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Some fixed point theorems in multiplicative metric spaces via compatible of type (E) and weakly sub-sequentially continuous mappings

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Abstract

In this paper, we established some common fixed point theorems for two pairs of self mappings by using the notion of compatibility of type (E) and weak sub-sequential continuity in multiplicative metric spaces. We deduce important results in this line by restricting the number of mappings involved. The proven results are the improved one in the sense that the closedness, completeness of the whole space and continuity of the mappings are relaxed.

Keywords: Coincidence point, fixed point, multiplicative metric space, compatible maps of type (E), weak sub-sequential continuous mappings.

2010 MSC: 47H10, 54H25.

1. Introduction

Bashirov et al. [2] brought up the multiplicative calculus into the attention of researchers and demonstrated its usefulness in their article titled multiplicative calculus and its applications. They introduced the concept of multiplicative metric space and established unique solution for the multiplicative differential equations. Thereafter, many common fixed point theorems had been established by different authors in multiplicative metric spaces. In 2011, Bashirov et al. [3] has shown that various problems from distinct fields can be modeled more effectively using multiplicative calculus. Jungck [8] established some fixed point results for compatible maps. Jungck and Rhoades [9] proved some common fixed point theorems without taking under consideration the continuity of any mappings. Bouhadjera and Thobie [6] proved some common fixed point theorems for two self pair of mapping using the notion of sub-compatible and sub-sequential maps in metric spaces. Beloul [4] established some fixed point theorems for two pairs of self mappings satisfying contractive conditions by using the notion of weak sub-sequential mappings and compatibility of type (E) in metric spaces. Bouhadjera [5] gave a general common fixed point theorem for two weakly sub-sequentially mappings along with subcompatible maps satisfying a significant estimated implicit function in a metric space. Recently, Afrah [1] established some fixed point results for two pairs of compatible mappings along with sub-sequential mappings in a multiplicative metric space. For detailed topology of multiplicative metric spaces, we refer to [7, 10, 11, 13].

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2. Preliminaries

Definition 2.1 ([2]). Let X be any nonempty set. A multiplicative metric is a mapping $d : X \times X \to \mathbb{R}^+$ satisfying the following conditions:

(MM1) $d(x, y) \geqslant 1$ for every $x, y \in X$ and $d(x, y) = 1$ if and only if $x = y$;
(MM2) $d(x, y) = d(y, x)$ for every $x, y \in X$;
(MM3) $d(x, y) \leqslant d(x, z).d(z, y)$ for every $x, y, z \in X$ (multiplicative triangle inequality). Then (X, d) is a multiplicative metric space.

Example 2.2 ([13]). Let $\mathbb{R}^+_n$ be the collection of all n-tuples of positive real numbers and let $d : \mathbb{R}^+_n \times \mathbb{R}^+_n \to \mathbb{R}$ be defined as $d(x, y) = |\frac{x_1}{y_1}|.|\frac{x_2}{y_2}|\cdots|\frac{x_n}{y_n}|$, where $x = (x_1, x_2, \dots, x_n), x = (y_1, y_2, \dots, y_n) \in \mathbb{R}^+_n$ and $|.| : \mathbb{R}_+ \to \mathbb{R}_+$ is defined as follows: $|a| = \begin{cases} a, & \text{if } a \geqslant 1, \\ \frac{1}{a}, & \text{if } a > 1. \end{cases}$

It is clear that all the conditions of a multiplicative metric are satisfied.

In 2012, Özavsar and Cevikel [13] gave the concept of multiplicative contraction mappings and proved some fixed point theorems in a multiplicative metric spaces.

Definition 2.3 ([13]). Let f be a mapping of a multiplicative metric space (X, d) into itself. Then f is called a multiplicative contraction if there exists a real constant $\lambda \in [0, 1)$ such that $d(fx, fy) \leqslant d^{\lambda}(x, y)$ for all $x, y \in X$.

Singh and Mahendra [12] introduced the notion of compatibility of type (E) in metric spaces, in the setting of multiplicative metric space, it becomes as follows.

