Estimation of type II truncated Fréchet inverse exponential distribution under censored data

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Abstract

This paper proposes a new probability distribution called as type II truncated Fréchet inverse exponential distribution. Fundamental statistical properties like moments, incomplete moments, and quantile function of the proposed model are studied. The estimation of the parameter of the new model is approached by maximum likelihood method based on complete and censored samples. Asymptotic confidence interval of model parameters is investigated. We assess the numerical results to study the theoretical results. Superiority of the type II truncated Fréchet inverse exponential model over some known distributions is showed through one real data set.

1. Introduction

In the last years, many authors concerned with generated families of distributions. Some of the authors used the truncated models as the generators. Abid and Abdulrazak [1] used truncated Fréchet distribution on (0, 1) to get the truncated Fréchet-G family, Najarzadegan et al. [7] who proposed the truncated Weibull-G family by using the (0, 1) truncated Weibull model, Bantan et al. [4] who studied the truncated inverted Kumaraswamy -G family by using the (0,1) truncated inverted Kumaraswamy distribution. Recently, Aldahlan [2] has used (0, 1) truncated Fréchet distribution to get the type II truncated Fréchet generated family of distributions and it has the following distribution function (cdf) and probability density function (pdf)

$$F(x; b, \xi) = 1 - ee^{-(1-G(x;\xi))^{-b}}, \ x \in \mathbb{R}, \ \ b > 0. \tag{1.1}$$

and

$$f(x; b, \xi) = beg(x; \xi)(1 - G(x; \xi))^{-b-1}e^{-(1-G(x;\xi))^{-b}}. \tag{1.2}$$

If $X \sim \text{exponential}(E)$ then the distribution of a random variable (r.v) $Y = 1/X$ follows an inverse expo-

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nential (IE) model. The pdf and cdf of the IE model are

$$g\left(x{:}\alpha\right)=\frac{\alpha}{x^2}e^{-\frac{\alpha}{x}},\quad x,\alpha>0,\tag{1.3}$$

and

$$G\left(x{:}\alpha\right)=e^{-\frac{\alpha}{x}},\quad x,\alpha>0.\tag{1.4}$$

The main objective here is to introduce the new model with two parameters which called type II truncated Fréchet inverse exponential (TIITFIE) distribution, study different statistical properties, maximum likelihood (ML) method is employed to estimate the population parameters of TIITFIE based on complete and censored samples.

By inserting (1.4) in (1.1) we get the cdf of TIITFIE model

$$F\left(x;\ b,\ \alpha\right)=\ 1-ee^{-\left(1-e^{-\frac{\alpha}{x}}\right)^{-b}},\quad x,\ b,\ \alpha>0.\tag{1.5}$$

The corresponding pdf to (1.5) is getting by inserting (1.3) and (1.4) in (1.2), then

$$f\left(x;\ b,\ \alpha\right)=\frac{\alpha be}{x^2}e^{-\frac{\alpha}{x}}\left(1-e^{-\frac{\alpha}{x}}\right)^{-b-1}e^{-\left(1-e^{-\frac{\alpha}{x}}\right)^{-b}},\quad x,\ b,\ \alpha>0,$$

where b is positive shape parameter and α is positive scale parameter.

Figures 1 and 2 show some pdf and cdf plots of X~ TIITFIE($\omega=b,\ \alpha$) for various parameter choices of ω.
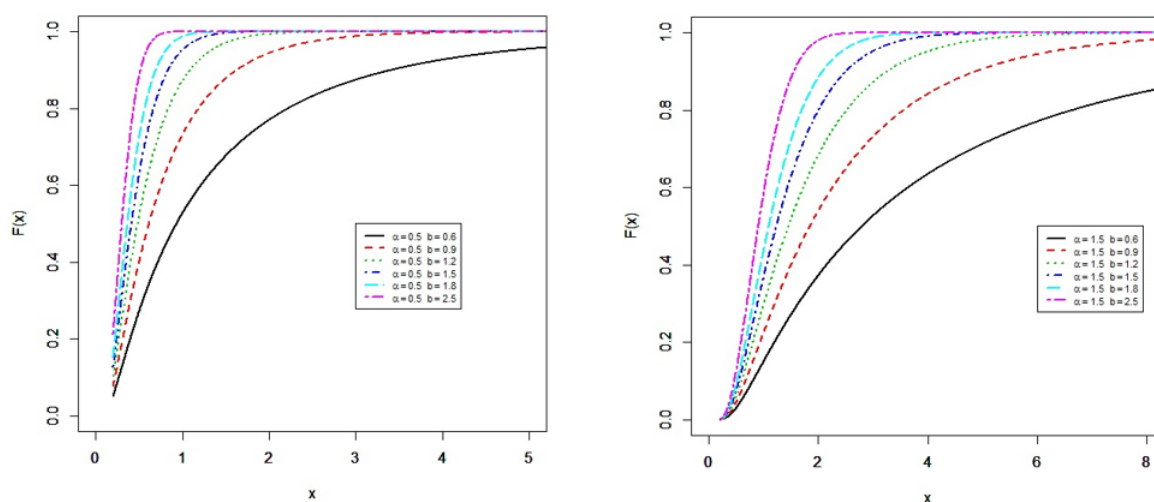

