



A classification of minimal translation surfaces in Minkowski space



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Abstract

Minimal surfaces are well known as a class of surfaces with vanishing mean curvature which minimize area within a given boundary configuration since 19th century. This fact was implicitly proved by Lagrange for nonparametric surfaces in 1760, and then by Meusnier in 1776 who used the analytic expression for the mean curvature. Mathematically, a minimal surface corresponds to the solution of a nonlinear partial differential equation. By solving some differential equations, in this paper we give a complete and explicit classification of minimal translation surfaces in an n -dimensional Minkowski space.

Keywords: Minimal surfaces, translation surfaces, Minkowski space.

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1. Introduction

Minimal surfaces are among the most natural objects in differential geometry, and have been studied during the last two and half centuries since J. L. Lagrange. In particular, minimal surfaces have encountered striking applications in other fields, like mathematical physics, conformal geometry, computer aided design, among others. Even though it has been the subject of intense activity, many basic problems remain open, for instance, the complete classification of minimal surfaces in 3-dimensional space. In general, to consider the solutions of some differential equations is essential and important, for instance, [5, 7–10, 12]. In order to search for more minimal surfaces, some natural geometric assumptions arise. In the 3-dimensional Euclidean space \mathbb{E}^3 , a surface is called a translation surface if it is given by an immersion

$$r : U \subset \mathbb{E}^2 \rightarrow \mathbb{E}^3 : (x, y) \rightarrow (x, y, z),$$

where $z = f(x) + g(y)$, $f(x)$ and $g(y)$ are differentiable functions. The first study of translation surfaces

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is due to Scherk, in 1835 who proved that, besides the plane, the only minimal translation surface is the so-called Scherk surface given by

$$z = \frac{1}{a} \ln \left| \frac{\cos(ax)}{\cos(ay)} \right|,$$

where a is a nonzero constant. In 1991, the minimal translation surfaces were generalized to minimal translation hypersurfaces by F. Dillen, L. Verstraelen and G. Zafindratafa [4]. Since then, translation surfaces (hypersurfaces) have been investigated intensively by geometers (see [1, 6, 7, 10, 11]).

In 1994, L. Verstraelen, J. Walrave and S. Yaprak studied minimal translation surfaces in Euclidean space \mathbb{E}^n for arbitrary codimension by proving that, besides the plane, the only minimal translation surface in \mathbb{E}^n is given by

$$r(s, t) = \left(s, t, \frac{c_3}{m} \ln \left| \frac{\cos(\sqrt{ms})}{\cos(\sqrt{mt})} \right|, \dots, \frac{c_n}{m} \ln \left| \frac{\cos(\sqrt{ms})}{\cos(\sqrt{mt})} \right| \right),$$

where $m = \sum_{k=3}^n c_k^2$ and c_k are constant, see details in [13].

In n -dimensional Minkowski space \mathbb{E}_1^n , there are two types of translation surfaces as follows (c.f. [5, 7]):

I. $r(s, t) = (s, t, x_3(s, t), x_4(s, t), \dots, x_n(s, t)),$

II. $r(s, t) = (x_3(s, t), x_4(s, t), \dots, x_n(s, t), s, t),$

where $x_i(s, t) = f_i(s) + g_i(t)$, $3 \leq i \leq n$. The problem to classify minimal translation surfaces in an n -dimensional Minkowski space becomes interesting and important. Generally, the classification problems for submanifolds in Minkowski space are more complicated than the Euclidean case because of the indefiniteness of the metric. In this paper, we will deal with the similar problem for translation surfaces as L. Verstraelen et al.'s work [13] in Minkowski space \mathbb{E}_1^n . We obtain a complete classification of minimal translation surfaces with type I and type II, respectively, in the n -dimensional Minkowski space \mathbb{E}_1^n . Specifically, we get the following classification theorems.

