$

$$y_{8n+3} = \frac{ka^{2n+1}(1+cg)^{n+1}(1+ek)^{n}}{e^{2n+1}(-1+ag)(-1+kc)\prod_{i=0}^{n-1}(-1+(2i+2)ag)(-1+(2i+2)kc)(-1+(2i+3)ag)(-1+(2i+3)kc)},$$

where $x_{-4} = a,\ x_{-3} = b,\ x_{-2} = c,\ x_{-1} = d,\ x_0 = e,\ y_{-4} = f,\ y_{-3} = g,\ y_{-2} = h,\ y_{-1} = k,$ *and* $y_0 = l$ *with the initial value must be non-zero and* $ag,\ kc,\ hb,\ ld \notin \left\{\frac{1}{n},\ n = 1,\ 2, \ldots\right\}$, *and* $cg,\ ek,\ bf,\ dh \neq -1$.

Proof. For $n = 0$, the result holds. Now suppose $n > 0$ and our supposition hold for $n-1$. That is,

$$x_{8n-12} = \frac{e^{2n-2}\prod_{i=0}^{n-2}(-1+(2i+1)ag)(-1+(2i+1)kc)(-1+2iag)(-1+2ikc)}{a^{2n-3}(1+cg)^{n-1}(1+ek)^{n-1}},$$

$$x_{8n-11} = \frac{bf^{2n-2}\prod_{i=0}^{n-2}(-1+(2i+1)hb)(-1+(2i+1)ld)(-1+2ihb)(-1+2ild)}{l^{2n-2}(1+bf)^{n-1}(1+dh)^{n-1}},$$

$$x_{8n-10} = \frac{ce^{2n-2}\prod_{i=0}^{n-2}(-1+(2i+1)ag)(-1+(2i+1)kc)(-1+(2i+2)ag)(-1+2ikc)}{a^{2n-2}(1+cg)^{n-1}(1+ek)^{n-1}},$$

$$x_{8n-9} = \frac{df^{2n-2}\prod_{i=0}^{n-2}(-1+(2i+1)hb)(-1+(2i+1)ld)(-1+(2i+2)hb)(-1+2ild)}{l^{2n-2}(1+bf)^{n-1}(1+dh)^{n-1}},$$

$$x_{8n-8} = \frac{e^{2n-1}\prod_{i=0}^{n-2}(-1+(2i+1)ag)(-1+(2i+1)kc)(-1+(2i+2)ag)(-1+(2i+2)kc)}{a^{2n-2}(1+cg)^{n-1}(1+ek)^{n-1}},$$

$$x_{8n-7} = \frac{bf^{2n-1}\prod_{i=0}^{n-2}(-1+(2i+1)hb)(-1+(2i+1)ld)(-1+(2i+2)hb)(-1+(2i+2)ld)}{l^{2n-1}(1+bf)^{n}(1+dh)^{n-1}},$$

$$x_{8n-6} = \frac{ce^{2n-1}(-1+ag)\prod_{i=0}^{n-2}(-1+(2i+2)ag)(-1+(2i+1)kc)(-1+(2i+3)ag)(-1+(2i+2)kc)}{a^{2n-1}(1+cg)^{n}(1+ek)^{n-1}},$$

$$x_{8n-5} = \frac{df^{2n-1}(-1+hb)\prod_{i=0}^{n-2}(-1+(2i+2)hb)(-1+(2i+1)ld)(-1+(2i+3)hb)(-1+(2i+2)ld)}{l^{2n-1}(1+bf)^{n}(1+dh)^{n}},$$

and

$$y_{8n-12} = \frac{l^{2n-2}(1+bf)^{n-1}(1+dh)^{n-1}}{f^{2n-3}\prod_{i=0}^{n-2}(-1+(2i+1)hb)(-1+(2i+1)ld)(-1+2ihb)(-1+2ild)},$$

$$y_{8n-11} = \frac{ga^{2n-2}(1+cg)^{n-1}(1+ek)^{n-1}}{e^{2n-2}\prod_{i=0}^{n-2}(-1+(2i+1)ag)(-1+(2i+1)kc)(-1+(2i+2)ag)(-1+2ikc)},$$

$$y_{8n-10} = \frac{hl^{2n-2}(1+bf)^{n-1}(1+dh)^{n-1}}{f^{2n-2}\prod_{i=0}^{n-2}(-1+(2i+1)hb)(-1+(2i+1)ld)(-1+(2i+2)hb)(-1+2ild)},$$

$$y_{8n-9} = \frac{ka^{2n-2}(1+cg)^{n-1}(1+ek)^{n-1}}{e^{2n-2}\prod_{i=0}^{n-2}(-1+(2i+1)ag)(-1+(2i+1)kc)(-1+(2i+2)ag)(-1+(2i+2)kc)},$$

$$y_{8n-8} = \frac{l^{2n-1}(1+bf)^{n-1}(1+dh)^{n-1}}{f^{2n-2}\prod_{i=0}^{n-2}(-1+(2i+1)hb)(-1+(2i+1)ld)(-1+(2i+2)hb)(-1+(2i+2)ld)},$$

$$y_{8n-7} = \frac{ga^{2n-1}(1+cg)^{n-1}(1+ek)^{n-1}}{e^{2n-1}(1+ag)\prod_{i=0}^{n-2}(-1+(2i+2)ag)(-1+(2i+1)kc)(-1+(2i+3)ag)(-1+(2i+2)kc)},$$

$$y_{8n-6} = \frac{hl^{2n-1}(1+bf)^{n}(1+dh)^{n-2}}{f^{2n-1}(-1+hb)\prod_{i=0}^{n-2}(-1+(2i+2)hb)(-1+(2i+1)ld)(-1+(2i+3)hb)(-1+(2i+2)ld)},$$

$$y_{8n-5} = \frac{ka^{2n-1}(1+cg)^{n}(1+ek)^{n-1}}{e^{2n-1}(-1+ag)(-1+kc)\prod_{i=0}^{n-2}(-1+(2i+2)ag)(-1+(2i+2)kc)(-1+(2i+3)ag)(-1+(2i+3)kc)}.$$

Now, we deduce from system (4.1) that

$$\begin{aligned}
x_{8n-4} &= \frac{x_{8n-8}y_{8n-9}}{y_{8n-5}(-1+x_{8n-8}y_{8n-9})} \\
&= \frac{\left(\frac{e^{2n-1}\prod_{i=0}^{n-2}(-1+(2i+1)ag)(-1+(2i+1)kc)(-1+(2i+2)ag)(-1+(2i+2)kc)}{a^{2n-2}(1+cg)^{n-1}(1+ek)^{n-1}}\right)\left(\frac{ka^{2n-2}(1+cg)^{n-1}(1+ek)^{n-1}}{e^{2n-2}\prod_{i=0}^{n-2}(-1+(2i+1)ag)(-1+(2i+1)kc)(-1+(2i+2)ag)(-1+(2i+2)kc)}\right)}{\left(\frac{ka^{2n-1}(1+cg)^{n}(1+ek)^{n-1}}{e^{2n-1}(-1+ag)(-1+kc)\prod_{i=0}^{n-2}(-1+(2i+2)ag)(-1+(2i+2)kc)(-1+(2i+3)ag)(-1+(2i+3)kc)}\right)\left(-1+\left(\frac{e^{2n-1}\prod_{i=0}^{n-2}(-1+(2i+1)ag)(-1+(2i+1)kc)(-1+(2i+2)ag)(-1+(2i+2)kc)}{a^{2n-2}(1+cg)^{n-1}(1+ek)^{n-1}}\right)\left(\frac{ka^{2n-2}(1+cg)^{n-1}(1+ek)^{n-1}}{e^{2n-2}\prod_{i=0}^{n-2}(-1+(2i+1)ag)(-1+(2i+1)kc)(-1+(2i+2)ag)(-1+(2i+2)kc)}\right)\right)} \\
&= \left(\frac{e^{2n}}{a^{2n-1}(1+ek)}\right)\frac{\prod_{i=0}^{n-2}(-1+(2i+2)ag)(-1+(2i+2)kc)(-1+(2i+3)ag)(-1+(2i+3)kc)}{(1+cg)^{n}(1+ek)^{n-1}} \\
&= \frac{e^{2n}\prod_{i=0}^{n-1}(-1+(2i+1)ag)(-1+(2i+1)kc)(-1+2iag)(-1+2ikc)}{a^{2n-1}(1+cg)^{n}(1+ek)^{n}}.
\end{aligned}$$

Similarly,

$$y_{8n-4} = \frac{y_{8n-8}x_{8n-9}}{x_{8n-5}(1+y_{8n-8}x_{8n-9})}$$

$$
\begin{aligned}
&= \frac{\left(\dfrac{l^{2n-1}(1+bf)^{n-1}(1+dh)^{n-1}}{f^{2n-2}\prod\limits_{i=0}^{n-2}(-1+(2i+1)hb)(-1+(2i+1)ld)(-1+(2i+2)hb)(-1+(2i+2)ld)}\right)\left(\dfrac{df^{2n-2}\prod\limits_{i=0}^{n-2}(-1+(2i+1)hb)(-1+(2i+1)ld)(-1+(2i+2)hb)(-1+2ild)}{l^{2n-2}(1+bf)^{n-1}(1+dh)^{n-1}}\right)}{\left(\dfrac{df^{2n-1}(-1+hb)\prod\limits_{i=0}^{n-2}(-1+(2i+2)hb)(-1+(2i+1)ld)(-1+(2i+3)hb)(-1+(2i+2)ld)}{l^{2n-1}(1+bf)^{n}(1+dh)^{n}}\right)\left(1+\left(\dfrac{l^{2n-1}(1+bf)^{n-1}(1+dh)^{n-1}}{f^{2n-2}\prod\limits_{i=0}^{n-2}(-1+(2i+1)hb)(-1+(2i+1)ld)(-1+(2i+2)hb)(-1+(2i+2)ld)}\right)\left(\dfrac{df^{2n-2}\prod\limits_{i=0}^{n-2}(-1+(2i+1)hb)(-1+(2i+1)ld)(-1+(2i+2)hb)(-1+2ild)}{l^{2n-2}(1+bf)^{n-1}(1+dh)^{n-1}}\right)\right)}\\
&= \frac{\left(\frac{l^{2n}}{-1+(2n-2)ld}\right)}{df^{2n-1}(1+hb)\left(1+\frac{ld}{-1+(2n-2)ld}\right)}\frac{(1+bf)^{n}(1+dh)^{n}}{\prod\limits_{i=0}^{n-2}(-1+(2i+2)hb)(-1+(2i+1)ld)(-1+(2i+3)hb)(-1+(2i+2)ld)}\\
&= \left(\frac{l^{2n}}{df^{2n-1}(1+hb)(-1+(2n-1)ld)}\right)\frac{(1+bf)^{n-1}(1+dh)^{n-1}}{\prod\limits_{i=0}^{n-2}(-1+(2i+2)hb)(-1+(2i+1)ld)(-1+(2i+3)hb)(-1+(2i+2)ld)}\\
&= \frac{l^{2n}(1+bf)^{n}(1+dh)^{n}}{f^{2n-1}\prod\limits_{i=0}^{n-1}(-1+(2i+1)hb)(-1+(2i+1)ld)(-1+2ihb)(-1+2ild)}.
\end{aligned}
$$