Figure 1: Plots of the cdf of the TIITFIE model.

The rest of this paper can be organized as follows. Sections 2 and 3 provide the reliability analysis and fundamental properties of the new model. In Section 4, point and approximate confidence interval (CI) of model Parameters are investigated under complete and Type II censoring (TIIC). In Section 5, the behavior of the estimates is studied via simulation. Application to real data set is analyzed to demonstrate the flexibility of TIITFIE model against some known models in Section 3. Finally, the paper ends with conclusion.
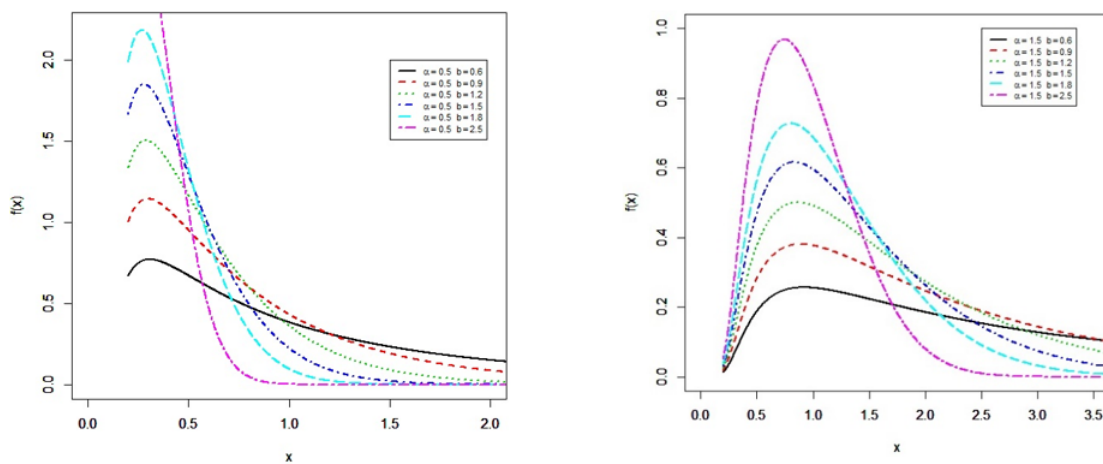
Figure 2: Plots of the pdf of the TIITFIE model.

2. The reliability analysis for TIITFIE model

The reliability function (rf), hazard rate function (hrf), reversed hrf and cumulative hrf of TIITFIE model are given by

$$R(x; b, \alpha) = e e^{-\left(1 - e^{-\frac{\alpha}{x}}\right)^{-b}},$$

$$h(x; b, \alpha) = \frac{\alpha b}{x^2} e^{-\frac{\alpha}{x}} \left(1 - e^{-\frac{\alpha}{x}}\right)^{-b-1},$$

$$\tau(x; b, \alpha) = \frac{\frac{\alpha b e}{x^2} e^{-\frac{\alpha}{x}} \left(1 - e^{-\frac{\alpha}{x}}\right)^{-b-1} e^{-\left(1 - e^{-\frac{\alpha}{x}}\right)^{-b}}}{1 - e e^{-\left(1 - e^{-\frac{\alpha}{x}}\right)^{-b}}},$$

