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Fixed points for a sequence of $\mathcal{L}$-fuzzy mappings in non-Archimedean ordered modified intuitionistic fuzzy metric spaces

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Abstract

In this paper, we obtain sufficient conditions for the existence of fixed points for a sequence of $\mathcal{L}$-fuzzy mappings in a non-Archimedean ordered modified intuitionistic fuzzy metric space. We use contractive conditions of implicit relation. Further, as an application, we also generalize our usual contractive conditions into integral contractive conditions.

Keywords: Ordered modified intuitionistic fuzzy metric, $\mathcal{L}$-fuzzy mappings, fixed points.

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1. Introduction

The metric fixed point theory has played a fundamental role in nonlinear analysis. It has traditionally involved an intertwining of geometrical and topological properties. After the celebrated Banach contraction principe, it has been extensively studied and refined by many leading researchers, either by changing the contractive condition or the underlying space (for more details see [10, 16]). Zadeh [23] in his seminal paper introduced the concept of fuzzy sets and Goguen [11] further generalized fuzzy sets to $\mathcal{L}$-fuzzy sets. Park [17] defined the concept of intuitionistic fuzzy metric spaces as a generalization of fuzzy metric spaces. In 2008, Saadati et al. [20] introduced the notion of modified intuitionistic fuzzy metric spaces. Recently Rashid et al. [19] proved an $\mathcal{L}$-fuzzy fixed point theorem in complete metric spaces. Afterward several results for fixed point of fuzzy and $\mathcal{L}$-fuzzy mappings in classic, ordered, fuzzy and intuitionistic fuzzy metric spaces are proved (see [1–6, 14, 15, 19, 22]).

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In this paper, we prove fixed point theorems for two sequences of self and $\mathcal{L}$-fuzzy mappings in a non-Archimedean ordered modified intuitionistic fuzzy metric space, we use contractive conditions of implicit relation. Further, as application, we generalize our usual contractive conditions into integral contractive conditions.

2. Preliminaries

Definition 2.1 ([5]). Let A and B be two nonempty subsets of an ordered set $(X, \preceq)$, the relation $\preceq_1$ between A and B is defined as $A \preceq_1 B$: if for every $a \in A$ there exists $b \in B$ such that $a \preceq b$.

Lemma 2.2 ([9]). *A complete lattice* $(L^*, \leqslant_{L^*})$ *is defined by*

$$L^* = \{(a_1, a_2) : (a_1, a_2) \in [0, 1]^2, a_1 + a_2 \leqslant 1\},$$

such that $(a_1, a_2) \leqslant_{L^*} (b_1, b_2) \Leftrightarrow a_1 \leqslant b_1$ *and* $a_2 \geqslant b_2$, *for all* $(a_1, a_2), (b_1, b_2) \in L^*$.

Definition 2.3 ([8]). A t-norm on L^* is a mapping $\mathcal{T} : (L^*)^2 \to L^*$ such that for all a, a', b, b'^*, $a \leqslant_{L^*} a'$ and $b \leqslant_{L^*} b'$:

(1) $\mathcal{T}(a, 1_{L^*}) = a$, (boundary condition);
(2) $\mathcal{T}(a, b) = \mathcal{T}(b, a)$, (commutativity);
(3) $\mathcal{T}(a, \mathcal{T}(b, c)) = \mathcal{T}(\mathcal{T}(a, b), c)$, (associativity);
(4) $\mathcal{T}(a, b) \leqslant_{L^*} \mathcal{T}(a', b')$, (monotonicity).

Definition 2.4 ([8, 9]). A continuous t-norm $\mathcal{T}$ on L^* is called continuous t-represent-able iff there is a continuous t-norm $*$ and a continuous t-conorm $\diamond$ on $[0, 1]$ such that, for all $a = (a_1, a_2)$ and $b = (b_{1,2})$ in L^*, $\mathcal{T}(a, b) = (a_1 * b_1, a_2 \diamond b_2)$.

Definition 2.5 ([8, 9]). A negator on L^* is any decreasing mapping $\mathcal{N} : L^* \to L^*$ satisfying $\mathcal{N}(0_{L^*}) = 1_{L^*}$ and $\mathcal{N}(1_{L^*}) = 0_{L^*}$. If $\mathcal{N}(\mathcal{N}(x)) = x$, for all $x \in L^*$, then $\mathcal{N}$ is called an involutive negator.

Definition 2.6 ([13]). A t-norm is said to be Hadžić type if the sequence $\{*_{m=0}^{\infty} s\}$ is equi-continuous at $s = 1$, i.e., for all $\epsilon \in (0, 1)$, there exists $\eta \in (0, 1)$ such that if $s \in (1 - \eta, 1]$, then $*^m s > 1 - \epsilon$ for all $m \in \mathbb{N}$.

Definition 2.7 ([21]). The t-norm $\mathcal{T}$ on L^* is called Hadžić type if for $\epsilon \in (0, 1)$, there exist $\delta \in (0, 1)$ such that

$$\mathcal{T}^m(\mathcal{N}_s(\delta), \dots, \mathcal{N}_s(\delta) \geqslant_{L^*} \mathcal{N}_s(\epsilon), \quad m \in N.$$

Definition 2.8 ([20]). Suppose that M and N are two fuzzy sets from $X \times X \times (0, \infty)$ into $(0, 1]$ such that $M(x, y, t) + N(x, y, t) \leqslant 1$ for all $x, y \in X$ and $t > 0$. The 3-tuple $(X, \mathcal{M}_{M,N}, \mathcal{T})$ is called a modified intuitionistic fuzzy metric space if X is a nonempty set, $\mathcal{T}$ is a continuous t-representable and $\mathcal{M}_{M,N}$ is a mapping $X \times X \times (0, \infty) \to L^*$, such that for every $x, y \in X$ and $t, s > 0$,

(M1) $\mathcal{M}_{M,N}(x, y, t) >_{L^*} 0_{L^*}$;
(M2) $\mathcal{M}_{M,N}(x, y, t) = 1_{L^*}$ if and only if $x = y$;
(M3) $\mathcal{M}_{M,N}(x, y, t) = \mathcal{M}_{M,N}(y, x, t)$;
(M4) $\mathcal{M}_{M,N}(x, y, t + s) \geqslant_{L^*} \mathcal{T}(\mathcal{M}_{M,N}(x, z, t), \mathcal{M}_{M,N}(z, y, s))$;
(M5) $\mathcal{M}_{M,N}(x, y, -) : (0, \infty) \to L^*$ is continuous.

Replace condition (M4) by $\mathcal{M}_{M,N}(x, y, \max\{t, s\}) \geqslant_{L^*} \mathcal{T}(\mathcal{M}_{M,N}(x, z, t), \mathcal{M}_{M,N}(z, y, s))$ or $\mathcal{M}_{M,N}(x, y, t) \geqslant_{L^*} \mathcal{T}(\mathcal{M}_{M,N}(x, z, t), \mathcal{M}_{M,N}(z, y, t))$, then $(X, \mathcal{M}_{M,N}, \mathcal{T})$ is called a non-Archimedean modified intuitionistic fuzzy metric space.

Definition 2.9 ([20]). A sequence $\{x_n\}$ in a modified intuitionistic fuzzy metric space $(X, \mathcal{M}_{M,N}, \mathcal{T})$ is called a Cauchy sequence if $\mathcal{M}_{M,N}(x_n, x_m, t) \to 1_{L^*}$ whenever $n, m \to \infty$ for every $t > 0$ and $m > n$. The sequence $\{x_n\}$ is said to be convergent to $x \in X$ in a modified intuitionistic fuzzy metric space $(X, \mathcal{M}_{M,N}, \mathcal{T})$ and denoted by $x_n \to x$ if $\mathcal{M}_{M,N}(x_n, x, t) \to 1_{L^*}$ whenever $n \to \infty$ for every $t > 0$. A modified intuitionistic fuzzy metric space is said to be complete if and only if every Cauchy sequence is convergent.

Lemma 2.10 ([20]). *Let* $(X, \mathcal{M}_{M,N}, \mathcal{T})$ *be a modified intuitionistic fuzzy metric space. Then* $\mathcal{M}_{M,N}$ *is continuous function on* $X \times X \times (0, \infty)$.

Definition 2.11 ([12]). Let $(X, M, N, *, \diamond)$ is an intuitionistic fuzzy metric space. Then we can construct the corresponding Hausdorff intuitionistic fuzzy metric as follows:

$$\mathcal{M}_{M,N}(A, B, t) = \min\{\inf_{a \in A} \mathcal{M}_{M,N}(a, B, t), \inf_{b \in B} \mathcal{M}_{M,N}(A, b, t)\}.$$

Definition 2.12 ([11]). An $\mathcal{L}$-fuzzy set A on a nonempty set X is a function $A : X \to L$, where L is complete distributive lattice with $1_{\mathcal{L}}$ and $0_{\mathcal{L}}$.