Theorem 1.1. *Let M be a translation surface with type I in Minkowski space \mathbb{E}_1^n . Then M is minimal if and only if M is a part of the following:*

1. a plane;
2. $r(s, t) = p(s) + q(t)$, where $p(s)$ and $q(t)$ are lightlike vectors in Minkowski space \mathbb{E}_1^n ;
3. a flat B-scroll over a null curve;
4. $r(s, t) = \left(s, t, \frac{c_3}{2}(t^2 - s^2), \dots, \frac{c_n}{2}(t^2 - s^2) \right)$;
5. $r(s, t) = \left(s, t, \frac{c_3}{m} \ln \left| \frac{\cos(\sqrt{ms})}{\cos(\sqrt{mt})} \right|, \dots, \frac{c_n}{m} \ln \left| \frac{\cos(\sqrt{ms})}{\cos(\sqrt{mt})} \right| \right)$ for $m > 0$;
6. $r(s, t) = \left(s, t, \frac{c_3}{m} \ln \left| \frac{\sinh(\sqrt{-ms})}{\sinh(\sqrt{-mt})} \right|, \dots, \frac{c_n}{m} \ln \left| \frac{\sinh(\sqrt{-ms})}{\sinh(\sqrt{-mt})} \right| \right)$ for $m < 0$;
7. $r(s, t) = \left(s, t, \frac{c_3}{m} \ln \left| \frac{\sinh(\sqrt{-ms})}{\cosh(\sqrt{-mt})} \right|, \dots, \frac{c_n}{m} \ln \left| \frac{\sinh(\sqrt{-ms})}{\cosh(\sqrt{-mt})} \right| \right)$ for $m < 0$;
8. $r(s, t) = \left(s, t, \frac{c_3}{m} \ln \left| \frac{\cosh(\sqrt{-ms})}{\cosh(\sqrt{-mt})} \right|, \dots, \frac{c_n}{m} \ln \left| \frac{\cosh(\sqrt{-ms})}{\cosh(\sqrt{-mt})} \right| \right)$ for $m < 0$,

where $m = c_3^2 + \dots + c_{n-1}^2 - c_n^2$ is constant.

Theorem 1.2. *Let M be a translation surface with type II in Minkowski space \mathbb{E}_1^n . Then M is minimal if and only if M is a part of the following:*

1. a plane;
2. a flat B-scroll over a null curve;
3. $r(s, t) = \left(\frac{c_3}{m} \ln \left| \frac{\cosh(\sqrt{mt})}{\cos(\sqrt{ms})} \right|, \dots, \frac{c_n}{m} \ln \left| \frac{\cosh(\sqrt{mt})}{\cos(\sqrt{ms})} \right|, s, t \right)$;
4. $r(s, t) = \left(\frac{c_3}{m} \ln \left| \frac{\sinh(\sqrt{mt})}{\cos(\sqrt{ms})} \right|, \dots, \frac{c_n}{m} \ln \left| \frac{\sinh(\sqrt{mt})}{\cos(\sqrt{ms})} \right|, s, t \right)$,

where $m = c_3^2 + \dots + c_n^2$ is a positive constant.

Remark 1.3. For $n = 3$, M is a minimal translation surface (spacelike or timelike) in Minkowski space \mathbb{E}_1^3 . In this case, Theorem 1.1 and Theorem 1.2 are generalizations of the 3-dimensional case (see [5, 7] for details).

Remark 1.4. Noting that $n > 3$ in the cases (4) and (5) in Theorem 1.1, there is no corresponding minimal translation surface in Minkowski 3-space \mathbb{E}_1^3 ([5]).