and

$$H(x; b, \alpha) = \left(1 - e^{-\frac{\alpha}{x}}\right)^{-b} - 1.$$

Figures 3 and 4 shows some rf and hrf plots of $X \sim$ TIITFIE$(\omega = b, \alpha)$ for various parameter choices of ω.
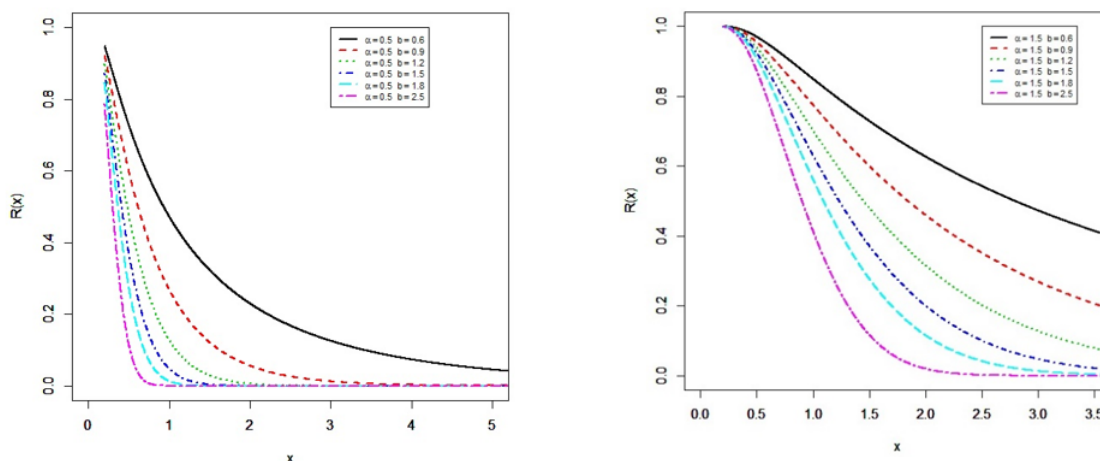

Figure 3: Plots of the rf of the TIITFIE model.

From Figures 2 and 4, we notice that the pdf of TIITFIE model can be uni-model, decreasing and right skewed. Also, the hrf of TIITFIE model can be increasing, constant and reversed J- shaped.
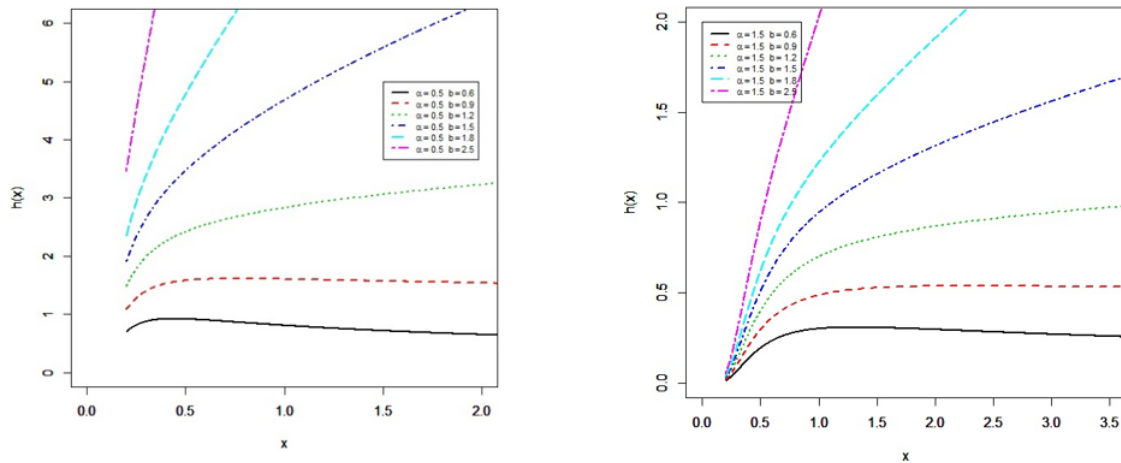
Figure 4: Plots of the hrf of the TIITFIE model.

3. Some fundamental statistical properties

In this section some fundamental statistical properties of the new model are derived.

3.1. Quantile and median

The quantile function, say $Q(u) = F^{-1}(u)$ of X is given by

$$Q_{(u)} = \frac{-\alpha}{\ln\left\{1 - \left[\ln\left(\frac{e}{1-u}\right)\right]^{\frac{-1}{b}}\right\}}, \qquad (3.1)$$

where, u is a uniform $(0, 1)$. The median (M) can be calculated by putting $u = 0.5$ in (3.1), then

$$M = \frac{-\alpha}{\ln\left\{1 - \left[\ln(2e)\right]^{\frac{-1}{b}}\right\}}.$$

3.2. Important expansion

In this subsection important expansions of the pdf for TIITFIE model are proposed.
Aldahlan [2] rewrited the equation (1.2) as

$$f(x) = \sum_{i=0}^{\infty} \eta_i g(x, \xi) G(x, \xi)^{i+1}, \qquad (3.2)$$

where $\eta_i = \sum_{j=0}^{\infty} \frac{be(-1)^{i+j}}{j!} \binom{b(j+1)+i}{i}$.