Definition 2.13 ([19]). The $\alpha_{\mathcal{L}}$-level set of $\mathcal{L}$-fuzzy set A is denoted by $A_{\alpha_{\mathcal{L}}}$, where

$$A_{\alpha_{\mathcal{L}}} = \{x : \alpha_{\mathcal{L}} \leqslant_L A(x)\} \quad \text{if} \quad \alpha_{\mathcal{L}} \in L \backslash \{0_{\mathcal{L}}\}, \quad A_{0_{\mathcal{L}}} = \overline{\{x : 0_{\mathcal{L}} \leqslant_L A(x)\}}.$$

Definition 2.14 ([19]). Let X and Y be two arbitrary nonempty sets. A mapping F is called $\mathcal{L}$-fuzzy mapping if $F : X \to \mathfrak{I}_{\mathcal{L}}(Y)$, where $\mathfrak{I}_{\mathcal{L}}(Y)$ is the collection of all $\mathcal{L}$-fuzzy sets of X. A point $z \in X$ is called a fixed point of an $\mathcal{L}$-fuzz mapping F if $z \in \{F_z\}_{\alpha_{\mathcal{L}}}$.

Definition 2.15 ([14]). A mapping $f : Y \subseteq X \to X$ on a non-Archime-dean modified intuitionistic fuzzy metric space $(X, \mathcal{M}_{M,N}, \mathcal{T})$ $Y \subseteq X$ is called occasionally coincidentally idempotent w.r.t. an L-fuzzy mapping $F : Y \to \mathfrak{I}_L(X)$ if $ffx = fx$ for some $x \in C(f, F)$, where $C(f, F)$ is the set of coincidences point of f and F. A mapping $f : Y \to X$ is said to be F-weakly commuting at $x \in Y$ if $ffx \in \{Ffx\}_{\alpha_{\mathcal{L}}}$ provided that $fx \in Y$ for all $x \in Y$.

Definition 2.16 ([2]). The mappings $f : X \to X$ and $F : X \to \mathfrak{I}_L(X)$ on a nonempty set X are said to be D-compatible if $f\{Fx\}_{\alpha_{\mathcal{L}}} \subset \{Ffx\}_{\alpha_{\mathcal{L}}}$, where $fx \in \{Fx\}_{\alpha_{\mathcal{L}}}$ for some $x \in X$.

3. Main results

Consider the collection Φ of all continuous functions $\phi : L^{*6} \to L^*$, which are non-decreasing in the first and second coordinate variable and non-increasing in the third, fourth, fifth, and sixth coordinate variable, which satisfy the property:

$$(\phi_1) \text{ If } \phi(a, b, b, a, \mathcal{T}(a, b), 1_{L^*}) \geqslant_{L^*} 0_{L^*} \text{ or } \phi(a, b, a, b, 1_{L^*}, \mathcal{T}(a, b)) \geqslant_{L^*} 0_{L^*}, \tag{3.1}$$

then we have $a \geqslant_{L^*} b$, for all $a, b \in L^*$.

Next, we rewrite the definition of D-compatible mappings for two sequences of $\mathcal{L}$-fuzzy mappings in modified intuitionistic fuzzy metric space.

Definition 3.1. Let $(X, \mathcal{M}_{M,N}, \mathcal{T})$ be a complete non-Archimedean modified intuitionistic fuzzy metric space. Suppose that $f_n : X \to X$ and $F_{n+1} : X \to \mathfrak{I}_{\mathcal{L}}(X)$ such that $\{f_{n+1}\}$ and $\{F_{n+1}\}$ are two sequences of self and $\mathcal{L}$-fuzzy mappings, where for each $x \in X$, $n \in \mathbb{N} \cup \{0\}$, $\alpha_{\mathcal{L}} \in L \backslash \{0_{\mathcal{L}}\}$, $f_{n+1}(X)$ and $\{F_{n+1}x\}_{\alpha_{\mathcal{L}}}$ are nonempty closed subsets of X. The pairs (f_{2n+1}, F_{2n+1}) and (f_{2n+2}, F_{2n+2}) are said to be D-compatible mappings if $f_{2n+1}\{F_{2n+1}x\}_{\alpha_{\mathcal{L}}} \subset \{F_{2n+1}f_{2n+1}x\}_{\alpha_{\mathcal{L}}}$ and $f_{2n+2}\{F_{2n+2}x\}_{\alpha_{\mathcal{L}}} \subset \{F_{2n+2}f_{2n+2}x\}_{\alpha_{\mathcal{L}}}$ where $f_{2n+1}x \in \{F_{2n+1}x\}_{\alpha_{\mathcal{L}}}$ and $f_{2n+2}x \in \{F_{2n+2}x\}_{\alpha_{\mathcal{L}}}$ for some $x \in X$.

Now, we introduce our main result as follows.

Theorem 3.2. *Let $(X, \mathcal{M}_{M,N}, \mathcal{T})$ be a complete non-Archimedean modified intuitionistic fuzzy metric space. Suppose that $f_n : X \to X$ and $F_{n+1} : X \to \mathfrak{I}_{\mathcal{L}}(X)$ such that $\{f_{n+1}\}$ and $\{F_{n+1}\}$ are two sequences of self and $\mathcal{L}$-fuzzy mappings, where for each $x \in X$, $n \in \mathbb{N} \cup \{0\}$, $\alpha_{\mathcal{L}} \in L \backslash \{0_{\mathcal{L}}\}$, $f_{n+1}(X)$ and $\{F_{n+1}x\}_{\alpha_{\mathcal{L}}}$ are nonempty closed subsets of X. Suppose that the pairs (f_{2n+1}, F_{2n+1}) and (f_{2n+2}, F_{2n+2}) are D-compatible mappings. Suppose that the condition: if for all $x, y \in X$ there exist $\phi \in \Phi$ such that*

$$\phi\begin{pmatrix} \mathcal{M}_{M,N}(\{F_{2n+1}x\}_{\alpha_{\mathcal{L}}}, \{F_{2n+2}y\}_{\alpha_{\mathcal{L}}}, t), \mathcal{M}_{M,N}(f_{2n+1}x, f_{2n+2}y, t), \\ \mathcal{M}_{M,N}(f_{2n+1}x, \{F_{2n+1}x\}_{\alpha_{\mathcal{L}}}, t), \mathcal{M}_{M,N}(f_{2n+2}y, \{F_{2n+2}y\}_{\alpha_{\mathcal{L}}}, t), \\ \mathcal{M}_{M,N}(f_{2n+1}x, \{F_{2n+2}y\}_{\alpha_{\mathcal{L}}}, t), \mathcal{M}_{M,N}(f_{2n+2}y, \{F_{2n+1}x\}_{\alpha_{\mathcal{L}}}, t) \end{pmatrix} \geqslant_{L^*} 0_{L^*}, \tag{3.2}$$

is satisfied, then the mappings $\{f_{n+1}\}$ and $\{F_{n+1}\}$ have a common fixed point.

Proof. Since X is nonempty and $f_n : X \to X$, then there exist $y_0 = f_0 x_0 \in X$. By $\{F_{n+1}x\}_{\alpha_{\mathcal{L}}}$ being nonempty closed subsets of X, we have $\{F_1 x_0\}_{\alpha_{\mathcal{L}}} \neq \phi$, then there exists $y_1 = f_1 x_1 \in X$ such that $f_1 x_1 \in \{F_1 x_0\}_{\alpha_{\mathcal{L}}}$. First, if $x_0 = x_1$, then $f_1 x_0 \in \{F_1 x_0\}_{\alpha_{\mathcal{L}}}$, i.e, x_0 is a coincidence point of f_1 and F_1. Again, since $\{F_2 x_1\}_{\alpha_{\mathcal{L}}} \neq \phi$, there exists $f_2 x_2 \in X$ such that $f_2 x_2 \in \{F_2 x_1\}_{\alpha_{\mathcal{L}}}$, if $x_1 = x_2$, then x_1 is a coincidence point of f_2 and F_2. By a similar way one may obtain that $f_{n+1} x_n \in \{F_{n+1} x_n\}_{\alpha_{\mathcal{L}}}$, then $\{x_n\}$ are coincidence points of $\{f_{n+1}\}$ and $\{F_{n+1}x\}_{\alpha_{\mathcal{L}}}$. By completing this way, it is easy to prove the theorem. Now, suppose that $x_0 \neq x_1 \neq x_2$ and $\mathcal{M}_{M,N}(\{F_1 x_0\}_{\alpha_{\mathcal{L}}}, \{F_2 x_1\}_{\alpha_{\mathcal{L}}}, t) \leqslant_{L^*} \mathcal{M}_{M,N}(f_1 x_1, f_2 x_2, t) = M(y_1, y_2, t)$. Since

