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Common fixed point theorems in intuitionistic fuzzy metric spaces and intuitionistic (ϕ, ψ)-contractive mappings

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Abstract

In this paper, we introduced the concept of intuitionistic (ϕ, ψ)-contractive mappings and proved some a common fixed point theorems in intuitionistic fuzzy metric space under (ϕ, ψ)-contractive mappings and weakly commuting intuitionistic fuzzy metric space.

Keywords: Intuitionistic fuzzy metric space, (ϕ, ψ)-contractive mapping, weakly commuting, fixed point.

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1. Introduction

In 1965, some author introduced the notion of the concept of fuzzy set theory which is one of the vital and useful branch of mathematics [12]. In 1986, the notion of an intuitionistic fuzzy set was introduced by Atanassov [2]. Kramosil and Michalek presented the definition of fuzzy metric space [5]. Many authors have studied fuzzy metric spaces (cf. [3, 11, 13]). In 2004, Park studied the concept of intuitionistic fuzzy metric space with $*$ continuous t-norm and $\diamondsuit$ continuous t-conorm [7]. A generalization of commutativity was introduced by Sessa [8]. Common fixed point theorem was proved by Jungck [4] and introduced the concept of weakly compatible. Fuzzy ψ-contractive mappings in non-Archiemedean fuzzy metric space introduced by Mihet [6] and proved fixed point theorem under fuzzy ψ-contractive mapping in fuzzy metric space. Vetro [10] got some Common fixed point results for a pair of generalization contractive type mappings.

In this paper, we introduce the concept of intuitionistic (ϕ, ψ)-contractive mappings and prove some a common fixed point theorems in intuitionistic fuzzy metric space under (ϕ, ψ)-contractive mappings and weakly commuting intuitionistic fuzzy metric space.

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2. Preliminaries

Definition 2.1 ([1]). A binary operation $* : [0,1]\times[0,1] \longrightarrow [0,1]$ is continuous t-norm if $*$ satisfies the following conditions:

(i) $*$ is commutative and associative;
(ii) $*$ is continuous;
(iii) $a*1 = a$ for all $a\in[0,1]$;
(iv) $a*b \leqslant c*d$ whenever $a\leqslant c$, $b\leqslant d$ and $a,b,c,d\in[0,1]$.

For examples of t-norms we have

- $a*b = \min\{a,b\}$;
- $a*b = a.b$;
- $a*b = \max\{0, a+b-1\}$;
- $a*b = \begin{cases} \min\{a,b\}, & \text{if } a+b>1,\\ 0, & \text{otherwise.}\end{cases}$

Definition 2.2 ([1]). A binary operation $\diamondsuit : [0,1]\times[0,1] \longrightarrow [0,1]$ is continuous t-conorm if $\diamondsuit$ satisfies the following conditions:

(i) $\diamondsuit$ is commutative and associative;
(ii) $\diamondsuit$ is continuous;
(iii) $a\diamondsuit 1 = a$ for all $a\in[0,1]$;
(iv) $a\diamondsuit b \leqslant c\diamondsuit d$ whenever $a\leqslant c$, $b\leqslant d$ and $a,b,c,d\in[0,1]$.

For examples of t-conorms we have

- $a\diamondsuit b = \max\{a,b\}$;
- $a\diamondsuit b = a+b-\ a.b$;
- $a\diamondsuit b = \min\{a+b,1\}$;
- $a\diamondsuit b = \begin{cases} \max\{a,b\}, & \text{if } a+b<1,\\ 1, & \text{otherwise.}\end{cases}$

Definition 2.3 ([1]). Let $(X,M,N,*,\diamondsuit)$ is said to be an intuitionistic fuzzy metric space if X is an arbitrary set, $*$ is a continuous t-norm, $\diamondsuit$ is a continuous t-conorm and M,N are fuzzy sets on $X\times X\times[0,\infty)$ satisfying the following conditions:

(i) $M(x,y,t)+N(x,y,t)\leqslant 1$ for all $x,y\in X$ and $t>0$;
(ii) $M(x,y,0)=0$ for all $x,y\in X$;
(iii) $M(x,y,t)=1$ for all $x,y\in X$ and $t>0$ if and only if $x=y$;
(iv) $M(x,y,t)=M(y,x,t)$ for all$x,y\in X$ and $t>0$;
(v) $M(x,y,t)*M(y,z,s)\leqslant M(x,z,t+s)$ for all $x,y,z\in X$ and $t,s>0$;
(vi) $M(x,y,.) : [0,\infty)\longrightarrow[0,1]$ is left continuous for all $x,y,\in X$;
(vii) $\lim\limits_{t\longrightarrow\infty} M(x,y,t)=1$ for all $x,y\in X$ and $t>0$;
(vii) $N(x,y,0)=1$ for all $x,y\in X$;
(viii) $N(x,y,t)=0$ for all $x,y\in X$ and $t>0$ if and only if $x=y$;
(ix) $N(x,y,t)=N(y,x,t)$ for all $x,y\in X$ and $t>0$;
(x) $N(x,y,t)\diamondsuit N(y,z,s)\geqslant N(x,z,t+s)$ for all $x,y,z\in X$ and $t,s>0$;
(xi) $N(x,y,.) : [0,\infty)\longrightarrow[0,1]$ is right continuous for all $x,y,\in X$;
(xii) $\lim\limits_{t\longrightarrow\infty} N(x,y,t)=0$ for all $x,y\in X$ and $t>0$.

Then (M, N) is called an intuitionistic fuzzy metric space on X. The functions $M(x, y, t)$ and $N(x, y, t)$ denote the degree of nearness and the degree of non-nearness between x and y w.r.t. t respectively.

Definition 2.4. Let $(X, M, N, *, \diamondsuit)$ be an intuitionistic fuzzy metric space. Then a sequence $\{x_n\}$ in X is said to be convergent to a point $x \in X$ for all $t > 0$, $\lim_{n\longrightarrow\infty} M(x_n, x, t) = 1$ and $N(x_n, x, t) = 0$.

Definition 2.5. Let $(X, M, N, *, \diamondsuit)$be an intuitionistic fuzzy metric space. Then a sequence $\{x_n\}$ in X is said to be Cauchy sequence if, for all $t > 0$ and $p > 0$, $\lim_{n\longrightarrow\infty} M(x_{n+p}, x_n, t) = 1$ and $N(x_{n+p}, x_n, t) = 0$.

Definition 2.6. An intuitionistic fuzzy metric space $(X, M, N, *, \diamondsuit)$ is said to be complete if and only if every Cauchy sequence in X is convergent.

Example 2.7. Let $X = \{\frac{1}{n}; n = 1, 2, 3, \ldots\} \bigcup \{0\}$ and let $*$ be the continuous t-norm and $\diamondsuit$ be the continuous t-conorm defined by $a * b = ab$ and $a \diamondsuit b = \min\{1, a + b\}$ for all $a, b \in [0, 1]$ for each $x, y \in X$ and $t > 0$, define (M, N) by

$$M(x, y, t) = \begin{cases} 0, & t = 0, \\ \frac{t}{t+d(x,y)}, & t > 0, \end{cases} \qquad N(x, y, t) = \begin{cases} 1, & t = 0, \\ \frac{d(x,y)}{t+d(x,y)}, & t > 0. \end{cases}$$

Then $(X, M, N, *, \diamondsuit)$ is an intuitionistic fuzzy metric space.

Definition 2.8. The class of all ϕ- functions will be denoted by Φ if $\phi : (0, 1] \longrightarrow (0, 1]$, is continuous non decreasing function, s.t. $\lim_{t\longrightarrow 0} \phi(t) = 0$, and $\phi(1) = 1$.

Definition 2.9. The class of all ψ- functions will be denoted by Ψ if $\psi : [0, \infty) \longrightarrow [0, \infty)$, is continuous non decreasing function, s.t. $\lim_{t\longrightarrow\infty} \psi(t) = \infty$, and $\psi(0) = 0$.

Definition 2.10 ([8])**.** A pair of self mappings (A, B) of an intuitionistic fuzzy metric space $(X, M, N, *, \diamondsuit)$ is said to be weakly commuting if $M(ABx, BAx, t) \geqslant M(Ax, Bx, t)$, $N(ABx, BAx, t) \leqslant N(Ax, Bx, t)$ for all $x \in X$ and $t > 0$.

Definition 2.11. Let $(X, M, N, *, \diamondsuit)$ be an intuitionistic fuzzy metric space and $\psi \in \Psi$. A mapping $A : X \longrightarrow X$ is called (ϕ, ψ)-contractive mapping if the following implication takes place:

$$M(x, y, t) > 0 \Longrightarrow M(A(x), A(y), t) \geqslant \phi(M(x, y, t)),$$
$$N(x, y, t) < 1 \Longrightarrow N(A(x), B(y), t) \leqslant \psi(N(x, y, t)).$$

Lemma 2.12. *If* $\phi \in \Phi$, *then* $\lim_{n\longrightarrow\infty} \phi^n(t) = 1$ *for all* $t \in (0, 1)$.