Again, we see from system (4.1) that

$$
\begin{aligned}
x_{8n-3} &= \frac{x_{8n-7}y_{8n-8}}{y_{8n-4}(-1+x_{8n-7}y_{8n-8})}\\
&= \frac{\left(\dfrac{bf^{2n-1}\prod\limits_{i=0}^{n-2}(-1+(2i+1)hb)(-1+(2i+1)ld)(-1+(2i+2)hb)(-1+(2i+2)ld)}{l^{2n-1}(1+bf)^{n}(1+dh)^{n-1}}\right)\left(\dfrac{l^{2n-1}(1+bf)^{n-1}(1+dh)^{n-1}}{f^{2n-2}\prod\limits_{i=0}^{n-2}(-1+(2i+1)hb)(-1+(2i+1)ld)(-1+(2i+2)hb)(-1+(2i+2)ld)}\right)}{\left(\dfrac{l^{2n}(1+bf)^{n}(1+dh)^{n}}{f^{2n-1}\prod\limits_{i=0}^{n-1}(-1+(2i+1)hb)(-1+(2i+1)ld)(-1+2ihb)(-1+2ild)}\right)\left(-1+\left(\dfrac{bf^{2n-1}\prod\limits_{i=0}^{n-2}(-1+(2i+1)hb)(-1+(2i+1)ld)(-1+(2i+2)hb)(-1+(2i+2)ld)}{l^{2n-1}(1+bf)^{n}(1+dh)^{n-1}}\right)\left(\dfrac{l^{2n-1}(1+bf)^{n-1}(1+dh)^{n-1}}{f^{2n-2}\prod\limits_{i=0}^{n-2}(-1+(2i+1)hb)(-1+(2i+1)ld)(-1+(2i+2)hb)(-1+(2i+2)ld)}\right)\right)}
\end{aligned}
$$

$$= \frac{bf^{2n}}{l^{2n}\left(1+bf\right)\left(\frac{(1+bf)^{n}(1+dh)^{n}}{\prod\limits_{i=0}^{n-1}(-1+(2i+1)hb)(-1+(2i+1)ld)(-1+2ihb)(-1+2ild)}\right)}$$

$$= \frac{bf^{2n}\prod\limits_{i=0}^{n-1}(-1+(2i+1)hb)(-1+(2i+1)ld)(-1+2ihb)(-1+2ild)}{l^{2n}(1+bf)^{n}(1+dh)^{n}},$$

and

$$y_{8n-3} = \frac{y_{8n-7}x_{8n-8}}{x_{8n-4}(1+y_{8n-7}x_{8n-8})}$$

$$= \frac{\left(\frac{ga^{2n-1}(1+cg)^{n-1}(1+ek)^{n-1}}{e^{2n-1}(-1+ag)\prod\limits_{i=0}^{n-2}(-1+(2i+2)ag)(-1+(2i+1)kc)(-1+(2i+3)ag)(-1+(2i+2)kc)}\right)\left(\frac{e^{2n-1}\prod\limits_{i=0}^{n-2}(-1+(2i+1)ag)(-1+(2i+1)kc)(-1+(2i+2)ag)(-1+(2i+2)kc)}{a^{2n-2}(1+cg)^{n-1}(1+ek)^{n-1}}\right)}{\left(\frac{e^{2n}\prod\limits_{i=0}^{n-1}(-1+(2i+1)ag)(-1+(2i+1)kc)(-1+2iag)(-1+2ikc)}{a^{2n-1}(1+cg)^{n}(1+ek)^{n}}\right)\left(1+\left(\frac{ga^{2n-1}(1+cg)^{n-1}(1+ek)^{n-1}}{e^{2n-1}(1+ag)\prod\limits_{i=0}^{n-2}(-1+(2i+2)ag)(-1+(2i+1)kc)(-1+(2i+3)ag)(-1+(2i+2)kc)}\right)\left(\frac{e^{2n-1}\prod\limits_{i=0}^{n-2}(-1+(2i+1)ag)(-1+(2i+1)kc)(-1+(2i+2)ag)(-1+(2i+2)kc)}{a^{2n-2}(1+cg)^{n-1}(1+ek)^{n-1}}\right)\right)}$$

$$= \frac{\left(\frac{ga^{2n}}{-1+(2n-1)ag}\right)}{e^{2n}\left(1+\frac{ag}{(-1+(2n-1)ag)}\right)\left(\frac{\prod\limits_{i=0}^{n-1}(-1+(2i+1)ag)(-1+(2i+1)kc)(-1+2iag)(-1+2ikc)}{(1+cg)^{n}(1+ek)^{n}}\right)}$$

$$= \frac{ga^{2n}(1+cg)^{n}(1+ek)^{n}}{e^{2n}(-1+2nag)\prod\limits_{i=0}^{n-1}(-1+(2i+1)ag)(-1+(2i+1)kc)(-1+2iag)(-1+2ikc)}$$

$$= \frac{ga^{2n}(1+cg)^{n}(1+ek)^{n}}{e^{2n}\prod\limits_{i=0}^{n-1}(-1+(2i+1)ag)(-1+(2i+1)kc)(-1+(2i+2)ag)(-1+2ikc)}.$$

Now, we see from system (4.1) that

$$x_{8n} = \frac{x_{8n-4}y_{8n-5}}{y_{8n-1}(-1+x_{8n-4}y_{8n-5})}$$

$$
= \frac{\left(\frac{e^{2n}\prod\limits_{i=0}^{n-1}(-1+(2i+1)ag)(-1+(2i+1)kc)(-1+2iag)(-1+2ikc)}{a^{2n-1}(1+cg)^n(1+ek)^n}\right)\left(\frac{ka^{2n-1}(1+cg)^n(1+ek)^{n-1}}{e^{2n-1}(-1+ag)(-1+kc)\prod\limits_{i=0}^{n-2}(-1+(2i+2)ag)(-1+(2i+2)kc)(-1+(2i+3)ag)(-1+(2i+3)kc)}\right)}{\left(\frac{ka^{2n}(1+cg)^n(1+ek)^n}{e^{2n}\prod\limits_{i=0}^{n-1}(-1+(2i+1)ag)(-1+(2i+1)kc)(-1+(2i+2)ag)(-1+(2i+2)kc)}\right)\left(-1+\left(\frac{e^{2n}\prod\limits_{i=0}^{n-1}(-1+(2i+1)ag)(-1+(2i+1)kc)(-1+2iag)(-1+2ikc)}{a^{2n-1}(1+cg)^n(1+ek)^n}\right)\left(\frac{ka^{2n-1}(1+cg)^n(1+ek)^{n-1}}{e^{2n-1}(-1+ag)(-1+kc)\prod\limits_{i=0}^{n-2}(-1+(2i+2)ag)(-1+(2i+2)kc)(-1+(2i+3)ag)(-1+(2i+3)kc)}\right)\right)}
$$

$$
= \frac{\left(\frac{e^{2n+1}}{1+ek}\right)}{a^{2n}\left(-1+\frac{ek}{(1+ek)}\right)}\frac{\prod\limits_{i=0}^{n-1}(-1+(2i+1)ag)(-1+(2i+1)kc)(-1+(2i+2)ag)(-1+(2i+2)kc)}{(1+cg)^n(1+ek)^n}
$$

$$
= \frac{e^{2n+1}\prod\limits_{i=0}^{n-1}(-1+(2i+1)ag)(-1+(2i+1)kc)(-1+(2i+2)ag)(-1+(2i+2)kc)}{a^{2n}(1+cg)^n(1+ek)^n}.
$$

Similarly,

$$
y_{8n} = \frac{y_{8n-4}x_{8n-5}}{x_{8n-1}(1+y_{8n-4}x_{8n-5})}
$$

$$
= \frac{\left(\frac{l^{2n}(1+bf)^n(1+dh)^n}{f^{2n-1}\prod\limits_{i=0}^{n-1}(-1+(2i+1)hb)(-1+(2i+1)ld)(-1+2ihb)(-1+2ild)}\right)\left(\frac{df^{2n-1}(-1+hb)\prod\limits_{i=0}^{n-2}(-1+(2i+2)hb)(-1+(2i+1)ld)(-1+(2i+3)hb)(-1+(2i+2)ld)}{l^{2n-1}(1+bf)^n(1+dh)^n}\right)}{\left(\frac{df^{2n}\prod\limits_{i=0}^{n-1}(-1+(2i+1)hb)(-1+(2i+1)ld)(-1+(2i+2)hb)(-1+2ild)}{l^{2n}(1+bf)^n(1+dh)^n}\right)\left(1+\left(\frac{l^{2n}(1+bf)^n(1+dh)^n}{f^{2n-1}\prod\limits_{i=0}^{n-1}(-1+(2i+1)hb)(-1+(2i+1)ld)(-1+2ihb)(-1+2ild)}\right)\left(\frac{df^{2n-1}(-1+hb)\prod\limits_{i=0}^{n-2}(-1+(2i+2)hb)(-1+(2i+1)ld)(-1+(2i+3)hb)(-1+(2i+2)ld)}{l^{2n+1}(1+bf)^n(1+dh)^n}\right)\right)}
$$

$$
= \frac{\left(\frac{l^{2n+1}}{-1+(2n-1)ld}\right)}{f^{2n}\left(1+\frac{ld}{(-1+(2n-1)ld)}\right)}\frac{(1+bf)^n(1+dh)^n}{\prod\limits_{i=0}^{n-1}(-1+(2i+1)hb)(-1+(2i+1)ld)(-1+(2i+2)hb)(-1+2ild)}
$$

$$= \frac{l^{2n+1}(1+bf)^n(1+dh)^n}{f^{2n}(-1+2nld)\prod_{i=0}^{n-1}(-1+(2i+1)hb)(-1+(2i+1)ld)(-1+(2i+2)hb)(-1+2ild)}$$

$$= \frac{l^{2n+1}(1+bf)^n(1+dh)^n}{f^{2n}\prod_{i=0}^{n-1}(-1+(2i+1)hb)(-1+(2i+1)ld)(-1+(2i+2)hb)(-1+(2i+2)ld)}.$$

Also, the other relations can be proved similarly. This completes the proof. □

Example 4.2. In order to verify the results of this section, we show a numerical example for the difference system (4.1) with the initial conditions $x_{-4} = 0.55$, $x_{-3} = 0.4$, $x_{-2} = 0.5$, $x_{-1} = 0.6$, $x_0 = 2$, $y_{-4} = 0.9$, $y_{-3} = 0.7$, $y_{-2} = 0.25$, $y_{-1} = 0.8$, and $y_0 = 0.31$ (see Figure 3).