Definition 2.4. Two self maps A and S on a multiplicative metric space (X, d) are said to be compatible of type (E), if $\lim_{n\to\infty} S^2x_n = \lim_{n\to\infty} SAx_n = Az$ and $\lim_{n\to\infty} A^2x_n = \lim_{n\to\infty} ASx_n = Sz$ whenever $\{x_n\}$ is a sequence in X such that $\lim_{n\to\infty} Ax_n = \lim_{n\to\infty} Sx_n = z$ for some $z \in X$.

Definition 2.5. Two self maps A and S on a multiplicative metric space (X, d) are said to be A-compatible of type (E), if $\lim_{n\to\infty} A^2x_n = \lim_{n\to\infty} ASx_n = Sz$ for some $z \in X$. On the other hand, pair of self maps A and S is said to be S-compatible of type (E), if $\lim_{n\to\infty} S^2x_n = \lim_{n\to\infty} SAx_n = Az$ for some $z \in X$.

Remark 2.6. It is also interesting to see that if A and S are compatible of type (E), then they are A-Compatible of type (E) and S-Compatible of type (E), but the converse is not true (see [4, Example 1]).

Bouhadjera and Godet Thobie [6] introduced the concept of sub-sequential continuity as follows.

Definition 2.7 ([6]). Two self maps A and S on a metric space (X, d) are said to be sub-sequentially continuous, if there exists a sequence $\{x_n\}$ such that $\lim_{n\to\infty} Ax_n = \lim_{n\to\infty} Sx_n = z$, for some $z \in X$ and $\lim_{n\to\infty} ASx_n = Az$, $\lim_{n\to\infty} SAx_n = Sz$.

Example 2.8 ([4]). Let $X = [0, 2]$ and d be the euclidean metric, define A, B, S and T as

$$A(x) = \begin{cases} \frac{x+1}{2}, & 0 \leqslant x \leqslant 1, \\ \frac{3}{4}, & 1 < x \leqslant 2, \end{cases} \qquad B(x) = \begin{cases} 1, & 0 \leqslant x \leqslant 1, \\ \frac{1}{2}, & 1 < x \leqslant 2 \end{cases}$$

$$S(x) = \begin{cases} 2 - x, & 0 \leqslant x \leqslant 1, \\ 0, & 1 < x \leqslant 2, \end{cases} \qquad T(x) = \begin{cases} x, & 0 \leqslant x \leqslant 1, \\ \frac{1}{4}, & 1 < x \leqslant 2. \end{cases}$$

Let us consider a sequence $\{x_n\}$ in X defined by $x_n = 1 - \frac{1}{n}$ for $n \in \mathbb{N}$ such that $\lim_{n\to\infty} Ax_n = 1 = \lim_{n\to\infty} Sx_n$ and $\lim_{n\to\infty} ASx_n = 1 = A(1)$; $\lim_{n\to\infty} A^2x_n = 1 = S(1)$, $\lim_{n\to\infty} S^2x_n = 1 = A(1)$. Hence (A, S) is weakly sub-sequentially continuous and compatible of type (E).

Motivated by the Definition 2.7, Beloul [4], and Bouhadjera [5], we redefine the following in the setting of a multiplicative metric space.

Definition 2.9. Two self maps A and S defined on a multiplicative metric space (X, d) are said to be weakly sub-sequentially continuous (in short wsc), if there exists a sequence $\{x_n\}$ such that $\lim_{n\to\infty} Ax_n = \lim_{n\to\infty} Sx_n = z$, for some $z \in X$ and $\lim_{n\to\infty} ASx_n = Az$ or $\lim_{n\to\infty} SAx_n = Sz$.

Definition 2.10. Two self maps A and S defined on a multiplicative metric space (X, d) are said to be A-sub-sequentially continuous, if there exists a sequence $\{x_n\}$ such that $\lim_{n\to\infty} Ax_n = \lim_{n\to\infty} Sx_n = z$, for some $z \in X$ and $\lim_{n\to\infty} ASx_n = Az$.

Definition 2.11. Two self maps A and S defined on a multiplicative metric space (X, d) are said to be S-sub-sequentially continuous, if there exists a sequence $\{x_n\}$ such that $\lim_{n\to\infty} Ax_n = \lim_{n\to\infty} Sx_n = z$, for some $z \in X$ and $\lim_{n\to\infty} SAx_n = Sz$.