Lemma 2.13. *If* $\psi \in \Psi$, *then* $\lim_{n\longrightarrow\infty} \psi^n(t) = 0$ *for all* $t \in (0, 1)$.

Example 2.14. Let $X = [0, \infty)$, $a * b = \min\{a, b\}$, $a \diamondsuit b = \max\{a, b\}$ $\forall a, b \in [0, 1]$ and define

$$M(x, y, t) = \begin{cases} 0, & t \leqslant d(x, y), \\ 1, & t > d(x, y), \end{cases} \qquad N(x, y, t) = \begin{cases} 1, & t \leqslant d(x, y), \\ 0, & t < d(x, y), \end{cases}$$

for all $x, y \in X$ and $t > 0$. Then $(X, M, N, *, \diamondsuit)$ is an intuitionistic fuzzy metric space.

Let ϕ be a mapping in Φ. Since $\phi(1) = 1$,

$$M(x, y, t) > 0 \Longrightarrow M(x, y, t) = 1 \Longrightarrow \phi(M(x, y, t)) = 1,$$

and let ψ be a mapping in Ψ. Since $\psi(0) = 0$,

$$N(x, y, t) < 1 \Longrightarrow N(x, y, t) = 0 \Longrightarrow \psi(N(x, y, t)) = 0,$$

therefore every mapping $A : X \longrightarrow X$ is an intuitionistic fuzzy (ϕ, ψ)-contractive mapping.

Definition 2.15. Let $(X, M, N, *, \diamondsuit)$ be an intuitionistic fuzzy metric space and let (A, B) is a pair of (ϕ, ψ)-contractive mapping if there exist $\psi \in \Psi$ such that for every $x, y \in X$ and $t \in (0, 1)$ with

$$M(x, y, t) > 0 \Longrightarrow M(A(x), B(y), t) \geqslant \phi(\min\{M(x, y, t), M(A(x), x, t), M(y, B(y), t)\}),$$
$$N(x, y, t) < 1 \Longrightarrow N(A(x), B(y), t) \leqslant \psi(\max\{N(x, y, t), N(A(x), x, t), N(y, B(y), t)\}).$$

3. Main results

Theorem 3.1. *Let* A *and* B *be self mappings of a complete intuitionistic fuzzy metric space* $(X, M, N, *, \diamondsuit)$ *such that* $a * b = \min\{a, b\}$ *and* $a \diamondsuit b = \max\{a, b\}$ *for all* $a, b \in X$. *Assume* $M(x_o, A(x_o), t) > 0$ *and* $N(x_o, A(x_o), t) < 1$ *for all* $x_o \in X$, $\forall t > 0$, *let* A, B *be self mappings of* X *satisfying the following conditions:*

(1) A, B *are two continuous and intuitionistic* ψ*-contractive mappings;*

(2)

$$M(A(x), B(y), t) \geqslant \phi(\min\{M(x, y, t), M(A(x), x, t), M(y, B(y), t)\}),$$
$$N(A(x), B(y), t) \leqslant \psi(\max\{N(x, y, t), N(A(x), x, t), N(y, B(y), t)\})$$

for all $x, y \in X$ *and* $t > 0$;

(3) $\{x_n\}$ *is* (A, B)*-sequence of initial point* x_o.

Then A *and* B *have a unique common fixed point in* X.

Proof. $\{x_n\}$ is (A, B)-sequence of initial point x_o. Then $x_o \in X$ and define the sequence $\{x_n\}$ by $x_1 = A(x_o), x_2 = B(x_1), \ldots, x_{2n+1} = A(x_{2n}), x_{2n+2} = B(x_{2n+1})$, from $M(x_o, A(x_o), t) = M(x_o, x_1, t) > 0$ and $N(x_o, A(x_o), t) = N(x_o, x_1, t) < 1$ it follows that,