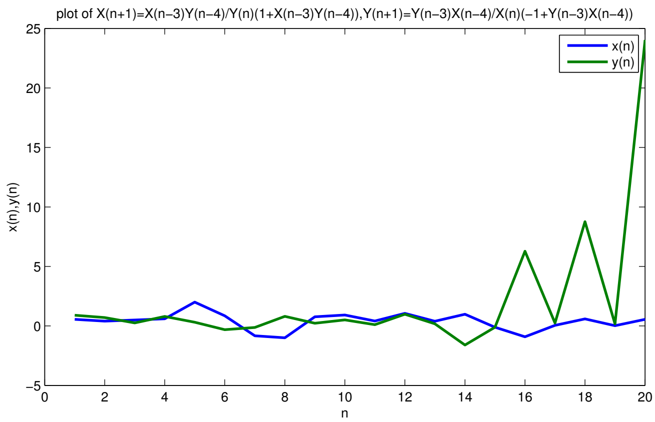

Figure 3

5. The fourth system: $x_{n+1} = x_{n-3}y_{n-4}/y_n(1 + x_{n-3}y_{n-4})$, $y_{n+1} = y_{n-3}x_{n-4}/x_n(-1 - y_{n-3}x_{n-4})$

In this section, we study the existence of analytical forms of the solutions for the following system of difference equations:

$$x_{n+1} = \frac{x_{n-3}y_{n-4}}{y_n(1+x_{n-3}y_{n-4})} \quad y_{n+1} = \frac{y_{n-3}x_{n-4}}{x_n(-1-y_{n-3}x_{n-4})}, \quad n = 0,\ 1,\ 2,\ldots, \tag{5.1}$$

with non-zero initial conditions x_{-4}, x_{-3}, x_{-2}, x_{-1}, x_0, y_{-4}, y_{-3}, y_{-2}, y_{-1} and y_0.

Theorem 5.1. *Suppose that* $\{x_n,\ y_n\}$ *is a solution for the system* (5.1) *then for* $n = 0, 1, 2, \ldots,$ *one obtains*

$$x_{8n-4} = \left(\frac{e^{2n}}{a^{2n-1}}\right)\prod_{i=0}^{n-1}\frac{(1+(2i+1)ag)(1+(2i+1)kc)(1+2iag)(1+2ikc)}{(1+(2i+1)cg)(1+(2i+1)ek)(1+2icg)(1+2iek)},$$
$$x_{8n-3} = \left(\frac{bf^{2n}}{l^{2n}}\right)\prod_{i=0}^{n-1}\frac{(1+(2i+1)hb)(1+(2i+1)ld)(1+2ihb)(1+2ild)}{(1+(2i+1)bf)(1+(2i+1)dh)(1+(2i+2)bf)(1+2idh)},$$
$$x_{8n-2} = \left(\frac{ce^{2n}}{a^{2n}}\right)\prod_{i=0}^{n-1}\frac{(1+(2i+1)ag)(1+(2i+1)kc)(1+(2i+2)ag)(1+2ikc)}{(1+(2i+1)cg)(1+(2i+1)ek)(1+(2i+2)cg)(1+2iek)},$$
$$x_{8n-1} = \left(\frac{df^{2n}}{l^{2n}}\right)\prod_{i=0}^{n-1}\frac{(1+(2i+1)hb)(1+(2i+1)ld)(1+(2i+2)hb)(1+2ild)}{(1+(2i+1)bf)(1+(2i+1)dh)(1+(2i+2)bf)(1+(2i+2)dh)},$$
$$x_{8n} = \left(\frac{e^{2n+1}}{a^{2n}}\right)\prod_{i=0}^{n-1}\frac{(1+(2i+1)ag)(1+(2i+1)kc)(1+(2i+2)ag)(1+(2i+2)kc)}{(1+(2i+1)cg)(1+(2i+1)ek)(1+(2i+2)cg)(1+(2i+2)ek)},$$

$$x_{8n+1}=\left(\frac{bf^{2n+1}}{l^{2n+1}(1+bf)}\right)\prod_{i=0}^{n-1}\frac{(1+(2i+1)hb)(1+(2i+1)ld)(1+(2i+2)hb)(1+(2i+2)ld)}{(1+(2i+2)bf)(1+(2i+1)dh)(1+(2i+3)bf)(1+(2i+2)dh)},$$
$$x_{8n+2}=\left(\frac{(-1)ce^{2n+1}(1+ag)}{a^{2n+1}(1+cg)}\right)\prod_{i=0}^{n-1}\frac{(1+(2i+2)ag)(1+(2i+1)kc)(1+(2i+3)ag)(1+(2i+2)kc)}{(1+(2i+2)cg)(1+(2i+1)ek)(1+(2i+3)cg)(1+(2i+2)ek)},$$
$$x_{8n+3}=\left(\frac{(-1)df^{2n+1}(1+hb)}{l^{2n+1}(1+bf)(1+dh)}\right)\prod_{i=0}^{n-1}\frac{(1+(2i+2)hb)(1+(2i+1)ld)(1+(2i+3)hb)(1+(2i+2)ld)}{(1+(2i+2)bf)(1+(2i+2)dh)(1+(2i+3)bf)(1+(2i+3)dh)},$$

and

$$y_{8n-4}=\left(\frac{l^{2n}}{f^{2n-1}}\right)\prod_{i=0}^{n-1}\frac{(1+(2i+1)bf)(1+(2i+1)dh)(1+2ibf)(1+2idh)}{(1+(2i+1)hb)(1+(2i+1)ld)(1+2ihb)(1+2ild)},$$
$$y_{8n-3}=\left(\frac{ga^{2n}}{e^{2n}}\right)\prod_{i=0}^{n-1}\frac{(1+(2i+1)cg)(1+(2i+1)ek)(1+2icg)(1+2iek)}{(1+(2i+1)ag)(1+(2i+1)ck)(1+(2i+2)ag)(1+2ick)},$$
$$y_{8n-2}=\left(\frac{hl^{2n}}{f^{2n}}\right)\prod_{i=0}^{n-1}\frac{(1+(2i+1)bf)(1+(2i+1)dh)(1+(2i+2)bf)(1+2idh)}{(1+(2i+1)hb)(1+(2i+1)ld)(1+(2i+2)hb)(1+2ild)},$$
$$y_{8n-1}=\left(\frac{ka^{2n}}{e^{2n}}\right)\prod_{i=0}^{n-1}\frac{(1+(2i+1)cg)(1+(2i+1)ek)(1+(2i+2)cg)(1+2iek)}{(1+(2i+1)ag)(1+(2i+1)ck)(1+(2i+2)ag)(1+(2i+2)ck)},$$
$$y_{8n}=\left(\frac{l^{2n+1}}{f^{2n}}\right)\prod_{i=0}^{n-1}\frac{(1+(2i+1)bf)(1+(2i+1)dh)(1+(2i+2)bf)(1+(2i+2)dh)}{(1+(2i+1)hb)(1+(2i+1)ld)(1+(2i+2)hb)(1+(2i+2)ld)},$$
$$y_{8n+1}=\left(\frac{(-1)ga^{2n+1}}{e^{2n+1}(1+ag)}\right)\prod_{i=0}^{n-1}\frac{(1+(2i+1)cg)(1+(2i+1)ek)(1+(2i+2)cg)(1+(2i+2)ek)}{(1+(2i+2)ag)(1+(2i+1)ck)(1+(2i+3)ag)(1+(2i+2)ck)},$$
$$y_{8n+2}=\left(\frac{(-1)hl^{2n+1}(1+bf)}{f^{2n+1}(1+hb)}\right)\prod_{i=0}^{n-1}\frac{(1+(2i+2)bf)(1+(2i+1)dh)(1+(2i+3)bf)(1+(2i+2)dh)}{(1+(2i+2)hb)(1+(2i+1)ld)(1+(2i+3)hb)(1+(2i+2)ld)},$$
$$y_{8n+3}=\left(\frac{ka^{2n+1}(1+cg)}{e^{2n+1}(1+ag)(1+ck)}\right)\prod_{i=0}^{n-1}\frac{(1+(2i+2)cg)(1+(2i+1)ek)(1+(2i+3)cg)(1+(2i+2)ek)}{(1+(2i+2)ag)(1+(2i+2)ck)(1+(2i+3)ag)(1+(2i+3)ck)},$$

where $x_{-4}=a,\ x_{-3}=b,\ x_{-2}=c,\ x_{-1}=d,\ x_0=e,\ y_{-4}=f,\ y_{-3}=g,\ y_{-2}=h,\ y_{-1}=k,$ *and* $y_0=l$ *with the initial value must be non-zero and* $ag,\ kc,\ hb,\ ld,\ cg,\ ek,\ bf,\ dh\notin\left\{-\frac{1}{n},\ n=1,\ 2,\dots\right\}$.

Proof. For $n=0$, the result holds. Now suppose $n>0$ and our supposition hold for $n-1$. That is,

$$x_{8n-12}=\left(\frac{e^{2n-2}}{a^{2n-3}}\right)\prod_{i=0}^{n-2}\frac{(1+(2i+1)ag)(1+(2i+1)kc)(1+2iag)(1+2ikc)}{(1+(2i+1)cg)(1+(2i+1)ek)(1+2icg)(1+2iek)},$$
$$x_{8n-11}=\left(\frac{bf^{2n-2}}{l^{2n-2}}\right)\prod_{i=0}^{n-2}\frac{(1+(2i+1)hb)(1+(2i+1)ld)(1+2ihb)(1+2ild)}{(1+(2i+1)bf)(1+(2i+1)dh)(1+(2i+2)bf)(1+2idh)},$$
$$x_{8n-10}=\left(\frac{ce^{2n-2}}{a^{2n-2}}\right)\prod_{i=0}^{n-2}\frac{(1+(2i+1)ag)(1+(2i+1)kc)(1+(2i+2)ag)(1+2ikc)}{(1+(2i+1)cg)(1+(2i+1)ek)(1+(2i+2)cg)(1+2iek)},$$
$$x_{8n-9}=\left(\frac{df^{2n-2}}{l^{2n-2}}\right)\prod_{i=0}^{n-2}\frac{(1+(2i+1)hb)(1+(2i+1)ld)(1+(2i+2)hb)(1+2ild)}{(1+(2i+1)bf)(1+(2i+1)dh)(1+(2i+2)bf)(1+(2i+2)dh)},$$
$$x_{8n-8}=\left(\frac{e^{2n-1}}{a^{2n-2}}\right)\prod_{i=0}^{n-2}\frac{(1+(2i+1)ag)(1+(2i+1)kc)(1+(2i+2)ag)(1+(2i+2)kc)}{(1+(2i+1)cg)(1+(2i+1)ek)(1+(2i+2)cg)(1+(2i+2)ek)},$$
$$x_{8n-7}=\left(\frac{bf^{2n-1}}{l^{2n-1}(1+bf)}\right)\prod_{i=0}^{n-2}\frac{(1+(2i+1)hb)(1+(2i+1)ld)(1+(2i+2)hb)(1+(2i+2)ld)}{(1+(2i+2)bf)(1+(2i+1)dh)(1+(2i+3)bf)(1+(2i+2)dh)},$$
$$x_{8n-6}=\left(\frac{(-1)ce^{2n-1}(1+ag)}{a^{2n-1}(1+cg)}\right)\prod_{i=0}^{n-2}\frac{(1+(2i+2)ag)(1+(2i+1)kc)(1+(2i+3)ag)(1+(2i+2)kc)}{(1+(2i+2)cg)(1+(2i+1)ek)(1+(2i+3)cg)(1+(2i+2)ek)},$$