$$\begin{aligned} \phi\begin{pmatrix} \mathcal{M}_{M,N}(y_1, y_2, t), \mathcal{M}_{M,N}(y_0, y_1, t), \\ \mathcal{M}_{M,N}(y_0, y_1, t), \mathcal{M}_{M,N}(y_1, y_2, t), \\ \mathcal{T}(\mathcal{M}_{M,N}(y_0, y_1, t), \mathcal{M}_{M,N}(y_1, y_2, t)), 1_{L^*} \end{pmatrix} &\geqslant_{L^*} \phi\begin{pmatrix} \mathcal{M}_{M,N}(y_1, y_2, t), \mathcal{M}_{M,N}(y_0, y_1, t), \\ \mathcal{M}_{M,N}(y_0, y_1, t), \mathcal{M}_{M,N}(y_1, y_2, t), \\ \mathcal{M}_{M,N}(y_0, y_2, t), 1_{L^*} \end{pmatrix} \\ &\geqslant_{L^*} \phi\begin{pmatrix} \mathcal{M}_{M,N}(\{F_1 x_0\}_{\alpha_L}, \{F_2 x_1\}_{\alpha_{\mathcal{L}}}, t), \mathcal{M}_{M,N}(f_0 x_0, f_1 x_1, t), \\ \mathcal{M}_{M,N}(f_0 x_0, \{F_1 x_0\}_{\alpha_{\mathcal{L}}}, t), \mathcal{M}_{M,N}(f_1 x_1, \{F_2 x_1\}_{\alpha_{\mathcal{L}}}, t), \\ \mathcal{M}_{M,N}(f_0 x_0, \{F_2 x_1\}_{\alpha_{\mathcal{L}}}, t), \mathcal{M}_{M,N}(f_1 x_1, \{F_1 x_0\}_{\alpha_{\mathcal{L}}}, t) \end{pmatrix} \\ &\geqslant_{L^*} 0_{L^*}. \end{aligned}$$

From (3.1), we have $\mathcal{M}_{M,N}(y_1, y_2, t) \geqslant_{L^*} \mathcal{M}_{M,N}(y_0, y_1, t)$. Similarly, there exists $x_3 \in X$ such that $y_3 = f_3 x_3 \in \{F_3 x_2\}_{\alpha_{\mathcal{L}}}$. Further, $\mathcal{M}_{M,N}(\{F_2 x_1\}_{\alpha_{\mathcal{L}}}, \{F_3 x_2\}_{\alpha_{\mathcal{L}}}, t) \leqslant_{L^*} \mathcal{M}_{M,N}(f_2 x_2, f_3 x_3, t)$, which gives $\mathcal{M}_{M,N}(y_2, y_3, t) \geqslant_{L^*} \mathcal{M}_{M,N}(y_1, y_2, t)$, then we have $\mathcal{M}_{M,N}(y_{n+1}, y_{n+2}, t) \geqslant_{L^*} \mathcal{M}_{M,N}(y_n, y_{n+1}, t)$. Now, we have a sequence $\{y_n\}$ such that

$$\{y_{2n+1}\} = \{f_{2n+1} x_{2n+1}\} \subseteq \{F_{2n+1} x_{2n}\}_{\alpha_{\mathcal{L}}}, \quad \{y_{2n+2}\} = \{f_{2n+2} x_{2n+2}\} \subseteq \{F_{2n+2} x_{2n+1}\}_{\alpha_{\mathcal{L}}}.$$

Now, $\{y_n\}$ is a Cauchy sequence, suppose not, i.e, $\lim_{n,m \to \infty} \mathcal{M}_{M,N}(y_n, y_m, t) \neq 1_{L^*}$, for all $m > n$ where $n, m \in \mathbb{N} \cup \{0\}$, since

$$\begin{aligned} &\phi\big(\mathcal{M}_{M,N}(y_n, y_m, t), 1_{L^*}, 1_{L^*}, \mathcal{M}_{M,N}(y_n, y_m, t), \mathcal{M}_{M,N}(y_n, y_m, t), 1_{L^*} \big) \\ &= \phi\big(\mathcal{M}_{M,N}(y_n, y_m, t), 1_{L^*}, 1_{L^*}, \mathcal{M}_{M,N}(y_n, y_m, t), \mathcal{T}(\mathcal{M}_{M,N}(y_n, y_m, t), 1_{L^*}), 1_{L^*} \big) \geqslant_{L^*} 0_{L^*}. \end{aligned}$$

Take the limit at $n, m \to \infty$. By continuity of Φ, we have

$$\phi\begin{pmatrix} \lim_{n,m\to\infty} \mathcal{M}_{M,N}(y_n, y_m, t), 1_{L^*}, \\ 1_{L^*}, \lim_{n,m\to\infty} \mathcal{M}_{M,N}(y_n, y_m, t), \\ \lim_{n,m\to\infty} \mathcal{M}_{M,N}(y_n, y_m, t), 1_{L^*} \end{pmatrix} \geqslant_{L^*} 0_{L^*}.$$

By (3.1), $\lim_{n,m\to\infty} \mathcal{M}_{M,N}(y_n, y_m, t) \geqslant_{L^*} 1_{L^*}$. It is a contradiction, so $\{y_n\}$ is a Cauchy sequence, since X is complete, then there exists $z \in X$ such that $\lim_{n\to\infty} \mathcal{M}_{M,N}(y_n, z, t) = 1_{L^*}$. As $y_{2n+1} \to z$, $y_{2n+1} =$

$f_{2n+1}x_{2n+1} \to z$ and $y_{2n+2} = f_{2n+2}x_{2n+2} \to z$, since $\{f_{n+1}(X)\}$ are closed subsets of X, then there exist $v, w \in X$ such that $z = f_{2n+1}v = f_{2n+2}w$. We show that $f_{2n+1}v \in \{F_{2n+1}v\}_{\alpha_{\mathcal{L}}}$. Since

$$\phi\begin{pmatrix} \mathcal{M}_{M,N}(\{F_{2n+1}v\}_{\alpha_{\mathcal{L}}}, y_{2n+2}, t), \mathcal{M}_{M,N}(y_{2n+1}, y_{2n+2}, t), \\ \mathcal{M}_{M,N}(y_{2n+1}, \{F_{2n+1}v\}_{\alpha_{\mathcal{L}}}, t), \mathcal{M}_{M,N}(y_{2n+2}, y_{2n+2}, t), \\ \mathcal{T}(\mathcal{M}_{M,N}(y_{2n+1}, y_{2n+1}, t), \mathcal{M}_{M,N}(y_{2n+1}, y_{2n+2}, t)), \\ \mathcal{M}_{M,N}(y_{2n+1}, \{F_{2n+1}v\}_{\alpha_{\mathcal{L}}}, t) \end{pmatrix}$$
$$\geqslant_{L^*} \phi\begin{pmatrix} \mathcal{M}_{M,N}(\{F_{2n+1}v\}_{\alpha_{\mathcal{L}}}, \{F_{2n+2}x_{2n+1}\}_{\alpha_{\mathcal{L}}}, t), \mathcal{M}_{M,N}(f_{2n+1}v, f_{2n+2}x_{2n+2}, t), \\ \mathcal{M}_{M,N}(f_{2n+1}v, \{F_{2n+1}v\}_{\alpha_{\mathcal{L}}}, t), \mathcal{M}_{M,N}(f_{2n+2}x_{2n+2}, \{F_{2n+2}x_{2n+1}\}_{\alpha_{\mathcal{L}}}, t), \\ \mathcal{M}_{M,N}(f_{2n+1}v, \{F_{2n+2}x_{2n+1}\}_{\alpha_{\mathcal{L}}}, t), \mathcal{M}_{M,N}(f_{2n+1}x_{2n+1}, \{F_{2n+1}v\}_{\alpha_{\mathcal{L}}}, t) \end{pmatrix} \geqslant_{L^*} 0_{L^*}.$$

By Lemma 2.10, $\mathcal{M}_{M,N}$ is continues and by continuity of $\mathcal{T}$, letting $n \to \infty$, we have

$$\phi\begin{pmatrix} \mathcal{M}_{M,N}(\{F_{2n+1}v\}_{\alpha_{\mathcal{L}}}, z, t), 1_{L^*}, \\ \mathcal{M}_{M,N}(z, \{F_{2n+1}v\}_{\alpha_{\mathcal{L}}}, t), 1_{L^*}, \\ 1_{L^*}, \mathcal{M}_{M,N}(z, \{F_{2n+1}v\}_{\alpha_{\mathcal{L}}}, t) \end{pmatrix} \geqslant_{L^*} 0_{L^*}.$$