$$\begin{aligned}
M(x_2, x_1, t) &= M(A(x_o), B(x_1), t)\\
&\geqslant \phi(\min\{M(x_o, x_1, t), M(A(x_o), x_o, t), M(x_1, B(x_1), t)\})\\
&\geqslant (M(x_o, x_1, t)) > 0,\\
N(x_2, x_1, t) &= N(A(x_o), B(x_1), t)\\
&\leqslant \psi(\max\{N(x_o, x_1, t), N(A(x_o), x_o, t), N(x_1, B(x_1), t)\})\\
&\leqslant (N(x_o, x_1, t)) < 1,\\
M(x_3, x_2, t) &= M(A(x_2), B(x_1), t)\\
&\geqslant \phi(\min\{M(x_2, x_1, t), M(A(x_2), x_2, t), M(x_1, B(x_1), t)\})\\
&\geqslant \phi(M(x_2, x_1, t))\\
&\geqslant \phi^2(M(x_o, x_1, t)) > 0,\\
N(x_3, x_2, t) &= N(A(x_2), B(x_1), t)\\
&\leqslant \psi(\max\{N(x_2, x_1, t), N(A(x_2), x_2, t), N(x_1, B(x_1), t)\})\\
&\leqslant \psi(N(x_2, x_1, t))\\
&\leqslant \psi^2(N(x_o, x_1, t)) < 1.
\end{aligned}$$

Generally, for each n, we get $M(x_{n+1}, x_n, t) \geqslant \phi^n(M(x_o, x_1, t)) > 0$ and $N(x_{n+1}, x_n, t) \leqslant \psi^n(N(x_o, x_1, t)) < 1$. By Lemmas 2.12 and 2.13 as $n \longrightarrow \infty$ we get $\lim_{n \longrightarrow \infty} M(x_{n+1}, x_n, t) = 1$, $\lim_{n \longrightarrow \infty} N(x_{n+1}, x_n, t) = 0$. Then $\{x_n\}$ is a Cauchy. Since X is Complete, then there exists $x \in X$ such that $x_n \longrightarrow x$, for all $t > 0$.

$$M(x_{2n+1}, A(x), t) = M(A(x_{2n}), A(x), t) \geqslant \phi(M(x_{2n}, x, t)),$$
$$N(x_{2n+1}, A(x), t) = N(A(x_{2n}), A(x), t) \leqslant \psi(N(x_{2n}, x, t)).$$

Since A is continuous, letting $n \longrightarrow \infty$ it follows, $M(x, A(x), t) \geqslant \phi(1) = 1$ and $N(x, A(x), t) \leqslant \psi(0) = 0$, hence $A(x) = x$. Similarity we get $B(x) = x$ and x is common fixed point of A and B. Assume that $x, y \in X$ are two common fixed points of A and B. If $x \neq y$, there exists $t > 0$, such that $0 < M(x, y, t) < 1$ and $0 > N(x, y, t) > 1$. Then

$$\begin{aligned}
M(x, y, t) &= M(A(x), B(y), t)\\
&\geqslant \phi(\min\{M(x, y, t), M(A(x), x, t), M(y, B(y), t)\}) = \phi(M(x, y, t)) > M(x, y, t),
\end{aligned}$$

$$N(x, y, t) = N(A(x), B(y), t) \leqslant \psi(\max\{N(x, y, t), N(A(x), x, t), N(y, B(y), t)\}) = \psi(N(x, y, t)) < N(x, y, t).$$

This is a contradiction, therefore $x = y$. □

Theorem 3.2. *Let A, B and T be self mappings of a Complete intuitionistic fuzzy metric space $(X, M, N, *, \diamondsuit)$ such that $a * b = \min\{a, b\}$, $a \diamondsuit b = \max\{a, b\}$ for all $a, b \in X$, let A, B and T be self mappings of X satisfying the following conditions:*

(1) *A is intuitionistic ψ-contractive mapping and B, T are two continuous mappings;*
(2) *$A(x) \subset B(x) \cap T(x)$ and $\{A, B\}, \{A, T\}$ are weakly commuting;*
(3)

$$\begin{aligned} M(Ax, Ay, t) &\geqslant \phi(\min\{M(Bx, Ty, t), M(Bx, Ax, t), M(Bx, Ay, t), M(Ty, Ay, t)\}), \\ N(Ax, Ay, t) &\leqslant \psi(\max\{N(Bx, Ty, t), N(Bx, Ax, t), N(Bx, Ay, t), N(Ty, Ay, t)\}), \end{aligned}$$

for all $x, y \in X$ and $t > 0$.

Then A, B and T have a unique Common fixed point in X.