By substituting (1.3) and (1.4) in (3.2), then the pdf of TIITFIE distribution is

$$f(x) = \sum_{i=0}^{\infty} \frac{w_i}{x^2} e^{-\frac{\alpha(i+2)}{x}}, \qquad (3.3)$$

where $w_k = \alpha\eta_i$.

3.3. Moments

If X has the pdf (3.3), then its rth moment can be derived as

$$\mu'_r = E(X^r) = \int_{-\infty}^{\infty} x^r f(x; \varphi) dx. \tag{3.4}$$

Inserting (3.3) into (3.4) yields:

$$\mu'_r = E(X^r) = \sum_{i=0}^{\infty} w_i \int_0^{\infty} x^{r-2} e^{-\alpha(i+2)x^{-1}} dx.$$

Let $y = x^{-1}$, then

$$\mu'_r = \sum_{i=0}^{\infty} w_i \int_0^{\infty} y^{-r} e^{-\alpha(i+2)y} dx.$$

Then, μ'_r becomes

$$\mu'_r = \sum_{i=0}^{\infty} \frac{w_i \Gamma(1-r)}{[\alpha(i+2)]^{1-r}}, \quad r < 1.$$

The moment generating function of TIITFIE distribution is

$$M_X(t) = \sum_{r=0}^{\infty} \frac{t^r}{r!} E(X^r) = \sum_{r,i=0}^{\infty} \frac{t^r}{r!} \frac{w_i \Gamma(1-r)}{[\alpha(i+2)]^{1-r}}, \quad r < 1.$$

The incomplete moments, say $\varphi_s(t)$, is

$$\varphi_s(t) = \int_0^t x^s f(x; \varphi) dx.$$

Using (3.2), then $\varphi_s(t)$ is given by

$$\varphi_s(t) = \sum_{i=0}^{\infty} w_i \int_0^t x^{s-2} e^{-\alpha(i+2)x^{-1}} dx,$$

then,

$$\varphi_s(t) = \sum_{i=0}^{\infty} w_k \frac{\nu\left(1-s, \alpha(i+2)t^{-1}\right)}{(\alpha(i+2))^{1-s}}, \quad s < 1,$$

where $\nu(s, t) = \int_0^t x^{s-1} e^{-x} dx$ is the lower incomplete gamma function.

Further, the conditional moments, say $\tau_s(t)$, is given by

$$\tau_s(t) = \int_t^{\infty} x^s f(x; \varphi) dx.$$

Hence, by using pdf (3.2), we can write

$$\tau_s(t) = \sum_{i=0}^{\infty} w_i \int_t^{\infty} x^{s-2} e^{-\alpha(i+2)x^{-1}} dx,$$

then,

$$\tau_s(t) = \sum_{i=0}^{\infty} w_i \frac{\Gamma\left(1-s, \alpha(i+2)t^{-1}\right)}{(\alpha(i+2))^{1-s}}, \quad s < 1,$$

where $\Gamma(s, t) = \int_t^{\infty} x^{s-1} e^{-x} dx$ is the upper incomplete gamma function.

4. ML estimation uder type II censored

Let $X_{(1)}, \ldots, X_{(n)}$ be observed values from the TIITFIE model with set of parameters $\varphi = (\alpha, b)^{\mathsf{T}}$. The likelihood function under type II censored data can be expressed as

$$L(\varphi) = K \prod_{i=1}^{r} f(x_i)[1 - F(x_r)]^{n-r},$$

where, and $K = \frac{n!}{(n-r)!}$ is constant and the total log-likelihood function under type II censored is

$$\ln L(\varphi) = \ln K + r\ln(\alpha b e) - 2\sum_{i=1}^{r} \ln x_i - \alpha \sum_{i=1}^{r} \frac{1}{x_i} - \sum_{i=1}^{r}\left(1 - e^{\frac{-\alpha}{x_i}}\right)^{-b}$$

$$- (b+1)\sum_{i=1}^{n} \ln\left(1 - e^{\frac{-\alpha}{x_i}}\right) + n - r - (n-r)\left(1 - e^{\frac{-\alpha}{x_r}}\right)^{-b}.$$