$$x_{8n-5}=\left(\frac{(-1)df^{2n-1}(1+hb)}{l^{2n-1}(1+bf)(1+dh)}\right)\prod_{i=0}^{n-2}\frac{(1+(2i+2)hb)(1+(2i+1)ld)(1+(2i+3)hb)(1+(2i+2)ld)}{(1+(2i+2)bf)(1+(2i+2)dh)(1+(2i+3)bf)(1+(2i+3)dh)},$$

and

$$\begin{aligned}
y_{8n-12}&=\left(\frac{l^{2n-2}}{f^{2n-3}}\right)\prod_{i=0}^{n-2}\frac{(1+(2i+1)bf)(1+(2i+1)dh)(1+2ibf)(1+2idh)}{(1+(2i+1)hb)(1+(2i+1)ld)(1+2ihb)(1+2ild)},\\
y_{8n-11}&=\left(\frac{ga^{2n-2}}{e^{2n-2}}\right)\prod_{i=0}^{n-2}\frac{(1+(2i+1)cg)(1+(2i+1)ek)(1+2icg)(1+2iek)}{(1+(2i+1)ag)(1+(2i+1)ck)(1+(2i+2)ag)(1+2ick)},\\
y_{8n-10}&=\left(\frac{hl^{2n-2}}{f^{2n-2}}\right)\prod_{i=0}^{n-2}\frac{(1+(2i+1)bf)(1+(2i+1)dh)(1+(2i+2)bf)(1+2idh)}{(1+(2i+1)hb)(1+(2i+1)ld)(1+(2i+2)hb)(1+2ild)},\\
y_{8n-9}&=\left(\frac{ka^{2n-2}}{e^{2n-2}}\right)\prod_{i=0}^{n-2}\frac{(1+(2i+1)cg)(1+(2i+1)ek)(1+(2i+2)cg)(1+2iek)}{(1+(2i+1)ag)(1+(2i+1)ck)(1+(2i+2)ag)(1+(2i+2)ck)},\\
y_{8n-8}&=\left(\frac{l^{2n-1}}{f^{2n-2}}\right)\prod_{i=0}^{n-2}\frac{(1+(2i+1)bf)(1+(2i+1)dh)(1+(2i+2)bf)(1+(2i+2)dh)}{(1+(2i+1)hb)(1+(2i+1)ld)(1+(2i+2)hb)(1+(2i+2)ld)},\\
y_{8n-7}&=\left(\frac{(-1)ga^{2n-1}}{e^{2n-1}(1+ag)}\right)\prod_{i=0}^{n-2}\frac{(1+(2i+1)cg)(1+(2i+1)ek)(1+(2i+2)cg)(1+(2i+2)ek)}{(1+(2i+2)ag)(1+(2i+1)ck)(1+(2i+3)ag)(1+(2i+2)ck)},\\
y_{8n-6}&=\left(\frac{(-1)hl^{2n-1}(1+bf)}{f^{2n-1}(1+hb)}\right)\prod_{i=0}^{n-2}\frac{(1+(2i+2)bf)(1+(2i+1)dh)(1+(2i+3)bf)(1+(2i+2)dh)}{(1+(2i+2)hb)(1+(2i+1)ld)(1+(2i+3)hb)(1+(2i+2)ld)},\\
y_{8n-5}&=\left(\frac{ka^{2n-1}(1+cg)}{e^{2n-1}(1+ag)(1+ck)}\right)\prod_{i=0}^{n-2}\frac{(1+(2i+2)cg)(1+(2i+1)ek)(1+(2i+3)cg)(1+(2i+2)ek)}{(1+(2i+2)ag)(1+(2i+2)ck)(1+(2i+3)ag)(1+(2i+3)ck)}.
\end{aligned}$$

Deducing from system (5.1)

$$\begin{aligned}
x_{8n-4}&=\frac{x_{8n-8}y_{8n-9}}{y_{8n-5}(1+x_{8n-8}y_{8n-9})}\\
&=\frac{\left(\left(\frac{e^{2n-1}}{a^{2n-2}}\right)\prod_{i=0}^{n-2}\frac{(1+(2i+1)ag)(1+(2i+1)kc)(1+(2i+2)ag)(1+(2i+2)kc)}{(1+(2i+1)cg)(1+(2i+1)ek)(1+(2i+2)cg)(1+(2i+2)ek)}\right)\left(\left(\frac{ka^{2n-2}}{e^{2n-2}}\right)\prod_{i=0}^{n-2}\frac{(1+(2i+1)cg)(1+(2i+1)ek)(1+(2i+2)cg)(1+2iek)}{(1+(2i+1)ag)(1+(2i+1)ck)(1+(2i+2)ag)(1+(2i+2)ck)}\right)}{\left(\left(\frac{ka^{2n-1}(1+cg)}{e^{2n-1}(1+ag)(1+ck)}\right)\prod_{i=0}^{n-2}\frac{(1+(2i+2)cg)(1+(2i+1)ek)(1+(2i+3)cg)(1+(2i+2)ek)}{(1+(2i+2)ag)(1+(2i+2)ck)(1+(2i+3)ag)(1+(2i+3)ck)}\right)\left(1+\left(\left(\frac{e^{2n-1}}{a^{2n-2}}\right)\prod_{i=0}^{n-2}\frac{(1+(2i+1)ag)(1+(2i+1)kc)(1+(2i+2)ag)(1+(2i+2)kc)}{(1+(2i+1)cg)(1+(2i+1)ek)(1+(2i+2)cg)(1+(2i+2)ek)}\right)\left(\left(\frac{a^{2n-2}}{e^{2n-2}}\right)\prod_{i=0}^{n-2}\frac{(1+(2i+1)cg)(1+(2i+1)ek)(1+(2i+2)cg)(1+2iek)}{(1+(2i+1)ag)(1+(2i+1)ck)(1+(2i+2)ag)(1+(2i+2)ck)}\right)\right)}\\
&=\frac{\left(\frac{e^{2n}}{(1+(2n-2)ek)}\right)(1+ag)(1+ck)}{a^{2n-1}(1+cg)\left(1+\frac{ek}{1+(2n-2)ek}\right)}\left(\prod_{i=0}^{n-2}\frac{(1+(2i+2)ag)(1+(2i+2)ck)(1+(2i+3)ag)(1+(2i+3)ck)}{(1+(2i+2)cg)(1+(2i+1)ek)(1+(2i+3)cg)(1+(2i+2)ek)}\right)\\
&=\left(\frac{e^{2n}(1+ag)(1+ck)}{a^{2n-1}(1+cg)(1+(2n-1)ek)}\right)\prod_{i=0}^{n-2}\frac{(1+(2i+2)ag)(1+(2i+2)ck)(1+(2i+3)ag)(1+(2i+3)ck)}{(1+(2i+2)cg)(1+(2i+1)ek)(1+(2i+3)cg)(1+(2i+2)ek)}\\
&=\left(\frac{e^{2n}}{a^{2n-1}}\right)\prod_{i=0}^{n-1}\frac{(1+(2i+1)ag)(1+(2i+1)kc)(1+2iag)(1+2ikc)}{(1+(2i+1)cg)(1+(2i+1)ek)(1+2icg)(1+2iek)}.
\end{aligned}$$

and

$$y_{8n-4}=\frac{y_{8n-8}x_{8n-9}}{x_{8n-5}(-1-y_{8n-8}x_{8n-9})}$$

$$
\begin{aligned}
&= \frac{\left(\left(\frac{l^{2n-1}}{f^{2n-2}}\right)\prod_{i=0}^{n-2}\frac{(1+(2i+1)bf)(1+(2i+1)dh)(1+(2i+2)bf)(1+(2i+2)dh)}{(1+(2i+1)hb)(1+(2i+1)ld)(1+(2i+2)hb)(1+(2i+2)ld)}\right)\left(\left(\frac{df^{2n-2}}{l^{2n-2}}\right)\prod_{i=0}^{n-2}\frac{(1+(2i+1)hb)(1+(2i+1)ld)(1+(2i+2)hb)(1+2ild)}{(1+(2i+1)bf)(1+(2i+1)dh)(1+(2i+2)bf)(1+(2i+2)dh)}\right)}{\left(\left(\frac{(-1)df^{2n-1}(1+hb)}{l^{2n-1}(1+bf)(1+dh)}\right)\prod_{i=0}^{n-2}\frac{(1+(2i+2)hb)(1+(2i+1)ld)(1+(2i+3)hb)(1+(2i+2)ld)}{(1+(2i+2)bf)(1+(2i+2)dh)(1+(2i+3)bf)(1+(2i+3)dh)}\right)\left(\begin{array}{c}-1-\left(\left(\frac{l^{2n-1}}{f^{2n-2}}\right)\prod_{i=0}^{n-2}\frac{(1+(2i+1)bf)(1+(2i+1)dh)(1+(2i+2)bf)(1+(2i+2)dh)}{(1+(2i+1)hb)(1+(2i+1)ld)(1+(2i+2)hb)(1+(2i+2)ld)}\right)\\ \left(\left(\frac{df^{2n-2}}{l^{2n-2}}\right)\prod_{i=0}^{n-2}\frac{(1+(2i+1)hb)(1+(2i+1)ld)(1+(2i+2)hb)(1+2ild)}{(1+(2i+1)bf)(1+(2i+1)dh)(1+(2i+2)bf)(1+(2i+2)dh)}\right)\end{array}\right)} \\
&= \frac{\left(\frac{l^{2n}}{(1+(2n-2)ld)}\right)(1+bf)(1+dh)}{f^{2n-1}(1+hb)\left(1+\frac{ld}{1+(2n-2)ld}\right)}\left(\prod_{i=0}^{n-2}\frac{(1+(2i+2)bf)(1+(2i+2)dh)(1+(2i+3)bf)(1+(2i+3)dh)}{(1+(2i+2)hb)(1+(2i+1)ld)(1+(2i+3)hb)(1+(2i+2)ld)}\right) \\
&= \left(\frac{l^{2n}(1+bf)(1+dh)}{f^{2n-1}(1+hb)(1+(2n-1)ld)}\right)\prod_{i=0}^{n-2}\frac{(1+(2i+2)bf)(1+(2i+2)dh)(1+(2i+3)bf)(1+(2i+3)dh)}{(1+(2i+2)hb)(1+(2i+1)ld)(1+(2i+3)hb)(1+(2i+2)ld)} \\
&= \left(\frac{l^{2n}}{f^{2n-1}}\right)\prod_{i=0}^{n-1}\frac{(1+(2i+1)bf)(1+(2i+1)dh)(1+2ibf)(1+2idh)}{(1+(2i+1)hb)(1+(2i+1)ld)(1+2ihb)(1+2ild)}.
\end{aligned}
$$