3. Main results

Theorem 3.1. *Let A, B, S and T be four self mappings of a multiplicative metric space X. If the pairs (A, S) and (B, T) are weakly sub sequentially continuous and compatible of type (E), then*

(i) *A and S have a coincidence point;*
(ii) *B and T have a coincidence point.*

Further, if there exists a real constant $\lambda \in (0, \frac{1}{2})$ such that for all $x, y \in X$, we have:

$$d(Sx, Ty) \leqslant \left[\phi[\max\{d(Ax, By), \frac{d(Ax, Sx)d(By, Ty)}{1 + d(Ax, By)}, \frac{d(Ax, Ty)d(By, Sx)}{1 + d(Ax, By)}\}]\right]^{\lambda}, \tag{3.1}$$

where $\phi : [0, \infty) \to [0, \infty)$ is a monotonic increasing function such that $\phi(0) = 0$ and $\phi(t) < t$ for all $t > 0$, then A, B, S and T have a unique common fixed point in X.

Proof. Since the pair (A, S) is weakly sub-sequentially continuous and compatible of type (E), therefore, there exists a sequence $\{x_n\}$ in X such that $\lim_{n\to\infty} Ax_n = \lim_{n\to\infty} Sx_n = z$, for some $z \in X$ and $\lim_{n\to\infty} ASx_n = Az$, $\lim_{n\to\infty} SAx_n = Sz$. The compatibility of type (E) implies that $\lim_{n\to\infty} A^2x_n = \lim_{n\to\infty} ASx_n = Sz$, for some $z \in X$ and $\lim_{n\to\infty} S^2x_n = \lim_{n\to\infty} SAx_n = Az$. Therefore $Az = Sz$, whereas in respect of the pair (B, T) being weakly sub-sequentially continuous, there exists a sequence $\{y_n\}$ in X such that $\lim_{n\to\infty} By_n = \lim_{n\to\infty} Ty_n = w$, for some $w \in X$ and $\lim_{n\to\infty} BTy_n = Bw$. Also, the pair (B, T) is compatible of type (E), therefore $\lim_{n\to\infty} B^2y_n = \lim_{n\to\infty} BTy_n = Tw$, for some $w \in X$ and $\lim_{n\to\infty} T^2y_n = \lim_{n\to\infty} TBy_n = Bw$. Therefore, $Bw = Tw$. Hence z is a coincidence point of the pair (A, S) whereas w is a coincidence point of the pair (B, T). Now, we'll prove that $z = w$. By putting $x = x_n$ and $y = y_n$ in (3.1), we have

$$d(Sx_n, Ty_n) \leqslant \left[\phi\left[\max\{d(Ax_n, By_n), \frac{d(Ax_n, Sx_n)d(By_n, Ty_n)}{1 + d(Ax_n, By_n)}, \frac{d(Ax_n, Ty_n)d(By_n, Sx_n)}{1 + d(Ax_n, By_n)}\}\right]\right]^{\lambda},$$

taking the limit as $n \to \infty$, we get

$$d(z, w) \leqslant \left[\phi[\max\{d(z, w), \frac{d(z, z)d(w, w)}{1 + d(z, w)}, \frac{d(z, w)d(w, z)}{1 + d(z, w)}\}]\right]^{\lambda} = [\phi[d(z, w)]]^{\lambda} \leqslant [d(z, w)]^{\lambda}.$$

Thus, $d(z, w) = 1$ and so $z = w$. Now we prove that $Sz = z$. By putting $x = z$ and $y = y_n$ in (3.1), we get

$$d(Sz, Ty_n) \leqslant \left[\phi[\max\{d(Az, By_n), \frac{d(Az, Sz)d(By_n, Ty_n)}{1 + d(Az, By_n)}, \frac{d(Az, Ty_n)d(By_n, Sz)}{1 + d(Az, By_n)}\}]\right]^{\lambda},$$

taking the limit as $n \to \infty$, we get

$$d(Sz,z) \leqslant \left[\phi[\max\{d(Sz,z), \frac{d(Sz,Sz)d(z,z)}{1+d(Sz,z)}, \frac{d(Sz,z)d(z,Sz)}{1+d(Sz,z)}\}]\right]^{\lambda} = [\phi[d(Sz,z)]]^{\lambda} \leqslant [d(Sz,z)]^{\lambda}.$$