By (3.1), this gives $\mathcal{M}_{M,N}(z, \{F_{2n+1}v\}_{\alpha_{\mathcal{L}}}, t) \geqslant_{L^*} 1_{L^*}$, then $z = f_{2n+1}v \in \{F_{2n+1}v\}_{\alpha_{\mathcal{L}}}$. Similarly, $z = f_{2n+2}w \in \{F_{2n+2}w\}_{\alpha_{\mathcal{L}}}$. Now, (f_{2n+1}, F_{2n+1}) is D-compatible mapping, therefore we have $f_{2n+1}z = f_{2n+1}f_{2n+1}v \in \{f_{2n+1}F_{2n+1}v\}_{\alpha_{\mathcal{L}}} \subset \{F_{2n+1}f_{2n+1}v\}_{\alpha_{\mathcal{L}}} = \{F_{2n+1}z\}_{\alpha_{\mathcal{L}}}$. Also, (f_{2n+2}, F_{2n+2}) is D-compatible mapping, then we obtain $f_{2n+2}z = f_{2n+2}f_{2n+2}w \in \{f_{2n+2}F_{2n+2}w\}_{\alpha_{\mathcal{L}}} \subset \{F_{2n+2}f_{2n+2}w\}_{\alpha_{\mathcal{L}}} = \{F_{2n+2}z\}_{\alpha_{\mathcal{L}}}$. Next we show that $z = f_{2n+1}z$. Suppose otherwise, i.e., $\mathcal{M}_{M,N}((z, f_{2n+1}z, t) \neq 1_{L^*}$, while

$$\phi\begin{pmatrix} \mathcal{M}_{M,N}(f_{2n+1}z, z, t), 1_{L^*}, \\ 1_{L^*}, \mathcal{M}_{M,N}(z, f_{2n+1}z, t), \\ \mathcal{M}_{M,N}(z, f_{2n+1}z, t), \mathcal{T}(\mathcal{M}_{M,N}(z, f_{2n+1}z, t), 1_{L^*}) \end{pmatrix}$$
$$\geqslant_{L^*} \phi\begin{pmatrix} \mathcal{M}_{M,N}(\{F_{2n+1}z\}_{\alpha_{\mathcal{L}}}, \{F_{2n+2}w\}_{\alpha_{\mathcal{L}}}, t), \mathcal{M}_{M,N}(f_{2n+1}z, f_{2n+2}w, t), \\ \mathcal{M}_{M,N}(f_{2n+1}z, \{F_{2n+1}z\}_{\alpha_{\mathcal{L}}}, t), \mathcal{M}_{M,N}(f_{2n+2}w, \{F_{2n+2}w\}_{\alpha_{\mathcal{L}}}, t), \\ \mathcal{M}_{M,N}(f_{2n+1}z, \{F_{2n+2}w\}_{\alpha_{\mathcal{L}}}, t), \mathcal{M}_{M,N}(f_{2n+2}w, \{F_{2n+1}z\}_{\alpha_{\mathcal{L}}}, t) \end{pmatrix} \geqslant_{L^*} 0_{L^*}.$$

By (3.1), we have $\mathcal{M}_{M,N}(z, \{f_{2n+1}z\}_{\alpha_{\mathcal{L}}}, t) \geqslant_{L^*} 1_{L^*}$, contradiction, then $z = f_{2n+1}z$. Similarly, $z = f_{2n+2}z$. Now, $z = f_{2n+1}z = f_{2n+2}z \in \{F_{2n+1}z\}_{\alpha_{\mathcal{L}}}$ and $z = f_{2n+1}z = f_{2n+2}z \in \{F_{2n+2}z\}_{\alpha_{\mathcal{L}}}$. This proves that z is a common fixed point of f_{2n+1}, f_{2n+2}, F_{2n+1} and F_{2n+2}. □

Example 3.3. Let $X = L = L^* = [0, 1]$, where $(L^*, \leqslant_{L^*})$ is defined by $L^* = \{a = (1_1, a_2) : (a_1, a_2) \in [0, 1]^2, a_1 + a_2 \leqslant 1\}$ such that for all $a = (a_1, a_2) \in L^*$ and $b = (b_1, b_2) \in L^*$, $(a_1, a_2) \leqslant_{L^*} (b_1, b_2) \Leftrightarrow a_1 \leqslant b_1$ and $a_2 \geqslant b_2$. Let $\mathcal{M}_{M,N}(x, y, t)$ be an intuitionistic fuzzy mapping on $X^2 \times (0, \infty)$ defined as $\mathcal{M}_{M,N}(x, y, t) = (M(x, y, t), N(x, y, t))$, where

$$(M(x, y, t), N(x, y, t)) = \begin{cases} \left(\frac{x}{y}, \frac{y-x}{y}\right), & \text{if } x \leqslant y, \\ \left(\frac{y}{x}, \frac{x-y}{x}\right), & \text{if } x \geqslant y, \end{cases}$$

for all $x, y \in X$ and $t > 0$. Suppose that $\mathcal{T}(a, b) = (\max\{0, a_1 + b_1 - 1\}, a_2 + b_2 - a_2 b_2)$. Then $(X, \mathcal{M}_{M,N}, \mathcal{T})$ is a modified intuitionistic fuzzy metric space. Set $\alpha = 0.2$ and define the mappings $f_{2n+1}, f_{2n+2}, F_{2n+1}$ and F_{2n+2} on X as

$$f_{2n+1}x = \frac{x}{2n+1}, \quad \{F_{2n+1}x\}_{0.2} = [\frac{x}{2n+2}, 1], \quad f_{2n+2}x = \frac{x}{2n+2}, \quad \text{and} \quad \{F_{2n+2}x\}_{0.2} = [\frac{x}{2n+3}, 1].$$

Define the sequences x_n and y_n in X such that for $n \in \mathbb{N} \cup \{0\}$,

$$x_{2n} = \{\frac{1}{2n+1}\}, \quad x_{2n+1} = \{\frac{1}{2n+2}\}, \quad \text{and} \quad x_{2n+2} = \{\frac{1}{2n+3}\},$$

then we have

$$y_{2n+1} = f_{2n+1}x_{2n+1} = \frac{1}{(2n+1)(2n+2)} \in [\frac{1}{(2n+1)(2n+2)}, 1] = \{F_{2n+1}x_{2n}\}_{0.2}$$

and

$$y_{2n+2} = f_{2n+2}x_{2n+2} = \frac{1}{(2n+2)(2n+3)} \in [\frac{1}{(2n+2)(2n+3)}, 1] = \{F_{2n+2}x_{2n+1}\}_{0.2}.$$

Letting $n \to \infty$, we have

$$\lim_{n\to\infty} y_{2n+1} = \lim_{n\to\infty} f_{2n+1}x_{2n+1} = f_{2n+1}0 = 0 \in [0,1] = \lim_{n\to\infty}\{F_{2n+1}x_{2n}\}_{0.2}$$

and

$$\lim_{n\to\infty} y_{2n+2} = \lim_{n\to\infty} f_{2n+2}x_{2n+2} = f_{2n+2}0 = 0 \in [0,1] = \lim_{n\to\infty}\{F_{2n+1}x_{2n}\}_{0.2}.$$

Further, applying condition (3.1) gives

$$\phi\begin{pmatrix} \mathcal{M}_{M,N}(\{F_{2n+1}x_{2n}\}_{\alpha_{\mathcal{L}}}, \{F_{2n+2}x_{2n+1}\}_{\alpha_{\mathcal{L}}}, t), \mathcal{M}_{M,N}(f_{2n+1}x_{2n+1}, f_{2n+2}x_{2n+2}, t), \\ \mathcal{M}_{M,N}(f_{2n+1}x_{2n+1}, \{F_{2n+1}x_{2n}\}_{\alpha_{\mathcal{L}}}, t), \mathcal{M}_{M,N}(f_{2n+2}x_{2n+2}, \{F_{2n+2}x_{2n+1}\}_{\alpha_{\mathcal{L}}}, t), \\ \mathcal{M}_{M,N}(f_{2n+1}x_{2n+1}, \{F_{2n+2}x_{2n+1}\}_{\alpha_{\mathcal{L}}}, t), \mathcal{M}_{M,N}(f_{2n+2}x_{2n+2}, \{F_{2n+1}x_{2n}\}_{\alpha_{\mathcal{L}}}, t) \end{pmatrix} \geqslant 0.$$

Finally, $f_{2n+1}f_{2n+1}0 = f_{2n+1}0 = 0$, $f_{2n+2}f_{2n+2}0 = f_{2n+2}0 = 0$, $f_{2n+1}f_{2n+1}0 = 0 \in [0,1] = \{F_{2n+1}0\}_{0.2}$ and $f_{2n+2}f_{2n+2}0 = 0 \in [0,1] = \{F_{2n+2}0\}_{0.2}$ $(f_{2n+1}0, \{F_{2n+1}0\}_{0.2})$, that is the pairs $(f_{2n+2}0, \{F_{2n+2}0\}_{0.2})$ are weakly commuting and occasionally coincidentally idempotent. Now, $0 = f_{2n+1}0 \in \{F_{2n+1}0\}_{0.2}$ and $0 = f_{2n+2}0 \in \{F_{2n+2}0\}_{0.2}$ is a common fixed point.