Again, extracting from system (5.1) we get,

$$
\begin{aligned}
x_{8n-2} &= \frac{x_{8n-6}y_{8n-7}}{y_{8n-3}(1+x_{8n-6}y_{8n-7})} \\
&= \frac{\left(\left(\frac{(-1)ce^{2n-1}(1+ag)}{a^{2n-1}(1+cg)}\right)\prod_{i=0}^{n-2}\frac{(1+(2i+2)ag)(1+(2i+1)kc)(1+(2i+3)ag)(1+(2i+2)kc)}{(1+(2i+2)cg)(1+(2i+1)ek)(1+(2i+3)cg)(1+(2i+2)ek)}\right)\left(\left(\frac{(-1)ga^{2n-1}}{e^{2n-1}(1+ag)}\right)\prod_{i=0}^{n-2}\frac{(1+(2i+1)cg)(1+(2i+1)ek)(1+(2i+2)cg)(1+(2i+2)ek)}{(1+(2i+2)ag)(1+(2i+1)kc)(1+(2i+3)ag)(1+(2i+2)kc)}\right)}{\left(\left(\frac{ga^{2n}}{e^{2n}}\right)\prod_{i=0}^{n-1}\frac{(1+(2i+1)cg)(1+(2i+1)ek)(1+2icg)(1+2iek)}{(1+(2i+1)ag)(1+(2i+1)kc)(1+(2i+2)ag)(1+2ikc)}\right)\left(\begin{array}{c}1+\left(\left(\frac{(-1)ce^{2n-1}(1+ag)}{a^{2n-1}(1+cg)}\right)\prod_{i=0}^{n-2}\frac{(1+(2i+2)ag)(1+(2i+1)kc)(1+(2i+3)ag)(1+(2i+2)kc)}{(1+(2i+2)cg)(1+(2i+1)ek)(1+(2i+3)cg)(1+(2i+2)ek)}\right)\\ \left(\left(\frac{(-1)ga^{2n-3}}{e^{2n-3}(1+ag)}\right)\prod_{i=0}^{n-2}\frac{(1+(2i+1)cg)(1+(2i+1)ek)(1+(2i+2)cg)(1+(2i+2)ek)}{(1+(2i+2)ag)(1+(2i+1)kc)(1+(2i+3)ag)(1+(2i+2)kc)}\right)\end{array}\right)} \\
&= \frac{\left(\frac{ce^{2n}}{(1+2(n-1)cg}\right)}{a^{2n}\left(1+\frac{cg}{1+2(n-1)cg}\right)\left(\prod_{i=0}^{n-1}\frac{(1+(2i+1)cg)(1+(2i+1)ek)(1+2icg)(1+2iek)}{(1+(2i+1)ag)(1+(2i+1)kc)(1+(2i+2)ag)(1+2ikc)}\right)} \\
&= \left(\frac{ce^{2n}}{a^{2n}(1+2ncg)}\right)\prod_{i=0}^{n-1}\frac{(1+(2i+1)ag)(1+(2i+1)ck)(1+(2i+2)ag)(1+2ick)}{(1+(2i+1)cg)(1+(2i+1)ek)(1+2icg)(1+2iek)} \\
&= \left(\frac{ce^{2n}}{a^{2n}}\right)\prod_{i=0}^{n-1}\frac{(1+(2i+1)ag)(1+(2i+1)kc)(1+(2i+2)ag)(1+2ikc)}{(1+(2i+1)cg)(1+(2i+1)ek)(1+(2i+2)cg)(1+2iek)}.
\end{aligned}
$$

and

$$
y_{8n-2} = \frac{y_{8n-6}x_{8n-7}}{x_{8n-3}(-1-y_{8n-6}x_{8n-7})}
$$

$$= \frac{\left(\left(\frac{(-1)hl^{2n-1}(1+bf)}{f^{2n-1}(1+hb)}\right)\prod_{i=0}^{n-2}\frac{(1+(2i+2)bf)(1+(2i+1)dh)(1+(2i+3)bf)(1+(2i+2)dh)}{(1+(2i+2)hb)(1+(2i+1)ld)(1+(2i+3)hb)(1+(2i+2)ld)}\right)\left(\left(\frac{bf^{2n-1}}{l^{2n-1}(1+bf)}\right)\prod_{i=0}^{n-2}\frac{(1+(2i+1)hb)(1+(2i+1)ld)(1+(2i+2)hb)(1+(2i+2)ld)}{(1+(2i+2)bf)(1+(2i+1)dh)(1+(2i+3)bf)(1+(2i+2)dh)}\right)}{\left(\left(\frac{bf^{2n}}{l^{2n}}\right)\prod_{i=0}^{n-1}\frac{(1+(2i+1)hb)(1+(2i+1)ld)(1+2ihb)(1+2ild)}{(1+(2i+1)bf)(1+(2i+1)dh)(1+(2i+2)bf)(1+2idh)}\right)\left(-1-\left(\left(\frac{(-1)hl^{2n-1}(1+bf)}{f^{2n-1}(1+hb)}\right)\prod_{i=0}^{n-2}\frac{(1+(2i+2)bf)(1+(2i+1)dh)(1+(2i+3)bf)(1+(2i+2)dh)}{(1+(2i+2)hb)(1+(2i+1)ld)(1+(2i+3)hb)(1+(2i+2)ld)}\right)\left(\left(\frac{bf^{2n-1}}{l^{2n-1}(1+bf)}\right)\prod_{i=0}^{n-2}\frac{(1+(2i+1)hb)(1+(2i+1)ld)(1+(2i+2)hb)(1+(2i+2)ld)}{(1+(2i+2)bf)(1+(2i+1)dh)(1+(2i+3)bf)(1+(2i+2)dh)}\right)\right)}$$

$$= \frac{\left(\frac{hb}{1+2(n-1)hb}\right)}{\left(1-\frac{hb}{1+2(n-1)hb}\right)\left(\left(\frac{bf^{2n}}{l^{2n}}\right)\prod_{i=0}^{n-1}\frac{(1+(2i+1)hb)(1+(2i+1)ld)(1+2ihb)(1+2ild)}{(1+(2i+1)bf)(1+(2i+1)dh)(1+(2i+2)bf)(1+2idh)}\right)}$$

$$= \left(\frac{hl^{2n}}{f^{2n}(1+(2n-2)hb)}\right)\prod_{i=0}^{n-1}\frac{(1+(2i+1)bf)(1+(2i+1)dh)(1+(2i+2)bf)(1+2idh)}{(1+(2i+1)hb)(1+(2i+1)ld)(1+2ihb)(1+2ild)}$$

$$= \left(\frac{hl^{2n}}{f^{2n}}\right)\prod_{i=0}^{n-1}\frac{(1+(2i+1)bf)(1+(2i+1)dh)(1+(2i+2)bf)(1+2idh)}{(1+(2i+1)hb)(1+(2i+1)ld)(1+(2i+2)hb)(1+2ild)}.$$

From system (5.1),

$$x_{8n} = \frac{x_{8n-4}y_{8n-5}}{y_{8n-1}(1+x_{8n-4}y_{8n-5})}$$

$$= \frac{\left(\left(\frac{e^{2n}}{a^{2n-1}}\right)\prod_{i=0}^{n-1}\frac{(1+(2i+1)ag)(1+(2i+1)kc)(1+2iag)(1+2ikc)}{(1+(2i+1)cg)(1+(2i+1)ek)(1+2icg)(1+2iek)}\right)\left(\left(\frac{ka^{2n-1}(1+cg)}{e^{2n-1}(1+ag)(1+ck)}\right)\prod_{i=0}^{n-2}\frac{(1+(2i+2)cg)(1+(2i+1)ek)(1+(2i+3)cg)(1+(2i+2)ek)}{(1+(2i+2)ag)(1+(2i+2)ck)(1+(2i+3)ag)(1+(2i+3)ck)}\right)}{\left(\left(\frac{ka^{2n}}{e^{2n}}\right)\prod_{i=0}^{n-1}\frac{(1+(2i+1)cg)(1+(2i+1)ek)(1+(2i+2)cg)(1+2iek)}{(1+(2i+1)ag)(1+(2i+1)ck)(1+(2i+2)ag)(1+(2i+2)ck)}\right)\left(1+\left(\left(\frac{e^{2n}}{a^{2n-1}}\right)\prod_{i=0}^{n-1}\frac{(1+(2i+1)ag)(1+(2i+1)kc)(1+2iag)(1+2ikc)}{(1+(2i+1)cg)(1+(2i+1)ek)(1+2icg)(1+2iek)}\right)\left(\left(\frac{ka^{2n-1}(1+cg)}{e^{2n-1}(1+ag)(1+ck)}\right)\prod_{i=0}^{n-2}\frac{(1+(2i+2)cg)(1+(2i+1)ek)(1+(2i+3)cg)(1+(2i+2)ek)}{(1+(2i+2)ag)(1+(2i+2)ck)(1+(2i+3)ag)(1+(2i+3)ck)}\right)\right)}$$

$$= \frac{\left(\frac{e^{2n+1}}{1+2(n-1)ek}\right)}{a^{2n}\left(1+\frac{ek}{1+2(n-1)ek}\right)\left(\prod_{i=0}^{n-1}\frac{(1+(2i+1)cg)(1+(2i+1)ek)(1+(2i+2)cg)(1+2iek)}{(1+(2i+1)ag)(1+(2i+1)kc)(1+(2i+2)ag)(1+(2i+2)kc)}\right)}$$

$$= \left(\frac{e^{2n+1}}{a^{2n}(1+2nek)}\right)\prod_{i=0}^{n-1}\frac{(1+(2i+1)ag)(1+(2i+1)ck)(1+(2i+2)ag)(1+(2i+2)ck)}{(1+(2i+1)cg)(1+(2i+1)ek)(1+(2i+2)cg)(1+2iek)}$$