The elements of the score function $U_\varphi = (U_\alpha, U_b)$ are given by

$$U_\alpha = \frac{r}{\alpha} - \sum_{i=1}^{r}\frac{1}{x_i} + b\sum_{i=1}^{r}\frac{1}{x_i}\left(1 - e^{\frac{-\alpha}{x_i}}\right)^{-b-1}e^{\frac{-\alpha}{x_i}} - (b+1)\sum_{i=1}^{r}\frac{\frac{1}{x_i}e^{\frac{-\alpha}{x_i}}}{1 - e^{\frac{-\alpha}{x_i}}} + b(n-r)\frac{1}{x_r}\left(1 - e^{\frac{-\alpha}{x_r}}\right)^{-b-1}e^{\frac{-\alpha}{x_r}},$$

and

$$U_b = \frac{r}{b} + \sum_{i=1}^{r}\left(1 - e^{\frac{-\alpha}{x_i}}\right)^{-b}\ln\left(1 - e^{\frac{-\alpha}{x_i}}\right) - \sum_{i=1}^{r}\ln\left(1 - e^{\frac{-\alpha}{x_i}}\right) + (n-r)\left(1 - e^{\frac{-\alpha}{x_r}}\right)^{-b}\ln\left(1 - e^{\frac{-\alpha}{x_r}}\right).$$

Then the ML estimators of the parameters α and b are investigated by putting $U_\varphi = 0$ and solving them. An approximate CI of model parameters for TIITFIE model is obtained.

It is known that, the asymptotic distribution of ML estimates of elements of parameters α and b is given by $((\hat{\alpha} - \alpha), (\hat{b} - b)) \to N(0, I^{-1}(\alpha, b))$, where, I^{-1} is the variance covariance matrix of unknown parameters, hence, the approximates $100(1-\nu)$ % two sided CIs for α and b are

$$\hat{\alpha} \pm Z_{\nu/2}\sqrt{\text{var}(\hat{\alpha})}, \quad \hat{b} \pm Z_{\nu/2}\sqrt{\text{var}(\hat{b})},$$

where, $Z_{\nu/2}$ is the upper $\nu/2^{\text{th}}$ percentile of the standard normal distribution.

5. Numerical results

This Section provides a numerical results to assess the behavior of the estimators in case of complete and TIIC. Mean square errors (MSEs), lower bound (LB) of CI and upper bound (UB) of CI, and average length (AL) of 90% and 95% are calculated via Mathematica 9. The following algorithm is designed as follows

1. 3000 random samples of size $n = 30, 50, 100$, and 200 are generated from TIITFIE model.
2. Values of true parameters (α, b) are taken as $(0.5, 0.5), (0.5, 0.8)$, and $(0.8, 0.8)$
3. Three levels of censoring are chosen as $r = 60\%, 80\%(\text{TIIC})$, and 100% (complete sample).
4. The ML estimates (MLEs), MSEs, LB, UB and AL for selected values of parameters are calculated.
5. Numerical outcomes are listed in Tables 1-3 based on complete and TIIC.

Table 1: MLE, MSE, LB, UB and AL of TIITFIE model under TIIC for (b=0.5, α =0.5).

n	x_r	MLE	MSE	90%			95%		
				LB	UB	AL	LB	UB	AL
30	60%	0.6054	0.0439	0.2877	0.9232	0.6355	0.2269	0.9840	0.7572
		0.6290	0.0805	0.2931	0.9649	0.6718	0.2288	1.0292	0.8005
	80%	0.5886	0.0276	0.3620	0.8152	0.4533	0.3186	0.8586	0.5400
		0.6527	0.0654	0.3339	0.9715	0.6376	0.2728	1.0326	0.7597
	100%	0.5244	0.0108	0.3797	0.6691	0.2894	0.3520	0.6968	0.3448
		0.5085	0.0147	0.2753	0.7417	0.4665	0.2306	0.7864	0.5558
50	60%	0.6186	0.0248	0.3682	0.8691	0.5009	0.3202	0.9171	0.5969
		0.6282	0.0209	0.3674	0.8889	0.5215	0.3175	0.9388	0.6213
	80%	0.5477	0.0092	0.3896	0.7057	0.3161	0.3593	0.7360	0.3767
		0.5808	0.0318	0.3574	0.8042	0.4468	0.3146	0.8470	0.5324
	100%	0.5441	0.0067	0.4257	0.6624	0.2367	0.4031	0.6851	0.2820
		0.5325	0.0121	0.3442	0.7208	0.3766	0.3081	0.7569	0.4487
100	60%	0.5464	0.0110	0.3959	0.6969	0.3010	0.3671	0.7257	0.3586
		0.5711	0.0190	0.3996	0.7425	0.3430	0.3667	0.7754	0.4086
	80%	0.5336	0.0069	0.4246	0.6427	0.2181	0.4037	0.6635	0.2598
		0.5133	0.0062	0.3723	0.6543	0.2821	0.3453	0.6813	0.3361
	100%	0.4990	0.0021	0.4239	0.5741	0.1502	0.4095	0.5885	0.1790
		0.4868	0.0069	0.3617	0.6120	0.2503	0.3377	0.6360	0.2982
200	60%	0.5231	0.0033	0.4234	0.6228	0.1995	0.4043	0.6419	0.2376
		0.5063	0.0048	0.3979	0.6147	0.2168	0.3771	0.6355	0.2584
	80%	0.5026	0.0012	0.4320	0.5733	0.1413	0.4184	0.5868	0.1684
		0.4916	0.0025	0.3946	0.5886	0.1940	0.3760	0.6072	0.2312
	100%	0.5067	0.0014	0.4537	0.5597	0.1060	0.4435	0.5699	0.1264
		0.5077	0.0040	0.4161	0.5993	0.1831	0.3986	0.6168	0.2182