Corollary 3.4. *Let $(X, \mathcal{M}_{M,N}, \mathcal{T})$ be a complete non-Archimedean modified intuitionistic fuzzy metric space. Suppose that $f : X \to X$ and $F_{n+1} : X \to \mathfrak{I}_{\mathcal{L}}(X)$ such that $\{F_n\}$ is a sequences of $\mathcal{L}$-fuzzy mappings, where for each $x \in X$, $n \in \mathbb{N} \cup \{0\}$, $\alpha_{\mathcal{L}} \in L \backslash \{0_{\mathcal{L}}\}$, $f(X)$ and $\{F_{n+1}x\}_{\alpha_{\mathcal{L}}}$ are nonempty closed subsets of X. Suppose that the pairs (f, F_{n+1}) are D-compatible mappings. Suppose that for all $x, y \in X$ there exists $\phi \in \Phi$ such that*

$$\phi\begin{pmatrix} \mathcal{M}_{M,N}(\{F_{2n+1}x\}_{\alpha_{\mathcal{L}}}, \{F_{2n+2y}\}_{\alpha_{\mathcal{L}}}, t), \mathcal{M}_{M,N}(fx, fy, t), \\ \mathcal{M}_{M,N}(fx, \{F_{2n+1}x\}_{\alpha_{\mathcal{L}}}, t), \mathcal{M}_{M,N}(fy, \{F_{2n+2}y\}_{\alpha_{\mathcal{L}}}, t), \\ \mathcal{M}_{M,N}(fx, \{F_{2n+2}y\}_{\alpha_{\mathcal{L}}}, t), \mathcal{M}_{M,N}(fy, \{F_{2n+1}x\}_{\alpha_{\mathcal{L}}}, t) \end{pmatrix} \geqslant_{L^*} 0_{L^*}.$$

Then the mappings f and $\{F_{n+1}\}$ have a common fixed point.

Corollary 3.5. *Let $(X, \mathcal{M}_{M,N}, \mathcal{T})$ be a complete non-Archimedean modified intuitionistic fuzzy metric space. Suppose that $F_{n+1} : X \to \mathfrak{I}_{\mathcal{L}}(X)$ such that $\{F_{n+1}\}$ is a sequences of $\mathcal{L}$-fuzzy mappings, where for each $x \in X$, $n \in \mathbb{N} \cup \{0\}$, $\alpha_{\mathcal{L}} \in L \backslash \{0_{\mathcal{L}}\}$, $\{F_{n+1}x\}_{\alpha_{\mathcal{L}}}$ are nonempty closed subsets of X. Suppose that there exists $\phi \in \Phi$ such that for $x, y \in X$,*

$$\phi\begin{pmatrix} \mathcal{M}_{M,N}(\{F_{2n+1}x\}_{\alpha_{\mathcal{L}}}, \{F_{2n+2y}\}_{\alpha_{\mathcal{L}}}, t), \mathcal{M}_{M,N}(x, y, t), \\ \mathcal{M}_{M,N}(x, \{F_{2n+1}x\}_{\alpha_{\mathcal{L}}}, t), \mathcal{M}_{M,N}(y, \{F_{2n+2}y\}_{\alpha_{\mathcal{L}}}, t), \\ \mathcal{M}_{M,N}(x, \{F_{2n+2}y\}_{\alpha_{\mathcal{L}}}, t), \mathcal{M}_{M,N}(y, \{F_{2n+1}x\}_{\alpha_{\mathcal{L}}}, t) \end{pmatrix} \geqslant_{L^*} 0_{L^*}.$$

Then the mappings $\{F_{n+1}\}$ have a common fixed point.

Corollary 3.6. *Let $(X, \mathcal{M}_{M,N}, \mathcal{T})$ be a complete non-Archimedean modified intuitionistic fuzzy metric space. Suppose that $F : X \to \mathfrak{I}_{\mathcal{L}}(X)$, where for each $x \in X$, $\alpha_{\mathcal{L}} \in L \backslash \{0_{\mathcal{L}}\}$, $\{Fx\}_{\alpha_{\mathcal{L}}}$ is a closed subset of X. Suppose that there exists $\phi \in \Phi$ such that for $x, y \in X$,*

$$\phi\begin{pmatrix} \mathcal{M}_{M,N}(\{Fx\}_{\alpha_{\mathcal{L}}}, \{Fy\}_{\alpha_{\mathcal{L}}}, t), \mathcal{M}_{M,N}(x, y, t), \\ \mathcal{M}_{M,N}(x, \{Fx\}_{\alpha_{\mathcal{L}}}, t), \mathcal{M}_{M,N}(y, \{Fy\}_{\alpha_{\mathcal{L}}}, t), \\ \mathcal{M}_{M,N}(x, \{Fy\}_{\alpha_{\mathcal{L}}}, t), \mathcal{M}_{M,N}(y, \{Fx\}_{\alpha_{\mathcal{L}}}, t) \end{pmatrix} \geqslant_{L^*} 0_{L^*}.$$

Then the mapping F have a fixed point.

4. Further results

Let $(X, \mathcal{M}_{M,N}, \mathcal{T})$ be a complete non-Archimedean modified intuitionistic fuzzy metric space. Suppose that $f_n : X \to X$ and $F_{n+1} : X \to \mathfrak{I}_{\mathcal{L}}(X)$ such that $\{f_{n+1}\}$ and $\{F_{n+1}\}$ are two sequences of self and $\mathcal{L}$-fuzzy mappings, where for each $x \in X$, $n \in \mathbb{N} \cup \{0\}$, $\alpha_{\mathcal{L}} \in L\backslash\{0_{\mathcal{L}}\}$, $f_{n+1}(X)$ and $\{F_{n+1}x\}_{\alpha_{\mathcal{L}}}$ are nonempty closed subsets of X. First, we rewrite the definition of occasionally coincidentally idempotent and weakly commuting mappings for two sequences of $\mathcal{L}$-fuzzy mappings in modified intuitionistic fuzzy metric space.

Definition 4.1. The pairs (f_{2n+1}, F_{2n+1}) and (f_{2n+2}, F_{2n+2}) are said to be occasionally coincidentally idempotent w.r.t. an $\mathcal{L}$-fuzzy mapping if $f_{2n+1}f_{2n+1}x = f_{2n+1}x$ and $f_{2n+2}f_{2n+2}x = f_{2n+2}x$ for some $x \in C(f_{2n+1}, F_{2n+1})$ and for some $x \in C(f_{2n+2}, F_{2n+2})$.

Definition 4.2. The pairs (f_{2n+1}, F_{2n+1}) and (f_{2n+2}, F_{2n+2}) are called F-weakly commuting at $x \in X$ if $f_{2n+1}f_{2n+1}x \in \{F_{2n+1}f_{2n+1}x\}_{\alpha_{\mathcal{L}}}$ and $f_{2n+2}f_{2n+2}x \in \{F_{2n+2}f_{2n+2}x\}_{\alpha_{\mathcal{L}}}$, where $f_{2n+2}x \in X$ for all $x \in X$.

Now, we prove the following theorem.

Theorem 4.3. *Let $(X, \mathcal{M}_{M,N}, \mathcal{T})$ be a complete non-Archimedean modified intuitionistic fuzzy metric space with Hadžić type t-norm and $\lim_{t\to\infty} \mathcal{M}_{M,N}(y_0, y_1, t) = 1_{L^*}$, let $\preceq$ be a partial order defined on X. Suppose that $f_n : X \to X$ and $F_{n+1} : X \to \mathfrak{I}_{\mathcal{L}}(X)$ such that $\{f_{n+1}\}$ and $\{F_{n+1}\}$ are two sequences of self and $\mathcal{L}$-fuzzy mappings, where for each $x \in X$, $\alpha_{\mathcal{L}} \in L\backslash\{0_{\mathcal{L}}\}$, $f_{n+1}(X)$ and $\{F_{n+1}x\}_{\alpha_{\mathcal{L}}}$ are nonempty closed subsets of X. Suppose that the condition (3.2) is satisfied. Suppose that we have the following conditions for all $x, y \in X$, $n \in \mathbb{N} \cup \{0\}$:*

(1) $\{F_{2n+1}x\}_{\alpha_{\mathcal{L}}} \preceq_1 f_{2n+1}(X)$ *and* $\{F_{2n+2}x\}_{\alpha_{\mathcal{L}}} \preceq_1 f_{2n+2}(X)$;
(2) *if* $f_{2n+1}y \in \{F_{2n+1}x\}_{\alpha_{\mathcal{L}}}$ *or* $f_{2n+2}y \in \{F_{2n+2}x\}_{\alpha_{\mathcal{L}}}$ *implies* $x \preceq y$;
(3) *if* $y_n \to y$, *then* $y_n \preceq y$ *for all* n;
(4) (f_{2n+2}, F_{2n+1}) *and* (f_{2n+1}, F_{2n+2}) *are weakly commuting and occasionally coincidentally idempotent.*

Then the mappings $\{f_{n+1}\}$ and $\{F_{n+1}\}$ have a common fixed point.