$$= \left(\frac{e^{2n+1}}{a^{2n}}\right)\prod_{i=0}^{n-1}\frac{(1+(2i+1)ag)(1+(2i+1)kc)(1+(2i+2)ag)(1+(2i+2)kc)}{(1+(2i+1)cg)(1+(2i+1)ek)(1+(2i+2)cg)(1+(2i+2)ek)}.$$

and

$$y_{8n} = \frac{y_{8n-4}x_{8n-5}}{x_{8n-1}(-1-y_{8n-4}x_{8n-5})}$$

$$
\begin{aligned}
&= \frac{\genfrac{}{}{0pt}{}{\left(\left(\frac{l^{2n}}{f^{2n-1}}\right)\prod_{i=0}^{n-1}\frac{(1+(2i+1)bf)(1+(2i+1)dh)(1+2ibf)(1+2idh)}{(1+(2i+1)hb)(1+(2i+1)ld)(1+2ihb)(1+2ild)}\right)}{\left(\left(\frac{(-1)df^{2n-1}(1+hb)}{l^{2n-1}(1+bf)(1+dh)}\right)\prod_{i=0}^{n-2}\frac{(1+(2i+2)hb)(1+(2i+1)ld)(1+(2i+3)hb)(1+(2i+2)ld)}{(1+(2i+2)bf)(1+(2i+2)dh)(1+(2i+3)bf)(1+(2i+3)dh)}\right)}}{\left(\left(\frac{df^{2n}}{l^{2n}}\right)\prod_{i=0}^{n-1}\frac{(1+(2i+1)hb)(1+(2i+1)ld)(1+(2i+2)hb)(1+2ild)}{(1+(2i+1)bf)(1+(2i+1)dh)(1+(2i+2)bf)(1+(2i+2)dh)}\right)\left(\genfrac{}{}{0pt}{}{-1-\left(\left(\frac{l^{2n}}{f^{2n-1}}\right)\prod_{i=0}^{n-1}\frac{(1+(2i+1)bf)(1+(2i+1)dh)(1+2ibf)(1+2idh)}{(1+(2i+1)hb)(1+(2i+1)ld)(1+2ihb)(1+2ild)}\right)}{\left(\left(\frac{(-1)df^{2n-1}(1+hb)}{l^{2n-1}(1+bf)(1+dh)}\right)\prod_{i=0}^{n-2}\frac{(1+(2i+2)hb)(1+(2i+1)ld)(1+(2i+3)hb)(1+(2i+2)ld)}{(1+(2i+2)bf)(1+(2i+2)dh)(1+(2i+3)bf)(1+(2i+3)dh)}\right)}\right)} \\
&= \frac{\left(\frac{l^{2n+1}}{1+2(n-1)ld}\right)}{f^{2n}\left(1+\frac{ld}{1+2(n-1)ld}\right)\left(\prod_{i=0}^{n-1}\frac{(1+(2i+1)hb)(1+(2i+1)ld)(1+(2i+2)hb)(1+2ild)}{(1+(2i+1)bf)(1+(2i+1)dh)(1+(2i+2)bf)(1+(2i+2)dh)}\right)} \\
&= \frac{l^{2n+1}}{f^{2n}(1+2nld)}\prod_{i=0}^{n-1}\frac{(1+(2i+1)bf)(1+(2i+1)dh)(1+(2i+2)bf)(1+(2i+2)dh)}{(1+(2i+1)hb)(1+(2i+1)ld)(1+(2i+2)hb)(1+2ild)} \\
&= \left(\frac{l^{2n+1}}{f^{2n}}\right)\prod_{i=0}^{n-1}\frac{(1+(2i+1)bf)(1+(2i+1)dh)(1+(2i+2)bf)(1+(2i+2)dh)}{(1+(2i+1)hb)(1+(2i+1)ld)(1+(2i+2)hb)(1+(2i+2)ld)}.
\end{aligned}
$$

Also, the other relations can be proved similarly. This completes the proof. □

Example 5.2. In order to verify the results of this section, we deal with some numerical example for the difference system (5.1) with the initial conditions $x_{-4} = 0.17$, $x_{-3} = 0.34$, $x_{-2} = 0.4$, $x_{-1} = 0.5$, $x_0 = 0.25$, $y_{-4} = 0.9$, $y_{-3} = 0.7$; $y_{-2} = 0.2$, $y_{-1} = 0.6$, and $y_0 = 0.15$ (see Figure 4).

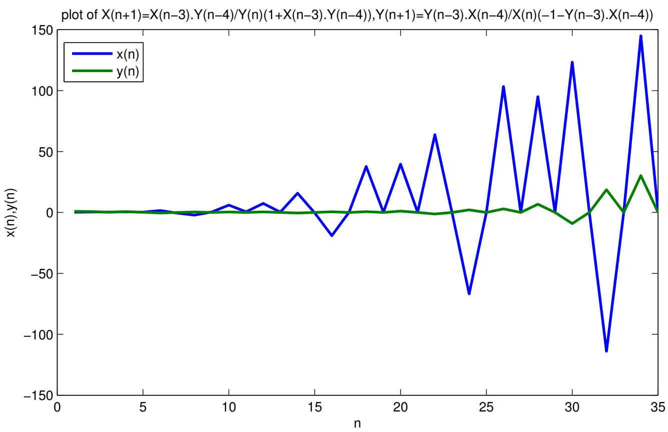

Figure 4

6. Other systems

The following theorem can be proved similarly.

Theorem 6.1. *Suppose that* $\{x_n, y_n\}$ *is a solution for the system*

$$
x_{n+1} = \frac{x_{n-3}y_{n-4}}{y_n(1 - x_{n-3}y_{n-4})}, \qquad y_{n+1} = \frac{y_{n-3}x_{n-4}}{x_n(1 + y_{n-3}x_{n-4})}, \qquad n = 0, 1, 2, \ldots,
$$

then for $n = 0,\ 1,\ 2, \ldots,$ *one obtains*

$$
x_{4n-3} = \left(\frac{bf^n}{l^n}\right)\prod_{i=0}^{n-1}\frac{(1+ihb)(1+ild)}{(1-(i+1)bf)(1-idh)}, \qquad x_{4n-2} = \left(\frac{ce^n}{a^n}\right)\prod_{i=0}^{n-1}\frac{(1+(i+1)ag)(1+ikc)}{(1-(i+1)cg)(1-iek)},
$$

$$x_{4n-1}=\left(\frac{df^n}{l^n}\right)\prod_{i=0}^{n-1}\frac{(1+(i+1)hb)(1+ild)}{(1-(i+1)bf)(1-(i+1)dh)},\qquad x_{4n}=\left(\frac{e^{n+1}}{a^n}\right)\prod_{i=0}^{n-1}\frac{(1+(i+1)ag)(1+(i+1)kc)}{(1-(i+1)cg)(1-(i+1)ek)},$$

and

$$y_{4n-3}=\left(\frac{ga^n}{e^n}\right)\prod_{i=0}^{n-1}\frac{(1-icg)(1-iek)}{(1+(i+1)ag)(1+ikc)},\qquad y_{4n-2}=\left(\frac{l^nh}{f^n}\right)\prod_{i=0}^{n-1}\frac{(1-(i+1)bf)(1-idh)}{(1+(i+1)hb)(1+ild)},$$

$$y_{4n-1}=\left(\frac{ka^n}{e^n}\right)\prod_{i=0}^{n-1}\frac{(1-(i+1)cg)(1-iek)}{(1+(i+1)ag)(1+(i+1)kc)},\qquad y_{4n}=\left(\frac{l^{n+1}}{f^n}\right)\prod_{i=0}^{n-1}\frac{(1-(i+1)bf)(1-(i+1)dh)}{(1+(i+1)hb)(1+(i+1)ld)},$$

where the initial value must be non-zero and $ag,\ kc,\ hb,\ ld\notin\left\{-\frac{1}{n},\ n=1,\ 2,\ldots\right\}$ *and* $cg,\ ek,\ bf,\ dh\notin\left\{\frac{1}{n},\ n=1,\ 2,\ldots\right\}$.

Theorem 6.2. *Suppose that* $\{x_n,\ y_n\}$ *is a solution for the system*

$$x_{n+1}=\frac{x_{n-3}y_{n-4}}{y_n(1-x_{n-3}y_{n-4})},\qquad y_{n+1}=\frac{y_{n-3}x_{n-4}}{x_n(1-y_{n-3}x_{n-4})},\qquad n=0,1,2,\ldots,$$

with non-zero initial conditions $x_{-4},\ x_{-3},\ x_{-2},\ x_{-1},\ x_0,\ y_{-3},\ y_{-3},\ y_{-2},\ y_{-1}$, *and* y_0, *then for* $n=0,1,2,\ldots,$

$$x_{4n-3}=\left(\frac{bf^n}{l^n}\right)\prod_{i=0}^{n-1}\frac{(1-ihb)(1-ild)}{(1-(i+1)bf)(1-idh)},\qquad x_{4n-2}=\left(\frac{ce^n}{a^n}\right)\prod_{i=0}^{n-1}\frac{(1-(i+1)ag)(1-ikc)}{(1-(i+1)cg)(1-iek)},$$

$$x_{4n-1}=\left(\frac{df^n}{l^n}\right)\prod_{i=0}^{n-1}\frac{(1-(i+1)hb)(1-ild)}{(1-(i+1)bf)(1-(i+1)dh)},\qquad x_{4n}=\left(\frac{e^{n+1}}{a^n}\right)\prod_{i=0}^{n-1}\frac{(1-(i+1)ag)(1-(i+1)kc)}{(1-(i+1)cg)(1-(i+1)ek)},$$

and

$$y_{4n-3}=\left(\frac{ga^n}{e^n}\right)\prod_{i=0}^{n-1}\frac{(1-icg)(1-iek)}{(1-(i+1)ag)(1-ikc)},\qquad y_{4n-2}=\left(\frac{l^nh}{f^n}\right)\prod_{i=0}^{n-1}\frac{(1-(i+1)bf)(1-idh)}{(1-(i+1)hb)(1-ild)},$$

$$y_{4n-1}=\left(\frac{ka^n}{e^n}\right)\prod_{i=0}^{n-1}\frac{(1-(i+1)cg)(1-iek)}{(1-(i+1)ag)(1-(i+1)kc)},\qquad y_{4n}=\left(\frac{l^{n+1}}{f^n}\right)\prod_{i=0}^{n-1}\frac{(1-(i+1)bf)(1-(i+1)dh)}{(1-(i+1)hb)(1-(i+1)ld)},$$

where $x_{-4}=a,\ x_{-3}=b,\ x_{-2}=c,\ x_{-1}=d,\ x_0=e,\ y_{-4}=f,\ y_{-3}=g,\ y_{-2}=h,\ y_{-1}=k$, *and* $y_0=l$ *with* $ag,\ kc,\ hb,\ ld,\ cg,\ ek,\ bf,\ dh\notin\left\{\frac{1}{n},\ n=1,\ 2,\ldots\right\}$.