2. Preliminaries

Let \mathbb{E}_1^n be n -dimensional Minkowski space with the metric

$$\langle , \rangle = dx_1^2 + dx_2^2 + \dots + dx_{n-1}^2 - dx_n^2.$$

We denote M by a surface immersed in Minkowski space \mathbb{E}_1^n parameterized by

$$r : U \subset \mathbb{E}^2 \rightarrow \mathbb{E}^3 : (s, t) \rightarrow (x_1(s, t), x_2(s, t), \dots, x_n(s, t)).$$

Then the components of the first fundamental form for surface M are given by

$$g_{11} = \langle r_s, r_s \rangle = r_s \cdot r_s, \quad g_{12} = \langle r_s, r_t \rangle = r_s \cdot r_t, \quad g_{22} = \langle r_t, r_t \rangle = r_t \cdot r_t.$$

A surface M is spacelike if the induced metric on surface M is positive, that is $g_{11}g_{22} - g_{12}^2 > 0$; a surface M is timelike if the induced metric on surface M is indefinite, that is, $g_{11}g_{22} - g_{12}^2 < 0$, respectively.

Let $\{\eta_3, \dots, \eta_n\}$ be normal frame field of the surface M immersed in Minkowski space \mathbb{E}_1^n . Then the components of the second fundamental form concerning the normal vector η_k are given by

$$\begin{aligned} b_{11}(\eta_k) &= \langle r_{ss}, \eta_k \rangle = r_{ss} \cdot \eta_k, \\ b_{12}(\eta_k) &= \langle r_{st}, \eta_k \rangle = r_{st} \cdot \eta_k, \\ b_{22}(\eta_k) &= \langle r_{tt}, \eta_k \rangle = r_{tt} \cdot \eta_k. \end{aligned}$$

Recall that the component of the mean curvature vector H_{η_k} is given by (c.f. [2, 3])

$$H_{\eta_k} = \frac{g_{22}b_{11} + g_{11}b_{22} - 2g_{12}b_{12}}{2(g_{11}g_{22} - g_{12}^2)}. \tag{2.1}$$

Hence a surface M is minimal (or maximal for a spacelike surface) if and only if $H_{\eta_k} = 0$ for all $k = 3, \dots, n$.

3. A proof of Theorem 1.1

In this section, by considering some differential equations we will give the concrete parameterizations of translation surfaces with type (I).

First of all, it is easy to check that

$$\begin{aligned} r_s &= (1, 0, f'_3(s), f'_4(s), \dots, f'_n(s)), \\ r_t &= (0, 1, g'_3(t), g'_4(t), \dots, g'_n(t)). \end{aligned}$$

We could build the normal frame fields in the normal space as follows

$$\begin{aligned} \eta_3 &= (-f'_3(s), -g'_3(t), 1, 0, \dots, 0, 0), \\ &\vdots \\ \eta_{n-1} &= (-f'_{n-1}(s), -g'_{n-1}(t), 0, 0, \dots, 1, 0), \\ \eta_n &= (f'_n(s), g'_n(t), 0, 0, \dots, 0, 1). \end{aligned}$$

It is easy to check that the components of the first fundamental form are in the following forms

$$\begin{aligned} g_{11} &= r_s \cdot r_s = 1 + f_3'^2 + \cdots + f_{n-1}'^2 - f_n'^2, \\ g_{22} &= r_t \cdot r_t = 1 + g_3'^2 + \cdots + g_{n-1}'^2 - g_n'^2, \\ g_{12} &= r_s \cdot r_t = f_3'g_3' + \cdots + f_{n-1}'g_{n-1}' - f_n'g_n', \end{aligned} \tag{3.1}$$

and the components of the second fundamental form are given by

$$\begin{aligned} b_{11}(\eta_k) &= r_{ss} \cdot \eta_k = f_k'', & b_{12}(\eta_k) &= r_{st} \cdot \eta_k = 0, & b_{22}(\eta_k) &= r_{tt} \cdot \eta_k = g_k'', \\ b_{11}(\eta_n) &= r_{ss} \cdot \eta_n = -f_n'', & b_{12}(\eta_n) &= r_{st} \cdot \eta_n = 0, & b_{22}(\eta_n) &= r_{tt} \cdot \eta_n = -g_n'' \end{aligned}$$

for $k = 3, \dots, n - 1$.