Proof. Let $x_0, y_0 \in X$ such that $y_0 = f_0x_0$. By assumption (1), there exist $x_1, x_2 \in X$ such that $y_1 = f_1x_1 \in \{F_1x_0\}_{\alpha_{\mathcal{L}}}$ and $y_2 = f_2x_2 \in \{F_2x_1\}_{\alpha_{\mathcal{L}}}$, from (2), $x_0 \preceq x_1 \preceq x_2$. Now, $\mathcal{M}_{M,N}(\{F_1x_0\}_{\alpha_{\mathcal{L}}}, \{F_2x_1\}_{\alpha_{\mathcal{L}}}, t) \leqslant_{L^*} \mathcal{M}_{M,N}(f_1x_1, f_2x_2, t) = M(y_1, y_2, t)$. By inequality (3.2) and property (3.1), we have $\mathcal{M}_{M,N}(y_1, y_2, t) \geqslant_{L^*} \mathcal{M}_{M,N}(y_0, y_1, t)$. Similarly, one can find $x_3 \in X$ and $x_2 \preceq x_3$ such that $y_3 = f_3x_3 \in \{F_3x_2\}_{\alpha_{\mathcal{L}}}$. Further, we have also $\mathcal{M}_{M,N}(\{F_2x_1\}_{\alpha_{\mathcal{L}}}, \{F_3x_2\}_{\alpha_{\mathcal{L}}}, t) \leqslant_{L^*} \mathcal{M}_{M,N}(f_2x_2, f_3x_3, t)$ and $\mathcal{M}_{M,N}(y_2, y_3, t) \geqslant_{L^*} \mathcal{M}_{M,N}(y_1, y_2, t)$, containing in this way, we have a sequence $\{y_n\}$ such that

$$\{y_{2n+1}\} = \{f_{2n+1}x_{2n+1}\} \subseteq \{F_{2n+1}x_{2n}\}_{\alpha_{\mathcal{L}}}, \quad \{y_{2n+2}\} = \{f_{2n+2}x_{2n+2}\} \subseteq \{F_{2n+2}x_{2n+1}\}_{\alpha_{\mathcal{L}}}.$$

By induction we obtain $\mathcal{M}_{M,N}(y_{n+1}, y_{n+2}, t) \geqslant_{L^*} \mathcal{M}_{M,N}(y_n, y_{n+1}, t)$. Since

$$\begin{aligned}
\mathcal{M}_{M,N}(y_n, y_m, t) &\geqslant_{L^*} \mathcal{T}(\mathcal{M}_{M,N}(y_n, y_{n+1}, t), \mathcal{M}_{M,N}(y_{n+1}, y_m, t)) \\
&\geqslant_{L^*} \mathcal{T}(\mathcal{M}_{M,N}(y_n, y_{n+1}, t), \mathcal{T}(\mathcal{M}_{M,N}(y_{n+1}, y_{n+2}, t), \mathcal{M}_{M,N}(y_{n+2}, y_m, t)) \\
&= \mathcal{T}^2(\mathcal{M}_{M,N}(y_n, y_{n+1}, t), \mathcal{M}_{M,N}(y_{n+1}, y_{n+2}, t), \mathcal{M}_{M,N}(y_{n+2}, y_m, t)) \\
&\geqslant_{L^*} \mathcal{T}^2(\mathcal{M}_{M,N}(y_n, y_{n+1}, t), \mathcal{M}_{M,N}(y_n, y_{n+1}, t), \mathcal{M}_{M,N}(y_{n+2}, y_m, t)) \\
&\geqslant_{L^*} \mathcal{T}^{m-n-1}(\mathcal{M}_{M,N}(y_n, y_{n+1}, t), \ldots, \mathcal{M}_{M,N}(y_n, y_{n+1}, t)) \\
&\geqslant_{L^*} \mathcal{T}^{m-n-1}(\mathcal{M}_{M,N}(y_0, y_1, t), \ldots, \mathcal{M}_{M,N}(y_0, y_1, t)).
\end{aligned}$$

Since $\mathcal{M}_{M,N}(y_0, y_1, t) = 1_{L^*}$ as $t \to \infty$ and $\mathcal{T}$ is a t-norm of Hadžić type, for any $\epsilon \in (0,1)$ and $\delta \in (0,1)$ we have $\mathcal{T}^{m-n-1}(\mathcal{N}(\delta), \ldots, \mathcal{N}(\delta)) \geqslant_{L^*} \mathcal{N}(\epsilon)$ for all $m > n$ where $n, m \in \mathbb{N} \cup \{0\}$, then $\lim_{n,m\to\infty} \mathcal{M}_{M,N}(y_n, y_m, t) = 1_{L^*}$, so $\{y_n\}$ is a Cauchy sequence, since X is complete, there exists $z \in X$ and $\lim_{n\to\infty} \mathcal{M}_{M,N}(y_n, z, t) = 1_{L^*}$.

As $y_{2n+1} \to z$, $y_{2n+2} = f_{2n+2}x_{2n+2} \to z$ and $y_{2n+1} = f_{2n+1}x_{2n+1} \to z$, since $f_{n+1}(X)$ are closed, then there exist $v, w \in X$ such that $z = f_{2n+1}v = f_{2n+2}w$. We show that $f_{2n+1}v \in \{F_{2n+1}v\}_{\alpha_{\mathcal{L}}}$. Since

$$\phi\begin{pmatrix} \mathcal{M}_{M,N}(\{F_{2n+1}v\}_{\alpha_{\mathcal{L}}}, y_{2n+2}, t), \mathcal{M}_{M,N}(y_{2n+1}, y_{2n+2}, t), \\ \mathcal{M}_{M,N}(y_{2n+1}, \{F_{2n+1}v\}_{\alpha_{\mathcal{L}}}, t), \mathcal{M}_{M,N}(y_{2n+2}, y_{2n+2}, t), \\ \mathcal{T}(\mathcal{M}_{M,N}(y_{2n+1}, y_{2n+1}, t), \mathcal{M}_{M,N}(y_{2n+1}, y_{2n+2}, t)), \\ \mathcal{M}_{M,N}(y_{2n+1}, \{F_{2n+1}v\}_{\alpha_{\mathcal{L}}}, t) \end{pmatrix}$$
$$\geqslant_{L^*} \phi\begin{pmatrix} \mathcal{M}_{M,N}(\{F_{2n+1}v\}_{\alpha_{\mathcal{L}}}, \{F_{2n+2}x_{2n+1}\}_{\alpha_{\mathcal{L}}}, t), \mathcal{M}_{M,N}(f_{2n+1}v, f_{2n+2}x_{2n+2}, t), \\ \mathcal{M}_{M,N}(f_{2n+1}v, \{F_{2n+1}v\}_{\alpha_{\mathcal{L}}}, t), \mathcal{M}_{M,N}(f_{2n+2}x_{2n+2}, \{F_{2n+2}x_{2n+1}\}_{\alpha_{\mathcal{L}}}, t), \\ \mathcal{M}_{M,N}(f_{2n+1}v, \{F_{2n+2}x_{2n+1}\}_{\alpha_{\mathcal{L}}}, t), \mathcal{M}_{M,N}(f_{2n+1}x_{2n+1}, \{F_{2n+1}v\}_{\alpha_{\mathcal{L}}}, t) \end{pmatrix} \geqslant_{L^*} 0_{L^*}.$$

When $n \to \infty$, we have that

$$\phi\begin{pmatrix} \mathcal{M}_{M,N}(\{F_{2n+1}v\}_{\alpha_{\mathcal{L}}}, z, t), 1_{L^*}, \\ \mathcal{M}_{M,N}(z, \{F_{2n+1}v\}_{\alpha_{\mathcal{L}}}, t), 1_{L^*}, \\ 1_{L^*}, \mathcal{M}_{M,N}(z, \{F_{2n+1}v\}_{\alpha_{\mathcal{L}}}, t) \end{pmatrix} \geqslant_{L^*} 0_{L^*}.$$

By (3.1), this gives $\mathcal{M}_{M,N}(z, \{F_{2n+1}v\}_{\alpha_{\mathcal{L}}}, t) \geqslant_{L^*} 1_{L^*}$, then $z = f_{2n+1}v \in \{F_{2n+1}v\}_{\alpha_{\mathcal{L}}}$. By a similar way one can find $z = f_{2n+2}w \in \{F_{2n+2}w\}_{\alpha_{\mathcal{L}}}$. Further, by weakly commuting and occasionally coincidentally idempotent of (f_{2n+1}, F_{2n+2}), we have $f_{2n+1}z = f_{2n+1}f_{2n+1}v = f_{2n+1}v = z$ and

$$z = f_{2n+1}z = f_{2n+1}f_{2n+1}v \in \{F_{2n+1}f_{2n+1}v\}_{\alpha_{\mathcal{L}}} = \{F_{2n+1}z\}_{\alpha_{\mathcal{L}}}.$$