Table 2: MLE, MSE, LB, UB and AL of TIITFIE model under TIIC for (b=0.8, α =0.5).

n	x_r	MLE	MSE	90%			95%		
				LB	UB	AL	LB	UB	AL
30	60%	1.2138	0.4326	0.4004	2.0272	1.6269	0.2446	2.1830	1.9384
		0.6448	0.0454	0.3366	0.9529	0.6163	0.2776	1.0120	0.7344
	80%	0.8859	0.1501	0.4880	1.2838	0.7958	0.4119	1.3600	0.9481
		0.5101	0.0237	0.2824	0.7379	0.4554	0.2388	0.7815	0.5426
	100%	0.8573	0.0356	0.5788	1.1358	0.5569	0.5255	1.1891	0.6636
		0.5075	0.0184	0.3038	0.7113	0.4075	0.2648	0.7503	0.4855
50	60%	1.0009	0.1827	0.5206	1.4811	0.9606	0.4286	1.5731	1.1445
		0.5938	0.0267	0.3688	0.8188	0.4500	0.3257	0.8619	0.5362
	80%	0.8710	0.0340	0.5777	1.1644	0.5866	0.5216	1.2205	0.6989
		0.5540	0.0180	0.3629	0.7451	0.3823	0.3263	0.7817	0.4555
	100%	0.8714	0.0159	0.6519	1.0910	0.4391	0.6098	1.1330	0.5232
		0.5408	0.0078	0.3732	0.7084	0.3352	0.3411	0.7405	0.3994
100	60%	0.9169	0.0371	0.6189	1.2149	0.5961	0.5618	1.2720	0.7102
		0.5607	0.0114	0.4085	0.7129	0.3044	0.3794	0.7420	0.3627
	80%	0.8567	0.0184	0.6526	1.0608	0.4083	0.6135	1.0999	0.4865
		0.5368	0.0050	0.4046	0.6691	0.2645	0.3793	0.6944	0.3151
	100%	0.8081	0.0049	0.6695	0.9466	0.2771	0.6430	0.9731	0.3301
		0.5123	0.0054	0.3985	0.6261	0.2276	0.3767	0.6479	0.2712
200	60%	0.7935	0.0050	0.6204	0.9665	0.3462	0.5872	0.9997	0.4124
		0.4931	0.0021	0.3961	0.5902	0.1941	0.3775	0.6087	0.2312
	80%	0.8203	0.0048	0.6854	0.9553	0.2698	0.6596	0.9811	0.3215
		0.5188	0.0022	0.4280	0.6096	0.1816	0.4106	0.6270	0.2164
	100%	0.8071	0.0012	0.7090	0.9051	0.1961	0.6902	0.9239	0.2337
		0.4929	0.0013	0.4153	0.5704	0.1551	0.4004	0.5853	0.1848

Table 3: MLE, MSE, LB, UB and AL of TIITFIE model under TIIC for (b=0.8, α =0.8).