Theorem 6.3. *Suppose that* $\{x_n,\ y_n\}$ *is a solution for the system*

$$x_{n+1}=\frac{x_{n-3}y_{n-4}}{y_n(-1-x_{n-3}y_{n-4})},\qquad y_{n+1}=\frac{y_{n-3}x_{n-4}}{x_n(1-y_{n-3}x_{n-4})},\qquad n=0,1,\ldots,$$

then for $n=0,\ 1,\ldots,$

$$x_{8n-4}=\frac{e^{2n}\prod_{i=0}^{n-1}(1-(2i+1)ag)(1-(2i+1)kc)(1-2iag)(1-2ikc)}{a^{2n-1}(1+cg)^n(1+ek)^n},$$

$$x_{8n-3}=\frac{bf^{2n}\prod_{i=0}^{n-1}(1-(2i+1)hb)(1-(2i+1)ld)(1-2ihb)(1-2ild)}{l^{2n}(1+bf)^n(1+dh)^n},$$

$$x_{8n-2} = \frac{ce^{2n}\prod_{i=0}^{n-1}(1-(2i+1)ag)(1-(2i+1)kc)(1-(2i+2)ag)(1-2ikc)}{a^{2n}(1+cg)^{n}(1+ek)^{n}},$$

$$x_{8n-1} = \frac{df^{2n}\prod_{i=0}^{n-1}(1-(2i+1)hb)(1-(2i+1)ld)(1-(2i+2)hb)(1-2ild)}{l^{2n}(1+bf)^{n}(1+dh)^{n}},$$

$$x_{8n} = \frac{e^{2n+1}\prod_{i=0}^{n-1}(1-(2i+1)ag)(1-(2i+1)kc)(1-(2i+2)ag)(1-(2i+2)kc)}{a^{2n}(1+cg)^{n}(1+ek)^{n}},$$

$$x_{8n+1} = \frac{(-1)bf^{2n+1}\prod_{i=0}^{n-1}(1-(2i+1)hb)(1-(2i+1)ld)(1-(2i+2)hb)(1-(2i+2)ld)}{l^{2n+1}(1+bf)^{n+1}(1+dh)^{n}},$$

$$x_{8n+2} = \frac{(-1)ce^{2n+1}(1-ag)\prod_{i=0}^{n-1}(1-(2i+2)ag)(1-(2i+1)kc)(1-(2i+3)ag)(1-(2i+2)kc)}{a^{2n+1}(1+cg)^{n+1}(1+ek)^{n}},$$

$$x_{8n+3} = \frac{df^{2n+1}(1-hb)\prod_{i=0}^{n-1}(1-(2i+2)hb)(1-(2i+1)ld)(1-(2i+3)hb)(1-(2i+2)ld)}{l^{2n+1}(1+bf)^{n+1}(1+dh)^{n+1}},$$

and

$$y_{8n-4} = \frac{l^{2n}(1+bf)^{n}(1+dh)^{n}}{f^{2n-1}\prod_{i=0}^{n-1}(1-(2i+1)hb)(1-(2i+1)ld)(1-2ihb)(1-2ild)},$$

$$y_{8n-3} = \frac{ga^{2n}(1+cg)^{n}(1+ek)^{n}}{e^{2n}\prod_{i=0}^{n-1}(1-(2i+1)ag)(1-(2i+1)kc)(1-(2i+2)ag)(1-2ikc)},$$

$$y_{8n-2} = \frac{hl^{2n}(1+bf)^{n}(1+dh)^{n}}{f^{2n}\prod_{i=0}^{n-1}(1-(2i+1)hb)(1-(2i+1)ld)(1-(2i+2)hb)(1-2ild)},$$

$$y_{8n-1} = \frac{ka^{2n}(1+cg)^{n}(1+ek)^{n}}{e^{2n}\prod_{i=0}^{n-1}(1-(2i+1)ag)(1-(2i+1)kc)(1-(2i+2)ag)(1-(2i+2)kc)},$$

$$y_{8n} = \frac{l^{2n+1}(1+bf)^{n}(1+dh)^{n}}{f^{2n}\prod_{i=0}^{n-1}(1-(2i+1)hb)(1-(2i+1)ld)(1-(2i+2)hb)(1-(2i+2)ld)},$$

$$y_{8n+1} = \frac{ga^{2n+1}(1+cg)^{n}(1+ek)^{n}}{e^{2n+1}(1-ag)\prod_{i=0}^{n-1}(1-(2i+2)ag)(1-(2i+1)kc)(1-(2i+3)ag)(1-(2i+2)kc)},$$

$$y_{8n+2} = \frac{(-1)hl^{2n+1}(1+bf)^{n+1}(1+dh)^{n-1}}{f^{2n+1}(1-hb)\prod_{i=0}^{n-1}(1-(2i+2)hb)(1-(2i+1)ld)(1-(2i+3)hb)(1-(2i+2)ld)},$$

$$y_{8n+3} = \frac{(-1)ka^{2n+1}(1+cg)^{n+1}(1+ek)^{n}}{e^{2n+1}(1-ag)(1-kc)\prod_{i=0}^{n-1}(1-(2i+2)ag)(1-(2i+2)kc)(1-(2i+3)ag)(1-(2i+3)kc)},$$

where $x_{-4}=a,\ x_{-3}=b,\ x_{-2}=c,\ x_{-1}=d,\ x_0=e,\ y_{-4}=f,\ y_{-3}=g,\ y_{-2}=h,\ y_{-1}=k,$ *and* $y_0=l$ *with the initial value must be non-zero and* $ag,\ kc,\ hb,\ ld\notin\left\{\frac{1}{n},\ n=1,\ 2,\dots\right\}$ *and* $cg,\ ek,\ bf,\ dh\neq-1.$

Theorem 6.4. *Suppose that* $\{x_n,\ y_n\}$ *is a solution for the system*

$$x_{n+1}=\frac{x_{n-3}y_{n-4}}{y_n(1+x_{n-3}y_{n-4})},\qquad y_{n+1}=\frac{y_{n-3}x_{n-4}}{x_n(-1+y_{n-3}x_{n-4})},\qquad n=0,1,2,\dots,$$

then for $n=0,1,2,\dots,$

$$\begin{aligned}
x_{8n-4}&=\left(\frac{e^{2n}}{a^{2n-1}}\right)\prod_{i=0}^{n-1}\frac{(1+(2i+1)ag)(1+(2i+1)kc)(1+2iag)(1+2ikc)}{(-1+(2i+1)cg)(-1+(2i+1)ek)(-1+2icg)(-1+2iek)},\\
x_{8n-3}&=\left(\frac{bf^{2n}}{l^{2n}}\right)\prod_{i=0}^{n-1}\frac{(1+(2i+1)hb)(1+(2i+1)ld)(1+2ihb)(1+2ild)}{(-1+(2i+1)bf)(-1+(2i+1)dh)(-1+(2i+2)bf)(-1+2idh)},\\
x_{8n-2}&=\left(\frac{ce^{2n}}{a^{2n}}\right)\prod_{i=0}^{n-1}\frac{(1+(2i+1)ag)(1+(2i+1)kc)(1+(2i+2)ag)(1+2ikc)}{(-1+(2i+1)cg)(-1+(2i+1)ek)(-1+(2i+2)cg)(-1+2iek)},\\
x_{8n-1}&=\left(\frac{df^{2n}}{l^{2n}}\right)\prod_{i=0}^{n-1}\frac{(1+(2i+1)hb)(1+(2i+1)ld)(1+(2i+2)hb)(1+2ild)}{(-1+(2i+1)bf)(-1+(2i+1)dh)(-1+(2i+2)bf)(-1+(2i+2)dh)},\\
x_{8n}&=\left(\frac{e^{2n+1}}{a^{2n}}\right)\prod_{i=0}^{n-1}\frac{(1+(2i+1)ag)(1+(2i+1)kc)(1+(2i+2)ag)(1+(2i+2)kc)}{(-1+(2i+1)cg)(-1+(2i+1)ek)(-1+(2i+2)cg)(-1+(2i+2)ek)},\\
x_{8n+1}&=\left(\frac{bf^{2n+1}}{l^{2n+1}(1+bf)}\right)\prod_{i=0}^{n-1}\frac{(1+(2i+1)hb)(1+(2i+1)ld)(1+(2i+2)hb)(1+(2i+2)ld)}{(-1+(2i+2)bf)(-1+(2i+1)dh)(-1+(2i+3)bf)(-1+(2i+2)dh)},\\
x_{8n+2}&=\left(\frac{(-1)ce^{2n+1}(1+ag)}{a^{2n+1}(1+cg)}\right)\prod_{i=0}^{n-1}\frac{(1+(2i+2)ag)(1+(2i+1)kc)(1+(2i+3)ag)(1+(2i+2)kc)}{(-1+(2i+2)cg)(-1+(2i+1)ek)(-1+(2i+3)cg)(-1+(2i+2)ek)},\\
x_{8n+3}&=\left(\frac{(-1)df^{2n+1}(1+hb)}{l^{2n+1}(1+bf)(1+dh)}\right)\prod_{i=0}^{n-1}\frac{(1+(2i+2)hb)(1+(2i+1)ld)(1+(2i+3)hb)(1+(2i+2)ld)}{(-1+(2i+2)bf)(-1+(2i+2)dh)(-1+(2i+3)bf)(-1+(2i+3)dh)},
\end{aligned}$$