Thus, $d(Sz,z)=1$ and so $Sz=z$. Therefore $Az=Sz=z$ and hence z is a common fixed point of (A,S). Now, we'll show that z is a common fixed point of (B,T). By putting $x=\{x_n\}$ and $y=z$ in (3.1), we get

$$d(Sx_n,Tz) \leqslant \left[\phi[\max\{d(Ax_n,Bz), \frac{d(Ax_n,Sx_n)d(Bz,Tz)}{1+d(Ax_n,Bz)}, \frac{d(Ax_n,Tz)d(Bz,Sx_n)}{1+d(Ax_n,Bz)}\}]\right]^{\lambda},$$

taking the limit as $n \to \infty$, we get

$$d(z,Tz) \leqslant \left[\phi[\max\{d(z,Tz), \frac{d(z,z)d(Tz,Tz)}{1+d(z,Tz)}, \frac{d(z,Tz)d(Tz,z)}{1+d(z,z)}\}]\right]^{\lambda} = [\phi[d(z,Tz)]]^{\lambda} \leqslant [d(z,Tz)]^{\lambda}.$$

Thus, $d(z,Tz)=1$ and so $Tz=z$. Therefore, $Bz=Tz=z$ and hence z is a common fixed point of (B,T). Therefore, in all $z=Az=Bz=Sz=Tz$, i.e., z is a common fixed point of A,B,S, and T. For uniqueness, let us asume that z and w be two common fixed points of A,B,S, and T. Then, by using (3.1) we have

$$\begin{aligned} d(Sz,Tw)=d(z,w) &\leqslant \left[\phi[\max\{d(Az,Bw), \frac{d(Az,Sz)d(Bw,Tw)}{1+d(Az,Bw)}, \frac{d(Az,Tw)d(Bw,Sz)}{1+d(Az,Bw)}\}]\right]^{\lambda} \\ &= [\phi(d(z,w))]^{\lambda} \leqslant [d(z,w)]^{\lambda}. \end{aligned}$$

This implies that $z=w$. Therefore, z is a unique common fixed point of S,T,A, and B. This completes the proof. □

If we put $A=B$ in Theorem 3.1 we have the following corollary for three mappings.

Corollary 3.2. *Let A, S, and T be three self mappings of a multiplicative metric space X. If the pairs (A,S) and (A,T) are weakly sub-sequentially continuous and compatible of type (A), then*

(i) *A and S have a coincidence point;*
(ii) *A and T have a coincidence point.*

Further, if there exists a real constant $\lambda \in (0,\frac{1}{2})$ such that for all $x,y \in X$, we have:

$$d(Sx,Ty) \leqslant \left[\phi[\max\{d(Ax,Ay), \frac{d(Ax,Sx)d(Ay,Ty)}{1+d(Ax,Ay)}, \frac{d(Ax,Ty)d(Ay,Sx)}{1+d(Ax,Ay)}\}]\right]^{\lambda},$$

where $\phi:[0,\infty)\to[0,\infty)$ is a continuous and monotonic increasing function such that $\phi(0)=0$ and $\phi(t)<t$ for all $t>0$, then A, S, and T have a unique common fixed point in X.

Alternatively, if we set $S=T$ in Theorem 3.1, we'll have the following corollary for three self mappings.

Corollary 3.3. *Let A,B, and S be three self mappings of a multiplicative metric space (X). If the pairs (A,S) and (B,S) are weakly sub-sequentially continuous and compatible of type (E), then*

(i) *A and S have a coincidence point;*
(ii) *B and S have a coincidence point.*

Further, if there exists a real constant $\lambda \in (0,\frac{1}{2})$ such that for all $x,y \in X$, we have:

$$d(Sx,Sy) \leqslant \left[\phi[\max\{d(Ax,By), \frac{d(Ax,Sx)d(By,Sy)}{1+d(Ax,By)}, \frac{d(Ax,Sy)d(By,Sx)}{1+d(Ax,By)}\}]\right]^{\lambda},$$

where $\phi:[0,\infty)\to[0,\infty)$ is a continuous and monotonic increasing function such that $\phi(0)=0$ and $\phi(t)<t$ for all $t>0$. Then A, B, and S have a unique common fixed point in X.