n	x_r	MLE	MSE	90%			95%		
				LB	UB	AL	LB	UB	AL
30	60%	1.0354	0.5166	0.3617	1.7090	1.3472	0.2328	1.8380	1.6052
		0.9628	0.2372	0.4951	1.4305	0.9353	0.4056	1.5200	1.1145
	80%	1.0058	0.1469	0.5360	1.4757	0.9398	0.4460	1.5657	1.1197
		0.9196	0.0685	0.5187	1.3205	0.8018	0.4419	1.3973	0.9553
	100%	0.9269	0.0685	0.6157	1.2382	0.6225	0.5561	1.2978	0.7417
		0.9290	0.0781	0.5623	1.2956	0.7333	0.4921	1.3658	0.8737
50	60%	1.0652	0.1509	0.5494	1.5809	1.0315	0.4507	1.6796	1.2290
		0.9990	0.0566	0.6241	1.3738	0.7497	0.5524	1.4456	0.8932
	80%	0.8997	0.0383	0.5939	1.2055	0.6116	0.5353	1.2640	0.7287
		0.8474	0.0271	0.5566	1.1383	0.5817	0.5009	1.1940	0.6931
	100%	0.7864	0.0156	0.5973	0.9754	0.3780	0.5612	1.0116	0.4504
		0.8063	0.0296	0.5512	1.0615	0.5103	0.5023	1.1104	0.6080
100	60%	0.9009	0.0517	0.6082	1.1936	0.5854	0.5522	1.2496	0.6974
		0.8185	0.0411	0.5959	1.0411	0.4452	0.5533	1.0837	0.5304
	80%	0.7873	0.0178	0.6060	0.9685	0.3626	0.5713	1.0032	0.4320
		0.7534	0.0155	0.5651	0.9417	0.3766	0.5291	0.9778	0.4487
	100%	0.8007	0.0074	0.6643	0.9372	0.2729	0.6382	0.9633	0.3251
		0.7762	0.0132	0.6034	0.9490	0.3456	0.5703	0.9821	0.4117
200	60%	0.7784	0.0064	0.6097	0.9472	0.3375	0.5774	0.9795	0.4021
		0.7679	0.0051	0.6163	0.9195	0.3033	0.5872	0.9485	0.3613
	80%	0.8334	0.0102	0.6953	0.9714	0.2762	0.6688	0.9979	0.3291
		0.8284	0.0095	0.6838	0.9730	0.2892	0.6561	1.0007	0.3446
	100%	0.7870	0.0034	0.6926	0.8814	0.1888	0.6746	0.8995	0.2250
		0.7876	0.0057	0.6630	0.9123	0.2493	0.6391	0.9361	0.2971

It is noticed from Tables 1-3 that the MSEs, LB, UB and AL are decreased when n is increased and when level of censoring is increased.

6. Application

Here, we fit the TIITFIE model to one real data. This data is proposed by Crowder et al. [5] contains the ball bearing data, which indicates the number of revolutions before failure for ball bearing.

We compare the fits of the TIITFIE model with some known models like; the beta IE (BIE) model (see Khan [6]) by keeping shape parameter is equal to one, logistic IE (LIE) model (see Oguntunde et al. [8]), inverse Weibull inverse exponential (IWIE) model (see Aldahlan [3]), and standard IE model.

We estimate the model Parameterss by using ML method. We compare the goodness-of-fit of the models using negative two $\log L(-2 \log L)$, Akaike information criterion (AIC), corrected Akaike information criterion (CAIC), Bayesian information criterion (BIC), Hannan Quinn information criterion (HQIC), Anderson Darling statistic (A*) and Cramér-von Mises statistic (W*).

Table 4: MLEs, SEs and measures of goodness-of-fit statistics for the data set.

Models	MLEs and (SEs)			-2LogL	AIC	CAIC	BIC	HQIC	A*	W*
TIITFIE (α, b)	94.631 (21.514)	1.945 (0.545)	-	234.911	238.911	239.482	237.671	239.536	0.238	0.041
BIE (θ, a, b)	11.722 (0.131)	2.825 (0.268)	1.667 (0.953)	237.43	243.43	244.63	241.571	244.368	0.357	0.05
LIE (a, b)	2.459 (0.279)	1.238 (1.226)	-	238.279	242.279	242.85	241.039	242.904	0.375	0.053
IWIE (α, β, θ)	1.247 (2.115)	1.281 (0.144)	14.33 (5.374)	241.346	247.346	248.546	245.487	248.284	0.640	0.091
IE(θ)	20.403 (2.55)	-	-	253.927	255.927				0.356	0.0498

The MLEs of the competitive models and their corresponding standard errors (SEs) are listed in Table 4 also, the values of AIC, CAIC, BIC, HQIC, A* and W* are showed in the same table.