Also, $f_{2n+2}z = f_{2n+2}f_{2n+2}v = f_{2n+2}w = z$ and

$$z = f_{2n+2}z = f_{2n+2}f_{2n+2}w \in \{F_{2n+2}f_{2n+2}w\}_{\alpha_{\mathcal{L}}} = \{F_{2n+2}z\}_{\alpha_{\mathcal{L}}}.$$

This completes the proof. □

Example 4.4. Define the triplet $(X, \mathcal{M}_{M,N}, \mathcal{T})$ as following, let $X = L = L^* = [0,1]$, where $(L^*, \leqslant_{L^*})$ is defined by $L^* = \{a = (1_1, a_2) : (a_1, a_2) \in [0,1]^2, a_1 + a_2 \leqslant 1\}$ such that for all $a = (a_1, a_2) \in L^*$ and $b = (b_1, b_2) \in L^*$, $(a_1, a_2) \leqslant_{L^*} (b_1, b_2) \Leftrightarrow a_1 \leqslant b_1$ and $a_2 \geqslant b_2$. Let $\mathcal{M}_{M,N}(x, y, t)$ be an intuitionistic fuzzy mapping on $X^2 \times (0, \infty)$ defined as $\mathcal{M}_{M,N}(x, y, t) = (M(x, y, t), N(x, y, t))$, where

$$(M(x,y,t), N(x,y,t)) = \begin{cases} \left(\frac{x}{y}, \frac{y-x}{y}\right), & \text{if } x \leqslant y, \\ \left(\frac{y}{x}, \frac{x-y}{x}\right), & \text{if } x \geqslant y, \end{cases}$$

for all $x, y \in X$ and $t > 0$. Suppose that $\mathcal{T}(a, b) = (\max\{0, a_1 + b_1 - 1\}, a_2 + b_2 - a_2 b_2)$. Then $(X, \mathcal{M}_{M,N}, \mathcal{T})$ is a modified intuitionistic fuzzy metric space. Set $\alpha = 0.2$ and define the mappings $f_{2n+1}, f_{2n+2}, F_{2n+1}$ and F_{2n+2} on X as $f_{2n+1}x = \frac{x}{5}$, $f_{2n+2}x = \frac{x}{3}$,

$$(F_{2n+1}x)(y) = \begin{cases} \frac{6}{7}, & \text{if } 0 \leqslant y < \frac{x}{20}, \\ \frac{4}{5}, & \text{if } \frac{x}{20} \leqslant y \leqslant 1, \end{cases} \quad \text{and} \quad (F_{2n+2}x)(y) = \begin{cases} \frac{1}{3}, & \text{if } 0 \leqslant y < \frac{x}{30}, \\ \frac{2}{5}, & \text{if } \frac{x}{30} \leqslant y \leqslant 1. \end{cases}$$

Now, $\{F_{2n+1}x\}_{\frac{4}{5}} = [\frac{x}{20}, 1]$ and $\{F_{2n+2}x\}_{\frac{2}{5}} = [\frac{x}{30}, 1]$. Consider the two sequences $\{x_n\}$ and $\{y_n\}$, where $x_{2n} = \frac{1}{2n}$, $x_{2n+1} = \frac{1}{2n+1}$ and $x_{2n+2} = \frac{1}{2n+2}$, then $y_{2n+1} = f_{2n+1}x_{2n+1} = \frac{1}{10n+5} \in [\frac{1}{40n}, 1] = \{F_{2n+1}x_{2n}\}_{\frac{4}{5}}$ and $y_{2n+2} = f_{2n+2}x_{2n+2} = \frac{1}{6n+6} \in [\frac{1}{60n+30}, 1] = \{F_{2n+2}x_{2n+1}\}_{\frac{2}{5}}$. Then (f_{2n+1}, F_{2n+1}) and (f_{2n+2}, F_{2n+2}) are D-compatible mappings. Now, $\lim_{n\to\infty} y_{2n+1} = \lim_{n\to\infty} f_{2n+1}x_{2n+2} = 0 \in [0,1] = \lim_{n\to\infty} \{F_{2n+1}x_{2n}\}_{\frac{4}{5}}$ and

$\lim_{n\to\infty} y_{2n+2} = \lim_{n\to\infty} f_{2n+2}x_{2n+2} = 0 \in [0,1] = \lim_{n\to\infty}\{F_{2n+2}x_{2n+1}\}_{\frac{2}{5}}$. We prove that 0 is a fixed point of F_{2n+1}, suppose not, since

$$\phi\begin{pmatrix} \mathcal{M}_{M,N}(\{F_{2n+1}0\}_{\frac{4}{5}}, y_{2n+2}, t), \mathcal{M}_{M,N}(y_{2n+1}, y_{2n+2}, t), \\ \mathcal{M}_{M,N}(y_{2n+1}, \{F_{2n+1}0\}_{\frac{4}{5}}, t), \mathcal{M}_{M,N}(y_{2n+2}, y_{2n+2}, t), \\ \mathcal{T}(\mathcal{M}_{M,N}(y_{2n+1}, y_{2n+1}, t), \mathcal{M}_{M,N}(y_{2n+1}, y_{2n+2}, t)), \\ \mathcal{M}_{M,N}(y_{2n+1}, \{F_{2n+1}0\}_{\frac{4}{5}}, t) \end{pmatrix}$$

$$\geqslant_{L^*} \phi\begin{pmatrix} \mathcal{M}_{M,N}(\{F_{2n+1}0\}_{\frac{4}{5}}, \{F_{2n+2}x_{2n+1}\}_{\frac{2}{5}}, t), \mathcal{M}_{M,N}(f_{2n+1}\nu, f_{2n+2}x_{2n+2}, t), \\ \mathcal{M}_{M,N}(f_{2n+1}\nu, \{F_{2n+1}0\}_{\alpha_{\frac{4}{5}}}, t), \mathcal{M}_{M,N}(f_{2n+2}x_{2n+2}, \{F_{2n+2}x_{2n+1}\}_{\frac{2}{5}}, t), \\ \mathcal{M}_{M,N}(f_{2n+1}0, \{F_{2n+2}x_{2n+1}\}_{\frac{2}{5}}, t), \mathcal{M}_{M,N}(f_{2n+1}x_{2n+1}, \{F_{2n+1}0\}_{\frac{4}{5}}, t) \end{pmatrix} \geqslant_{L^*} 0_{L^*}.$$

By continuity of $\mathcal{T}$ and $\mathcal{M}_{M,N}$, letting $n \to \infty$, we have

$$\phi\begin{pmatrix} \mathcal{M}_{M,N}(\{F_{2n+1}0\}_{\frac{4}{5}}, 0, t), 1_{L^*}, \mathcal{M}_{M,N}(0, \{F_{2n+1}0\}_{\frac{4}{5}}, t), \\ 1_{L^*}, 1_{L^*}, \mathcal{M}_{M,N}(0, \{F_{2n+1}0\}_{\frac{4}{5}}, t) \end{pmatrix} \geqslant_{L^*} 0_{L^*}.$$

By (3.1), this gives $\mathcal{M}_{M,N}(0, \{F_{2n+1}0\}_{\frac{4}{5}}, t) \geqslant_{L^*} 1_{L^*}$, then $0 = f_{2n+1}0 \in \{F_{2n+1}0\}_{\frac{4}{5}}$ and by similar way we have $0 = f_{2n+2}0 \in \{F_{2n+2}0\}_{\frac{2}{5}}$.