Let $H(\eta_i)$ be the mean curvature with respect to the normal vector η_i . Then the surface M is minimal if and only if $H(\eta_i) = 0$ ($i = 3, 4, \dots, n$), that is,

$$[1 + g_3'^2 + \cdots + g_{n-1}'^2 - g_n'^2]f_k'' + [1 + f_3'^2 + \cdots + f_{n-1}'^2 - f_n'^2]g_k'' = 0. \tag{3.2}$$

We first consider the case $1 + g_3'^2 + \cdots + g_{n-1}'^2 - g_n'^2 = 1 + f_3'^2 + \cdots + f_{n-1}'^2 - f_n'^2 = 0$. If we let

$$p(s) = (s, 0, f_3, \dots, f_n), \quad q(t) = (0, t, g_3, \dots, g_n),$$

then both $p(s)$ and $q(t)$ are two lightlike vectors in Minkowski space \mathbb{E}_1^n . So the case (2) in Theorem 1.1 is obtained.

If $1 + g_3'^2 + \cdots + g_{n-1}'^2 - g_n'^2 = 0$ and $1 + f_3'^2 + \cdots + f_{n-1}'^2 - f_n'^2 \neq 0$, by (3.2) we have that $g_k'' = 0$, so $g_k = c_k t$ for $k = 3, \dots, n$. Then the surface M takes the form

$$r(s, t) = (s, 0, f_3, \dots, f_n) + t(0, 1, c_3, \dots, c_n),$$

which implies that M is a flat B-scroll over a null curve. Hence, we obtain the case (3) in Theorem 1.1.

If $1 + g_3'^2 + \cdots + g_{n-1}'^2 - g_n'^2 \neq 0$ and $1 + f_3'^2 + \cdots + f_{n-1}'^2 - f_n'^2 = 0$, similarly we obtain the case (3) in Theorem 1.1.

We now consider the case $1 + g_3'^2 + \cdots + g_{n-1}'^2 - g_n'^2 \neq 0$ and $1 + f_3'^2 + \cdots + f_{n-1}'^2 - f_n'^2 \neq 0$. From (3.2), there exist constants c_k such that

$$-\frac{f_k''}{1 + f_3'^2 + \cdots + f_{n-1}'^2 - f_n'^2} = \frac{g_k''}{1 + g_3'^2 + \cdots + g_{n-1}'^2 - g_n'^2} = c_k \tag{3.3}$$

for $k = 3, \dots, n$. Hence

$$f_k'' = -c_k[1 + f_3'^2 + \cdots + f_{n-1}'^2 - f_n'^2], \tag{3.4}$$

$$g_k'' = c_k[1 + g_3'^2 + \cdots + g_{n-1}'^2 - g_n'^2]. \tag{3.5}$$

If $c_k = 0$, then from (3.4) and (3.5) we conclude that $f_k(s) = a_k s + b_k$, and $g_k(t) = c_k t + d_k$ for $k = 3, \dots, n$. Thus, M is a plane in the Minkowski space \mathbb{E}_1^n , which gives the case (1) in Theorem 1.1.

Now suppose that at least one of the $c_k \neq 0$. By re-arranging the indices (if necessary) we can assume that $c_3 \neq 0$. Then taking the ratio of the equation given in (3.5), we have

$$\frac{f_k''}{f_3''} = \frac{g_k''}{g_3''} = \frac{c_k}{c_3}, \quad k = 3, \dots, n. \tag{3.6}$$

Hence

$$f_k''(s) = \frac{c_k}{c_3} f_3''(s), \quad g_k''(t) = \frac{c_k}{c_3} g_3''(t), \quad k = 3, \dots, n.$$

After a translation of the variable s , we get

$$f'_k(s) = \frac{c_k}{c_3} f'_3(s), \quad g'_k(t) = \frac{c_k}{c_3} g'_3(t), \quad k = 3, \dots, n. \tag{3.7}$$

Putting $k = 3$ in (3.5), we have

$$f''_3 = -c_3[1 + f_3'^2 + \dots + f_{n-1}'^2 - f_n'^2], \tag{3.8}$$

$$g''_3 = c_3[1 + g_3'^2 + \dots + g_{n-1}'^2 - g_n'^2]. \tag{3.9}$$

Substituting (3.7) into (3.8) and (3.9), we have

$$f''_3 = -\frac{m}{c_3} (f'_3)^2 - c_3, \quad g''_3 = \frac{m}{c_3} (g'_3)^2 + c_3, \tag{3.10}$$

where $m = c_3^2 + \dots + c_{n-1}^2 - c_n^2$.