and

$$\begin{aligned}
y_{8n-4}&=\left(\frac{l^{2n}}{f^{2n-1}}\right)\prod_{i=0}^{n-1}\frac{(-1+(2i+1)bf)(-1+(2i+1)dh)(-1+2ibf)(-1+2idh)}{(1+(2i+1)hb)(1+(2i+1)ld)(1+2ihb)(1+2ild)},\\
y_{8n-3}&=\left(\frac{ga^{2n}}{e^{2n}}\right)\prod_{i=0}^{n-1}\frac{(-1+(2i+1)cg)(-1+(2i+1)ek)(-1+2icg)(-1+2iek)}{(1+(2i+1)ag)(1+(2i+1)ck)(1+(2i+2)ag)(1+2ick)},\\
y_{8n-2}&=\left(\frac{hl^{2n}}{f^{2n}}\right)\prod_{i=0}^{n-1}\frac{(-1+(2i+1)bf)(-1+(2i+1)dh)(-1+(2i+2)bf)(-1+2idh)}{(1+(2i+1)hb)(1+(2i+1)ld)(1+(2i+2)hb)(1+2ild)},\\
y_{8n-1}&=\left(\frac{ka^{2n}}{e^{2n}}\right)\prod_{i=0}^{n-1}\frac{(-1+(2i+1)cg)(-1+(2i+1)ek)(-1+(2i+2)cg)(-1+2iek)}{(1+(2i+1)ag)(1+(2i+1)ck)(1+(2i+2)ag)(1+(2i+2)ck)},\\
y_{8n}&=\left(\frac{l^{2n+1}}{f^{2n}}\right)\prod_{i=0}^{n-1}\frac{(-1+(2i+1)bf)(-1+(2i+1)dh)(-1+(2i+2)bf)(-1+(2i+2)dh)}{(1+(2i+1)hb)(1+(2i+1)ld)(1+(2i+2)hb)(1+(2i+2)ld)},\\
y_{8n+1}&=\left(\frac{(-1)ga^{2n+1}}{e^{2n+1}(1+ag)}\right)\prod_{i=0}^{n-1}\frac{(-1+(2i+1)cg)(-1+(2i+1)ek)(-1+(2i+2)cg)(-1+(2i+2)ek)}{(1+(2i+2)ag)(1+(2i+1)ck)(1+(2i+3)ag)(1+(2i+2)ck)},\\
y_{8n+2}&=\left(\frac{(-1)hl^{2n+1}(1+bf)}{f^{2n+1}(1+hb)}\right)\prod_{i=0}^{n-1}\frac{(-1+(2i+2)bf)(-1+(2i+1)dh)(-1+(2i+3)bf)(-1+(2i+2)dh)}{(1+(2i+2)hb)(1+(2i+1)ld)(1+(2i+3)hb)(1+(2i+2)ld)},\\
y_{8n+3}&=\left(\frac{ka^{2n+1}(1+cg)}{e^{2n+1}(1+ag)(1+ck)}\right)\prod_{i=0}^{n-1}\frac{(-1+(2i+2)cg)(-1+(2i+1)ek)(-1+(2i+3)cg)(-1+(2i+2)ek)}{(1+(2i+2)ag)(1+(2i+2)ck)(1+(2i+3)ag)(1+(2i+3)ck)},
\end{aligned}$$

where $x_{-4}=a,\ x_{-3}=b,\ x_{-2}=c,\ x_{-1}=d,\ x_0=e,\ y_{-4}=f,\ y_{-3}=g,\ y_{-2}=h,\ y_{-1}=k,$ *and* $y_0=$

l *with the initial value must be non-zero and* $ag,\ kc,\ hb,\ ld \notin \left\{-\frac{1}{n},\ n = 1,\ 2, \ldots\right\}$*, and* $cg,\ ek,\ bf,\ dh \notin \left\{\frac{1}{n},\ n = 1,\ 2, \ldots\right\}$.

The following systems can be considered as similar. Therefore, the form of expression and proof of the following systems can be obtained similarly to the above theorem.

$$x_{n+1} = \frac{x_{n-3}y_{n-4}}{y_n(-1 + x_{n-3}y_{n-4})}, \qquad y_{n+1} = \frac{y_{n-3}x_{n-4}}{x_n(-1 - y_{n-3}x_{n-4})}.$$
$$x_{n+1} = \frac{x_{n-3}y_{n-4}}{y_n(-1 - x_{n-3}y_{n-4})}, \qquad y_{n+1} = \frac{y_{n-3}x_{n-4}}{x_n(-1 + y_{n-3}x_{n-4})}.$$
$$x_{n+1} = \frac{x_{n-3}y_{n-4}}{y_n(-1 + x_{n-3}y_{n-4})}, \qquad y_{n+1} = \frac{y_{n-3}x_{n-4}}{x_n(-1 + y_{n-3}x_{n-4})}.$$
$$x_{n+1} = \frac{x_{n-3}y_{n-4}}{y_n(-1 - x_{n-3}y_{n-4})}, \qquad y_{n+1} = \frac{y_{n-3}x_{n-4}}{x_n(-1 - y_{n-3}x_{n-4})}.$$
$$x_{n+1} = \frac{x_{n-3}y_{n-4}}{y_n(1 - x_{n-3}y_{n-4})}, \qquad y_{n+1} = \frac{y_{n-3}x_{n-4}}{x_n(-1 - y_{n-3}x_{n-4})}.$$
$$x_{n+1} = \frac{x_{n-3}y_{n-4}}{y_n(1 - x_{n-3}y_{n-4})}, \qquad y_{n+1} = \frac{y_{n-3}x_{n-4}}{x_n(-1 + y_{n-3}x_{n-4})}.$$
$$x_{n+1} = \frac{x_{n-3}y_{n-4}}{y_n(-1 - x_{n-3}y_{n-4})}, \qquad y_{n+1} = \frac{y_{n-3}x_{n-4}}{x_n(1 + y_{n-3}x_{n-4})}.$$
$$x_{n+1} = \frac{x_{n-3}y_{n-4}}{y_n(-1 + x_{n-3}y_{n-4})}, \qquad y_{n+1} = \frac{y_{n-3}x_{n-4}}{x_n(1 - y_{n-3}x_{n-4})}.$$

Acknowledgment

This article was funded by the Deanship of Scientific Research (DSR), King Abdulaziz University, Jeddah. The authors, therefore, acknowledge with thanks DSR technical and financial support.

References

[1] E. O. Alzahrani, M. M. El-Dessoky, E. M. Elsayed, Y. Kuang, *Solutions and Properties of Some Degenerate Systems of Difference Equations*, J. Comput. Anal. Appl., **18** (2015), 321–333. 1
[2] H. Bao, *On a System of Second-Order Nonlinear Difference Equations*, J. Appl. Math. Phys., **3** (2015), 903–910.
[3] E. Camouzis, *Open problems and Conjectures*, J. Difference Equ. Appl., **15** (2009), 203–323. 1
[4] C. A. Clark, M. R. S. Kulenović, J. F. Selgrade, *On a system of rational difference equations*, J. Difference Equ. Appl., **11** (2005), 565–580.
[5] Q. Din, E. M. Elsayed, *Stability analysis of a discrete ecological model*, Comput. Ecol. Softw., **4** (2014), 89–103.
[6] Q. Din, M. N. Qureshi, A. Q. Khan, *Dynamics of a fourth-order system of rational difference equations*, Adv. Difference Equ., **2012** (2012), 15 pages. 1
[7] Q. Din, M. N. Qureshi, A. Q. Khan, *Qualitative behavior of an anti-competitive system of third-order rational difference equations*, Comput. Ecol. Softw., **4** (2014), 104–115.
[8] M. M. El-Dessoky, *On a systems of rational difference equations of Order Two*, Proc. Jangjeon Math. Soc., **19** (2016), 271–284.
[9] M. M. El-Dessoky, *Solution of a rational systems of difference equations of order three*, Mathematics, **2016** (2016), 12 pages.
[10] M. M. El-Dessoky, *The form of solutions and periodicity for some systems of third - order rational difference equations*, Math. Method Appl. Sci., **39** (2016), 1076–1092. 1
[11] M. M. El-Dessoky, E. M. Elsayed, *On a solution of system of three fractional difference equations*, J. Comput. Anal. Appl., **19** (2015), 760–769.
[12] M. M. El-Dessoky, E. M. Elsayed, M. Alghamdi, *Solutions and periodicity for some systems of fourth order rational difference equations*, J. Comput. Anal. Appl., **18** (2015), 179–194.
[13] M. M. El-Dessoky, M. Mansour, E. M. Elsayed, *Solutions of some rational systems of difference equations*, Util. Math., **92** (2013), 329–336.
[14] E. M. Elsayed, M. M. El-Dessoky, E. O. Alzahrani, *The Form of The Solution and Dynamics of a Rational Recursive Sequence*, J. Comput. Anal. Appl., **17** (2014), 172–186.

[15] E. M. Elsayed, M. Mansour, M. M. El-Dessoky, *Solutions of fractional systems of difference equations*, Ars Combin., **110** (2013), 469–479.
[16] A. Q. Khan, M. N. Qureshi, *Global dynamics of some systems of rational difference equations*, J. Egyptian Math. Soc., **24** (2016), 30–36.
[17] A. Q. Khan, M. N. Qureshi, Q. Din, *Global dynamics of some systems of higher-order rational difference equations*, Adv. Difference Equ., **2013** (2013), 23 pages.
[18] V. L. Kocić, G. Ladas, *Global Behavior of Nonlinear Difference Equations of Higher Order with Applications*, Kluwer Academic Publishers, Dordrecht, (1993).
[19] M. Kulenovic, G. Ladas, *Dynamics of Second Order Rational Difference Equations with Open Problems and Conjectures*, Chapman & Hall / CRC Press, U.S.A., (2001).
[20] A. S. Kurbanlı, *On the Behavior of Solutions of the System of Rational Difference Equations*, World Appl. Sci. J., **10** (2010), 1344–1350.
[21] A. S. Kurbanlı, C. Çinar, İ. Yalçinkaya, *On the behavior of positive solutions of the system of rational difference equations* $x_{n+1} = \frac{x_{n-1}}{y_n x_{n-1}+1}$, $y_{n+1} = \frac{y_{n-1}}{x_n y_{n-1}+1}$, Math. Comput. Model., **53** (2011), 1261–1267. 1
[22] M. Mansour, M. M. El-Dessoky, E. M. Elsayed, *On the solution of rational systems of difference equations*, J. Comput. Anal. Appl., **15** (2013), 967–976.
[23] A. Neyrameh, H. Neyrameh, M. Ebrahimi, A. Roozi, *Analytic solution diffusivity equation in rational form*, World Appl. Sci. J., **10** (2010), 764–768.
[24] A. Y. Özban, *On the positive solutions of the system of rational difference equations*, J. Math. Anal. Appl., **323** (2006), 26–32.
[25] G. Papaschinopoulos, C. J. Schinas, G. Stefanidou, *On a k-order system of Lyness-type difference equations*, Adv. Difference Equ., **2007** (2007), 13 pages.
[26] G. Papaschinopoulos, G. Stefanidou, *Asymptotic behavior of the solutions of a class of rational difference equations*, Int. J. Difference Equ., **5** (2010), 233–249.
[27] S. Stević, *On a system of difference equations*, Appl. Math. Comput., **218** (2011), 3372–3378. 1
[28] S. Stević, B. Iričanin, Z. Šmarda, *Boundedness character of a fourth-order system of difference equations*, Adv. Difference Equ., **2015** (2015), 11 pages.
[29] I. Yalçınkaya, *On the global asymptotic stability of a second-order system of difference equations*, Discrete Dyn. Nat. Soc., **2008** (2008), 12 pages.
[30] X. Yang, Y. Liu, S. Bai, *On the system of high order rational difference equations* $x_{n+1} = \frac{\alpha}{y_{n-p}}, y_{n+1} = \frac{b y_{n-p}}{x_n - q y_{n-p}}$, Appl. Math. Comput., **171** (2005), 853–856. 1