If we put $S = T$ in Corollary 3.2, we have the following result for two self mappings.

Corollary 3.4. *Let A and S be two self mappings of a multiplicative metric space X. If the pair (A, S) is weakly sub-sequentially continuous and compatible of type (E), then A and S have a coincidence point. Further, if there exists a real constant $\lambda \in (0, \frac{1}{2})$ such that for all $x, y \in X$, we have*

$$d(Sx, Sy) \leqslant \left[\phi[\max\{d(Ax, Ay), \frac{d(Ax, Sx)d(Ay, Sy)}{1 + d(Ax, Ay)}, \frac{d(Ax, Sy)d(Ay, Sx)}{1 + d(Ax, Ay)}\}]\right]^{\lambda},$$

where $\phi : [0, \infty) \to [0, \infty)$ is a continuous and monotonic increasing function such that $\phi(0) = 0$ and $\phi(t) < t$ for all $t > 0$, then A and S have a unique common fixed point.

The following result can be derived from Theorem 3.1 on setting $\phi(t) = kt$.

Corollary 3.5. *Let A, B, S and T be four self mappings of a multiplicative metric space X. If the pairs (A, S) and (B, T) are weakly sub-sequentially continuous and compatible of type (E), then*

(i) *A and S have a coincidence point;*
(ii) *B and T have a coincidence point.*

Further, if there exists a real constant $\lambda \in (0, \frac{1}{2})$ such that for all $x, y \in X$, we have

$$d(Sx, Ty) \leqslant \left[k \max\{d(Ax, By), \frac{d(Ax, Sx)d(By, Ty)}{1 + d(Ax, By)}, \frac{d(Ax, Ty)d(By, Sx)}{1 + d(Ax, By)}\}\right]^{\lambda},$$

then A, B, S, and T have a unique common fixed point in X.

Theorem 3.6. *Let A, B, S and T be four self mappings of a multiplicative metric space X. If the pair (A, S) is A-compatible of type(E) and A-sub-sequentially continuous and $\{B, T\}$ is B-compatible of type (E) and B-sub-sequentially continuous, then*

(i) *A and S have a coincidence point;*
(ii) *B and T have a coincidence point.*

Further, if there exists a real constant $\lambda \in (0, \frac{1}{2})$ such that for all $x, y \in X$, we have

$$d(Sx, Ty) \leqslant \left[\phi[\max\{d(Ax, By), \frac{d(Ax, Sx)d(By, Ty)}{1 + d(Ax, By)}, \frac{d(Ax, Ty)d(By, Sx)}{1 + d(Ax, By)}\}]\right]^{\lambda},$$

where $\phi : [0, \infty) \to [0, \infty)$ is a continuous and monotonic increasing function such that $\phi(0) = 0$ and $\phi(t) < t$ for all $t > 0$, then A, B, S, and T have a unique common fixed point in X.

Proof. The proof is obvious as on the lines of Theorem 3.1. □

Remark 3.7. The results similar to Corollaries 3.2-3.5 can also be obtained in the respect of Theorem 3.6.

If we set $k = 1$, $S = T$, and $A = B = I_X$ (the identity mapping on X) in the corollaries of Theorem 3.1 and Theorem 3.6 with completeness as an additional condition, then we have the Banach fixed theorem in a complete multiplicative metric space (X, d) as follows.

Corollary 3.8. *Let T be a mapping of a complete multiplicative metric space (X, d) into itself satisfying the following condition:*

$$d(Tx, Ty) \leqslant [d(x, y)]^{\lambda} \ \textit{for all} \ \ x, y \in X, \ \textit{where} \ \lambda \in (0, \frac{1}{2}).$$

Then T has a unique fixed point in X.

Now, we'll furnish with an example to illustrate the validity of claimed results in this article.