Table 4 shows that the TIITFIE model has the lowest values for AIC, CAIC, BIC, HQIC, A* and W* among all fitted models. So, the TIITFIE is the best model. The estimated pdf, pp plot, cdf and rf for the TIITFIE model are showed in Figures 5 and 6.
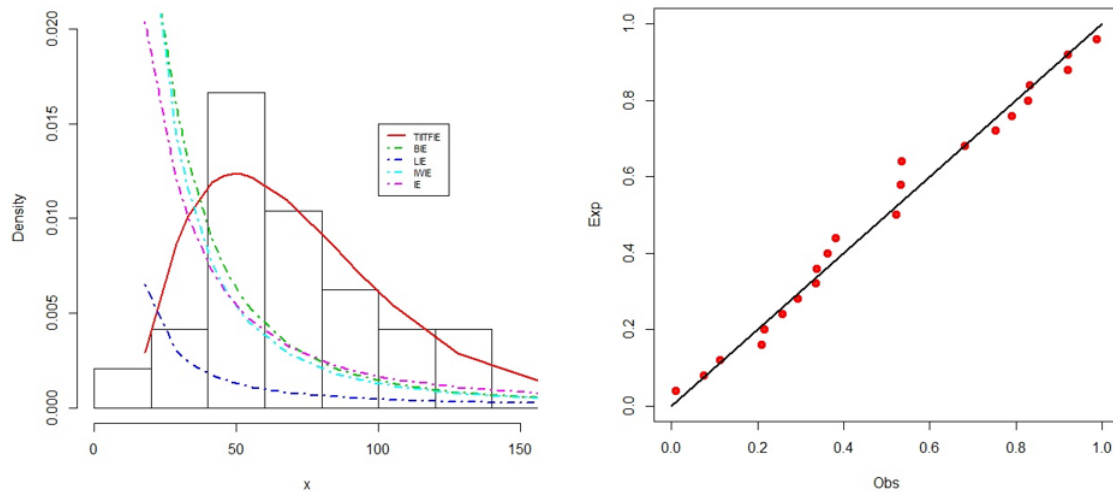

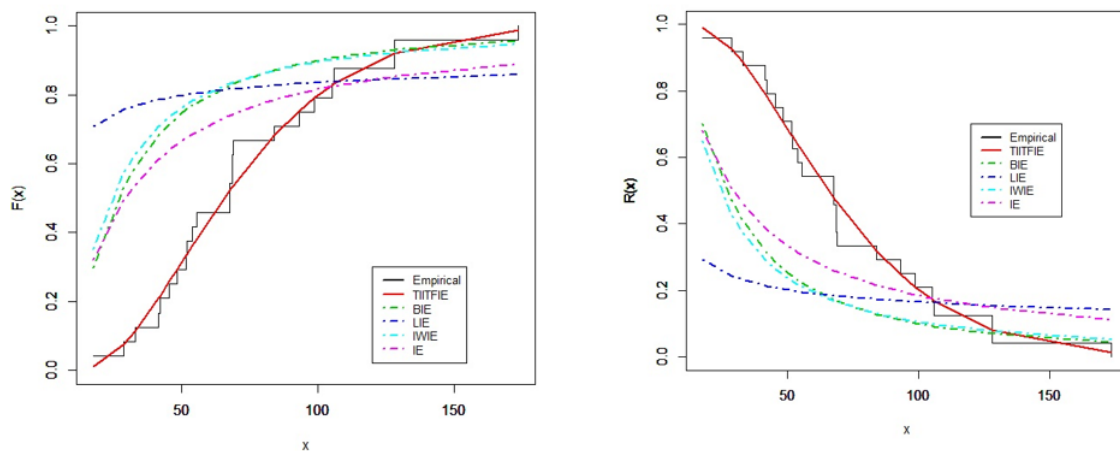Figure 5: The estimated pdf and pp plot of the TIITFIE model.

Figure 6: The estimated cdf and rf of the TIITFIE model.

7. Conclusions

A new probability distribution related to IE model, the so called TIITFIE distribution is proposed. Moments, incomplete moments, conditional moments and quantile function of the new model are calculated. Parameter estimation of model parameters is discussed via maximum likelihood method from complete and censored samples. Also, asymptotic confidence intervals of parameters are computed. Good performance and accuracy of the maximum likelihood estimators of the parameters are examined through a numerical results. Application to one real data set show that the fit of the new model is superior to some other known models.

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