We now distinguish the following three cases:

Case 1. $m = 0$. In this case, by (3.10) we have

$$f''_3 = -c_3, \quad g''_3 = c_3, \tag{3.11}$$

and hence

$$f'_3 = -c_3s + d_1, \quad g'_3 = c_3t + d_2. \tag{3.12}$$

It follows from (3.7) that

$$f'_k = -c_k s + \frac{c_k}{c_3} d_1, \quad g'_k = c_k t + \frac{c_k}{c_3} d_2.$$

By integration, and after an affine transformation, we have

$$f_k = -\frac{c_k}{2} s^2, \quad g_k = \frac{c_k}{2} t^2. \tag{3.13}$$

So the case (4) in Theorem 1.1 is obtained.

Case 2. $m > 0$. In this case, from (3.10) we have

$$\frac{m}{c_3} f''_3 = -\left[\left(\frac{m}{c_3} f'_3\right)^2 + m\right], \quad \frac{m}{c_3} g''_3 = \left(\frac{m}{c_3} g'_3\right)^2 + m. \tag{3.14}$$

By solving the equation (3.14), we have

$$f'_3 = -\frac{c_3}{\sqrt{m}} \tan(\sqrt{m}s), \quad g'_3 = \frac{c_3}{\sqrt{m}} \tan(\sqrt{m}t).$$

From (3.7), we have

$$f'_k = -\frac{c_k}{\sqrt{m}} \tan(\sqrt{m}s), \quad g'_k = \frac{c_k}{\sqrt{m}} \tan(\sqrt{m}t), \tag{3.15}$$

and hence

$$f_k = \frac{c_k}{m} \ln |\cos(\sqrt{m}s)|, \quad g_k = -\frac{c_k}{m} \ln |\cos(\sqrt{m}t)|,$$

which gives the case (5) in Theorem 1.1.

Case 3. $m < 0$. In this case from (3.10) we have

$$\frac{m}{c_3} f''_3 = -\left[\left(\frac{m}{c_3} f'_3\right)^2 - (\sqrt{-m})^2\right], \quad \frac{m}{c_3} g''_3 = \left(\frac{m}{c_3} g'_3\right)^2 - (\sqrt{-m})^2. \tag{3.16}$$

By solving the first equation of (3.16), we have

$$f'_3 = -\frac{c_3}{\sqrt{-m}} \coth(\sqrt{-ms}) \quad \text{or} \quad f'_3 = -\frac{c_3}{\sqrt{-m}} \tanh(\sqrt{-ms}), \tag{3.17}$$

which together with (3.7) gives

$$f'_k = -\frac{c_k}{\sqrt{-m}} \coth(\sqrt{-ms}) \quad \text{or} \quad f'_k = -\frac{c_k}{\sqrt{-m}} \tanh(\sqrt{-ms}),$$

and hence

$$f_k = \frac{c_k}{m} \ln |\sinh(\sqrt{-ms})| \quad \text{or} \quad f_k = \frac{c_k}{m} \ln |\cosh(\sqrt{-ms})|. \tag{3.18}$$

Similarly, by the second equation of (3.16), we have

$$g_k = -\frac{c_k}{m} \ln |\sinh(\sqrt{-mt})| \quad \text{or} \quad g_k = -\frac{c_k}{m} \ln |\cosh(\sqrt{-mt})|. \tag{3.19}$$

Combining (3.18) and (3.19), the cases (6), (7) and (8) in Theorem 1.1 are obtained. This completes the proof of Theorem 1.1.