Example 3.9. Let $X = [0, 2]$ with the usual multiplicative metric $d = |\frac{x}{y}|$. Set $A = B$ and $S = T$. Let us define the self mappings $A, B, S,$ and T as follows:

$$A(x) = B(x) = \begin{cases} 1, & 0 \leqslant x \leqslant 1, \\ \frac{3}{4}, & 1 < x \leqslant 2, \end{cases} \qquad S(x) = T(x) = \begin{cases} \frac{x+1}{2}, & 0 \leqslant x \leqslant 1, \\ 2, & 1 < x \leqslant 2. \end{cases}$$

Let us consider a sequence $\{x_n\}$ in X defined by $x_n = 1 - \frac{1}{n}$ for $n \in N$ such that $\lim_{n\to\infty} Ax_n = 1 = \lim_{n\to\infty} Sx_n$, $\lim_{n\to\infty} ASx_n = 1 = A(1)$, $\lim_{n\to\infty} A^2x_n = 1 = S(1)$, and $\lim_{n\to\infty} S^2x_n = 1 = A(1)$. Hence, (A, S) is weakly sub-sequentially continuous and compatible of type (E) mappings. Proceeding in the same way, we can easily show that (B, T) is weakly sub-sequentially continuous and compatible of type (E).

For $\lambda = \frac{1}{3}$, we have

$$d(Sx, Ty) \leqslant \left[\phi[\max\{d(Ax, By), \frac{d(Ax, Sx)d(By, Ty)}{1 + d(Ax, By)}, \frac{d(Ax, Ty)d(By, Sx)}{1 + d(Ax, By)}\}]\right]^{\lambda}.$$

Thus, all the conditions of Theorem 3.1 are satisfied and A, B, S, T have a unique common fixed point $x = 1$. Although, all the mappings taking under consideration are discontinuous at the common fixed point $x = 1$.

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References

[1] A. N. A. Afrah, *Common fixed point results for compatible-type mappings in multiplicative metric spaces*, J. Nonlinear Sci. Appl., **9** (2016), 2244–2257. 1
[2] A. E. Bashirov, E. M. Kurpinar, A. Özyapici, *Multiplicative calculus and its applications*, Math. Anal. Appl., **337** (2008), 36–48. 1, 2.1
[3] A. E. Bashirov, E. Misirli, Y. Tandogdu, A. Özyapici, *On modeling with multiplicative differential equations*, Appl. Math. J. Chinese Univ. Ser. B, **26** (2011), 425–438. 1
[4] S. Beloul, *Some fixed point theorems for weakly sub-sequentially continuous and compatible of type (E) mappings with application*, Int. J. Nonlinear Anal. Appl., **7** (2016), 53–62. 1, 2.6, 2.8, 2
[5] H. Bouhadjera, *More general common fixed point theorems under a new concept*, Demonstr. Math., **49** (2016), 64–78. 1, 2
[6] H. Bouhadjera, C. Godet-Thobie, *Common fixed point theorems for pairs of sub-compatible maps*, arXiv, **2009** (2009), 16 pages. 1, 2, 2.7
[7] X. J. He, M. M. Song, D. P. Chen, *Common fixed points for weak commutative mappings on a multiplicative metric space*, Fixed Point Theory Appl., **2014** (2014), 9 pages. 1
[8] G. Jungck, *Compatible mappings and common fixed points*, Internat. J. Math. Math. Sci., **9** (1986), 771–779. 1
[9] G. Jungck, B. E. Rhoades, *Fixed point for set valued functions without continuity*, Indian J. Pure Appl. Math., **29** (1998), 227–238. 1
[10] P. Kumar, S. Kumar, S. M. Kang, *Common Fixed Point Theorems for Subcompatible and Occasionally Weakly Compatible Mappings in Multiplicative Metric Spaces*, Int. J. Math. Anal., **9** (2015), 1785–1794. 1
[11] M. Sarwar, Badshah-e-Rome, *Some unique fixed point theorems in multiplicative metric space*, arXiv, **2014** (2014), 19 pages. 1
[12] M. R. Singh, S. Y. Mahendra, *Compatible mappings of type (E) and common fixed point theorems of Meir-Keller type*, Int. J. Math. Sci. Engg. Appl., **1** (2007), 299–315. 2
[13] M. Özavsar, A. C. Cevikel, *Fixed point of multiplicative contraction mappings on multiplicative metric space*, arXiv, **2012** (2012), 14 pages. 1, 2.2, 2, 2.3