Proof. Let $x_o \in X$ be an arbitrary point. Then there exist a point $x_1 \in X$ such that $Ax_o = Bx_1$. Since $A(x) \subset T(x)$, then there exists a point $x_2 \in X$ such that $Ax_1 = Tx_2$. In general, we get a sequence $\{y_n\}$ recursively as $y_n = Bx_{n+1} = Ax_n$ and $y_{n+1} = Tx_{n+2} = Ax_{n+1}$, $n \in N$.

Let $M_n = M(y_{n+1}, y_n, t) = M(Ax_{n+1}, Ax_n, t)$ and $N_n = N(y_{n+1}, y_n, t) = N(Ax_{n+1}, Ax_n, t)$ and $M(y_o, y_1, t) > 0$, $N(y_o, y_1, t) < 1$. Then

$$M_{n+1} = M(y_{n+2}, y_{n+1}, t) = M(Ax_{n+2}, Ax_{n+1}, t) \quad \text{and} \quad N_{n+1} = N(y_{n+2}, y_{n+1}, t) = N(Ax_{n+2}, Ax_{n+1}, t).$$

Using inequality (3), we get,

$$\begin{aligned} M_{n+1} &= M(Ax_{n+2}, Ax_{n+1}, t) \\ &\geqslant \phi(\min\{M(Bx_{n+2}, Tx_{n+1}, t), M(Bx_{n+2}, Ax_{n+2}, t), M(Bx_{n+2}, Ax_{n+1}, t), M(Tx_{n+1}, Ax_{n+1}, t)\}), \\ &= \phi(\min\{M(Ax_{n+1}, Ax_n, t), M(Ax_{n+1}, Ax_{n+2}, t), M(Ax_{n+1}, Ax_{n+1}, t), M(Ax_n, Ax_{n+1}, t)\}) \\ &= \phi(\min\{M_n, M_{n+1}, 1, M_n\}), \\ N_{n+1} &= N(Ax_{n+2}, Ax_{n+1}, t) \\ &\leqslant \psi(\max\{N(Bx_{n+2}, Tx_{n+1}, t), N(Bx_{n+2}, Ax_{n+2}, t), N(Bx_{n+2}, Ax_{n+1}, t), N(Tx_{n+1}, Ax_{n+1}, t)\}) \\ &= \psi(\max\{N(Ax_{n+1}, Ax_n, t), N(Ax_{n+1}, Ax_{n+2}, t), N(Ax_{n+1}, Ax_{n+1}, t), N(Ax_n, Ax_{n+1}, t)\}) \\ &= \psi(\max\{N_n, N_{n+1}, 0, N_n\}). \end{aligned}$$

If $M_n > M_{n+1}$ and $N_n < N_{n+1}$, then by definition of (ϕ, ψ)-we have

$$M_{n+1} \geqslant \phi(M_{n+1}) > M_{n+1}, \quad N_{n+1} \leqslant \psi(N_{n+1}) < N_{n+1}.$$

This is a contradiction, so,

$$M_{n+1} \geqslant \phi(M_n), \quad N_{n+1} \leqslant \psi(N_n).$$

We get, $M(y_{n+2}, y_{n+1}, t) \geqslant \phi(M(y_{n+1}, y_n, t))$ and $N(y_{n+2}, y_{n+1}, t) \leqslant \psi(N(y_{n+1}, y_n, t))$, $\forall n \in N$, $t > 0$. Hence, repeating this inequality n times we obtain,

$$M(y_n, y_{n+1}, t) \geqslant \phi^n(M(y_o, y_1, t)), \quad N(y_n, y_{n+1}, t) \leqslant \psi^n(N(y_o, y_1, t).$$

Letting $n \longrightarrow \infty$, we get $\lim_{n \longrightarrow \infty} M(y_n, y_{n+1}, t) = 1$ and $\lim_{n \longrightarrow \infty} N(y_n, y_{n+1}, t) = 0$. Then $\{y_n\}$ is a Cauchy. Since X is Complete, then there exists $z \in X$ such that $y_n \longrightarrow z$. Hence $(Ax_n) \longrightarrow z \in X$. Since A is intuitionistic (ϕ, ψ)-contractive mapping,

$$M(y_n, Az, t) = M(Ax_n, Az, t) \geqslant \phi(M(x_n, z, t)), \quad N(y_n, Az, t) = N(Ax_n, Az, t) \leqslant \psi(N(x_n, z, t)).$$