4. A proof of Theorem 1.2

In this section, we consider the translation surfaces with type (II). Similar to the discussion on the proof of Theorem 1.1, we give the normal vector field

$$\begin{aligned} \eta_3 &= (1, 0, \dots, -f'_3(s), g'_3(t)), \\ &\vdots \\ \eta_n &= (0, 0, \dots, 1, -f'_n(s), g'_n(t)). \end{aligned}$$

The components of the first fundamental form and the second fundamental form are written respectively as

$$g_{11} = f_3'^2 + \dots + f_n'^2 + 1, \quad g_{22} = g_3'^2 + \dots + g_n'^2 - 1, \quad g_{12} = f_3'g_3' + \dots + f_n'g_n',$$

and

$$b_{11}(\eta_k) = f_k'', \quad b_{12}(\eta_k) = 0, \quad b_{22}(\eta_k) = g_k''$$

for $k = 3, \dots, n$.

Hence, it is easy to check that the surface M is minimal if and only if

$$[g_3'^2 + \dots + g_n'^2 - 1]f_k'' + [f_3'^2 + \dots + f_n'^2 + 1]g_k'' = 0. \tag{4.1}$$

If $g_3'^2 + \dots + g_n'^2 - 1 = 0$, by (4.1) we have that $g_k'' = 0$. So $g(k) = c_k t$ for $k = 3, \dots, n$. Then the surface M is given by

$$r(s, t) = (f_3, \dots, f_n, s, 0) + t(c_3, \dots, c_n, 0, 1),$$

which implies that M is a flat B-scroll over a null curve. So the case (2) in Theorem 1.2 is given.

If $g_3'^2 + \dots + g_n'^2 - 1 \neq 0$, from (4.1), there exist constants c_k such that

$$\frac{f_k''}{f_3'^2 + \dots + f_n'^2 + 1} = -\frac{g_k''}{g_3'^2 + \dots + g_n'^2 - 1} = c_k \tag{4.2}$$

for $k = 3, \dots, n$. Hence

$$f_k'' = c_k[f_3'^2 + \dots + f_n'^2 + 1], \quad g_k'' = -c_k[g_3'^2 + \dots + g_n'^2 - 1]. \quad (4.3)$$

If $c_k = 0$, then from (4.3) we conclude that $f_k(s) = a_k s + b_k$, $g_k(t) = c_k t + d_k$ for $k = 3, \dots, n$. Thus M is a plane in the Minkowski space \mathbb{E}_1^n and the case (1) in Theorem 1.2 is obtained.

Now suppose that at least one of the $c_k \neq 0$, we also assume that $c_3 \neq 0$. Similar to the discussion of the surfaces of type (I), we have

$$f_k'(s) = \frac{c_k}{c_3} f_3'(s), \quad g_k'(t) = \frac{c_k}{c_3} g_3'(t), \quad k = 3, \dots, n. \quad (4.4)$$

Substituting (4.4) into (4.3) for $k = 3$, we have

$$f_3'' = \frac{m}{c_3} (f_3')^2 + c_3, \quad g_3'' = -\frac{m}{c_3} (g_3')^2 + c_3, \quad (4.5)$$

where $m = c_3^2 + \dots + c_n^2 > 0$. By solving the first equation of (4.5), we have

$$f_3' = \frac{c_3}{\sqrt{m}} \tan(\sqrt{ms}),$$

which together with (4.4) gives

$$f_k' = \frac{c_k}{\sqrt{m}} \tan(\sqrt{ms}),$$

and hence

$$f_k = -\frac{c_k}{m} \ln |\cos(\sqrt{ms})|.$$

Similarly, by the second equation of (4.5), we have

$$g_k = \frac{c_k}{m} \ln |\cosh(\sqrt{mt})|, \quad \text{or} \quad g_k = \frac{c_k}{m} \ln |\sinh(\sqrt{mt})|.$$

So the cases (3) and (4) in Theorem 1.2 are obtained. This completes the proof of Theorem 1.2.

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