By taking the limit as $n \longrightarrow \infty$ we obtain,

$$M(z, Az, t) \geqslant \phi(1) = 1, \quad N(z, Az, t) \leqslant \psi(0) = 0,$$

hence $Az = z$.Since $(Ax_n) \longrightarrow z \in X$, hence the sub-sequences $\{Bx_n\}$ and $\{Tx_n\}$ of $\{Ax_n\}$ have the same limit. Since B is continuous, in this case we have $BAx_n \longrightarrow Bz, BBx_n \longrightarrow Bz$. Also (A, B) is weakly commuting, we have $ABx_n \longrightarrow Bz$. Let $x = Bx_n, y = x_n$ in (3), we get

$$M(ABx_n, Ax_n, t) \geqslant \phi(\min\{M(BBx_n, Tx_n, t), M(BBx_n, ABx_n, t), M(BBx_n, Ax_n, t), M(Tx_n, Ax_n, t)\}),$$
$$N(ABx_n, Ax_n, t) \leqslant \psi(\max\{N(BBx_n, Tx_n, t), N(BBx_n, ABx_n, t), N(BBx_n, Ax_n, t), N(Tx_n, Ax_n, t)\}).$$

Taking limit $n \longrightarrow \infty$ we get,

$$\begin{aligned} M(Bz, z, t) &\geqslant \phi(\min\{M(Bz, z, t), M(Bz, Bz, t), M(Bz, z, t), M(z, z, t)\}) \\ &= \phi(\min\{M(Bz, z, t), 1, M(Bz, z, t), 1\}) \\ &= \phi(M(Bz, z, t)) > M(Bz, z, t), \\ N(Bz, z, t) &\leqslant \psi(\max\{N(Bz, z, t), N(Bz, Bz, t), N(Bz, z, t), N(z, z, t)\}) \\ &= \psi(\max\{N(Bz, z, t), 0, N(Bz, z, t), 0\}) \\ &= \psi(N(Bz, z, t)) < N(Bz, z, t). \end{aligned}$$

So we get, $Bz = z$. Since T is continuous, in this case we have $TTx_n \longrightarrow Tz$, $TAx_n \longrightarrow Tz$. Also (A, T) is weakly commuting, we have $ATx_n \longrightarrow Tz$, let $x = x_n, y = Tx_n$ in (3), we get,

$$M(Ax_n, ATx_n, t) \geqslant \phi(\min\{M(Bx_n, TTx_n, t), M(Bx_n, Ax_n, t), M(Bx_n, ATx_n, t), M(TTx_n, ATx_n, t)\}),$$
$$N(Ax_n, ATx_n, t) \leqslant \psi(\max\{N(Bx_n, TTx_n, t), N(Bx_n, Ax_n, t), N(Bx_n, ATx_n, t), N(TTx_n, ATx_n, t)\}).$$

Taking limit as $n \longrightarrow \infty$,

$$\begin{aligned} M(z, Tz, t) &\geqslant \phi(\min\{M(z, Tz, t), M(z, z, t), M(z, Tz, t), M(Tz, Tz, t)\}) \\ &= \phi(\min\{M(z, Tz, t), 1, M(z, Tz, t), 1\}) \\ &= \phi(M(z, Tz, t)) > M(z, Tz, t), \\ N(z, Tz, t) &\leqslant \psi(\max\{N(z, Tz, t), N(z, z, t), N(z, Tz, t), N(Tz, Tz, t)\}) \\ &= \psi(\max\{N(z, Tz, t), 0, N(z, Tz, t), 0\}) \\ &= \psi(N(z, Tz, t)) < N(z, Tz, t). \end{aligned}$$

So, we get $Tz = z$. Thus, we have $Az = Bz = Tz = z$, then z is a common fixed point of A, B, and T. Now we prove the uniqueness of the common fixed points of A, B, and T. Let v be another common fixed point of A, B, and T. Then $Av = Bv = Tv = v$. Take $x = z, y = v$ in(3), we get,

$$M(z, v, t) \geqslant \phi(\min\{M(Bz, Tv, t), M(Bz, Az, t), M(Bz, Av, t), M(Tv, Az, t)\}) = \phi(M(z, v, t)) > M(z, v, t),$$
$$N(z, v, t) \leqslant \psi(\max\{N(Bz, Tv, t), N(Bz, Az, t), N(Bz, Av, t), N(Tv, Az, t)\}) = \psi(N(z, v, t)) < N(z, v, t).$$

We get $z = v$. Therefore z is a unique common fixed point of A, B, and T. □

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