Corollary 4.5. *Let* $(X, \mathcal{M}_{M,N}, \mathcal{T})$ *be a complete non-Archimedean modified intuitionistic fuzzy metric space with Hadžić type t-norm and* $\lim_{t\to\infty} \mathcal{M}_{M,N}(y_0, y_1, t) = 1_{L^*}$, *let* $\preceq$ *be a partial order defined on* X. *Suppose that* $f : X \to X$ *and* $F_{n+1} : X \to \mathfrak{I}_{\mathcal{L}}(X)$ *such that* $\{F_{n+1}\}$ *is a sequence of* $\mathcal{L}$*-fuzzy mappings, where for each* $x \in X$, $\alpha_{\mathcal{L}} \in L\backslash\{0_{\mathcal{L}}\}$, $f(X)$ *and* $\{F_{n+1}x\}_{\alpha_{\mathcal{L}}}$ *are nonempty closed subsets of* X. *Suppose that we have the following conditions for all* $x, y \in X$, $n \in \mathbb{N} \cup \{0\}$:

(1) $\{F_{n+1}x\}_{\alpha_{\mathcal{L}}} \preceq_1 f(X)$;
(2) *if* $fy \in \{F_{n+1}x\}_{\alpha_{\mathcal{L}}}$ *implies* $x \preceq y$;
(3) *if* $y_n \to y$, *then* $y_n \preceq y$ *for all* n;
(4) (f, F_{2n+1}) *are weakly commuting and occasionally coincidentally idempotent.*

If for all comparable elements $x, y \in X$ *there exists* $\phi \in \Phi$ *such that*

$$\phi\begin{pmatrix} \mathcal{M}_{M,N}(\{F_{2n+1}x\}_{\alpha_{\mathcal{L}}}, \{F_{2n+2y}\}_{\alpha_{\mathcal{L}}}, t), \mathcal{M}_{M,N}(fx, fy, t), \\ \mathcal{M}_{M,N}(fx, \{F_{2n+1}x\}_{\alpha_{\mathcal{L}}}, t), \mathcal{M}_{M,N}(fy, \{F_{2n+2}y\}_{\alpha_{\mathcal{L}}}, t), \\ \mathcal{M}_{M,N}(fx, \{F_{2n+2}y\}_{\alpha_{\mathcal{L}}}, t), \mathcal{M}_{M,N}(fy, \{F_{2n+1}x\}_{\alpha_{\mathcal{L}}}, t) \end{pmatrix} \geqslant_{L^*} 0_{L^*},$$

then the mappings f *and* $\{F_{n+1}\}$ *have a common fixed point.*

Corollary 4.6. *Let* $(X, \mathcal{M}_{M,N}, \mathcal{T})$ *be a complete non-Archimedean modified intuitionistic fuzzy metric space with Hadžić type t-norm and* $\lim_{t\to\infty} \mathcal{M}_{M,N}(y_0, y_1, t) = 1_{L^*}$, *let* $\preceq$ *be a partial order defined on* X. *Suppose that* $F_{n+1} : X \to \mathfrak{I}_{\mathcal{L}}(X)$ *such that* $\{F_{n+1}\}$ *is a sequence of* $\mathcal{L}$*-fuzzy mappings, where for each* $x \in X$, $\alpha_{\mathcal{L}} \in L\backslash\{0_{\mathcal{L}}\}$, $\{F_{n+1}x\}_{\alpha_{\mathcal{L}}}$ *are nonempty closed subsets of* X, $n \in \mathbb{N} \cup \{0\}$. *If for all comparable elements* $x, y \in X$ *there exist* $\phi \in \Phi$ *such that*

$$\phi\begin{pmatrix} \mathcal{M}_{M,N}(\{F_{2n+1}x\}_{\alpha_{\mathcal{L}}}, \{F_{2n+2y}\}_{\alpha_{\mathcal{L}}}, t), \mathcal{M}_{M,N}(x, y, t), \\ \mathcal{M}_{M,N}(x, \{F_{2n+1}x\}_{\alpha_{\mathcal{L}}}, t), \mathcal{M}_{M,N}(y, \{F_{2n+2}y\}_{\alpha_{\mathcal{L}}}, t), \\ \mathcal{M}_{M,N}(x, \{F_{2n+2}y\}_{\alpha_{\mathcal{L}}}, t), \mathcal{M}_{M,N}(y, \{F_{2n+1}x\}_{\alpha_{\mathcal{L}}}, t) \end{pmatrix} \geqslant_{L^*} 0_{L^*},$$

then the mappings $\{F_{n+1}\}$ *have a common fixed point.*

Corollary 4.7. *Let* $(X, \mathcal{M}_{M,N}, \mathcal{T})$ *be a complete non-Archimedean modified intuitionistic fuzzy metric space with Hadžić type t-norm and* $\lim_{t\to\infty} \mathcal{M}_{M,N}(y_0, y_1, t) = 1_{L^*}$, *let* $\preceq$ *be a partial order defined on* X. *Suppose that* $F : X \to \mathfrak{I}_{\mathcal{L}}(X)$ *such that* F *is an* $\mathcal{L}$*-fuzzy mapping, where for each* $x \in X$, $\alpha_{\mathcal{L}} \in L\backslash\{0_{\mathcal{L}}\}$, $\{Fx\}_{\alpha_{\mathcal{L}}}$ *is nonempty closed subset of* X. *If for all comparable elements* $x, y \in X$ *there exists* $\phi \in \Phi$ *such that*

$$\phi\begin{pmatrix} \mathcal{M}_{M,N}(\{Fx\}_{\alpha_{\mathcal{L}}}, \{Fy\}_{\alpha_{\mathcal{L}}}, t), \mathcal{M}_{M,N}(x, y, t), \\ \mathcal{M}_{M,N}(x, \{Fx\}_{\alpha_{\mathcal{L}}}, t), \mathcal{M}_{M,N}(y, \{Fy\}_{\alpha_{\mathcal{L}}}, t), \\ \mathcal{M}_{M,N}(x, \{Fy\}_{\alpha_{\mathcal{L}}}, t), \mathcal{M}_{M,N}(y, \{Fx\}_{\alpha_{\mathcal{L}}}, t) \end{pmatrix} \geqslant_{L^*} 0_{L^*},$$

then the mapping F *have a fixed point.*

5. Integral type

Branciari [7] introduced the idea of integral contractive condition and later on several researchers used it to proved fixed point results in fuzzy metric spaces and other generalized spaces (see, for example, [2, 3, 6, 15, 18]). Recently Imdad et al. [14] and Sadaati et al. [20] also used this condition in modified intuitionistic fuzzy metric spaces.

In this section, we introduce a generalized version of our usual contractive condition with implicit relation for $\mathcal{L}$-fuzzy mappings in complete non-Archimedean modified intuitionistic fuzzy metric spaces.

Let Φ be the family of all continuous mappings $\phi : L^{*6} \to L^*$, which are non-increasing in the 3$^{\text{rd}}$, 4$^{\text{th}}$, 5$^{\text{rd}}$, 6$^{\text{rd}}$, non-decreasing in 1$^{\text{rd}}$ coordinate variable, and satisfying the following properties:

(ϕ_2)

$$\int_{0_{L^*}}^{\phi(a,b,b,a,\mathcal{T}(a,b),1_{L^*})} \varphi(s)ds \geqslant_{L^*} 0_{L^*} \quad \text{or} \quad \int_{0_{L^*}}^{\phi(a,b,b,a,1_{L^*},\mathcal{T}(a,b))} \varphi(s)ds \geqslant_{L^*} 0_{L^*};$$

(ϕ_3)

$$\int_{0_{L^*}}^{\phi(\int_{0_{L^*}}^{a} \psi(s)ds, \int_{0_{L^*}}^{b} \psi(s)ds, \int_{0_{L^*}}^{b} \psi(s)ds, \int_{0}^{a} \psi(s)ds, \int_{0_{L^*}}^{\mathcal{T}(a,b)} \psi(s)ds, 1_{L^*})} \varphi(s)ds \geqslant_{L^*} 0_{L^*},$$

or

$$\int_{0_{L^*}}^{\phi(\int_{0_{L^*}}^{a} \psi(s)ds, \int_{0_{L^*}}^{b} \psi(s)ds, \int_{0_{L^*}}^{a} \psi(s)ds, \int_{0_{L^*}}^{b} \psi(s)ds, 1_{L^*}, \int_{0_{L^*}}^{\mathcal{T}(a,b)} \psi(s)ds)} \varphi(s)ds \geqslant_{L^*} 0_{L^*},$$

for all $a, b \in L^*$ implies $a \geqslant_{L^*} b$, where $\varphi, \psi : L^* \to L^*$ are summable non negative lebesgue integrable functions such that for each $\epsilon \in L^*$, $\int_{0_{L^*}}^{\epsilon} \varphi(s)ds \geqslant_{L^*} 0_{L^*}$ and $\int_{0_{L^*}}^{\epsilon} \psi(s)ds \geqslant_{L^*} 0_{L^*}$.

Theorem 5.1. *The conclusion of Theorem 3.2 remains valid if we have the condition (3.2) as following:*

$$\int_{0_{L^*}}^{\phi(Q)} \varphi(s)ds \geqslant_{L^*} 0_{L^*}, \tag{5.1}$$

where

$$Q = \begin{pmatrix} \mathcal{M}_{M,N}(\{F_{2n+1}x\}_{\alpha_{\mathcal{L}}}, \{F_{2n+2}y\}_{\alpha_{\mathcal{L}}}, t), \mathcal{M}_{M,N}(f_{2n+1}x, f_{2n+2}y, t), \\ \mathcal{M}_{M,N}(f_{2n+1}x, \{F_{2n+1}x\}_{\alpha_{\mathcal{L}}}, t), \mathcal{M}_{M,N}(f_{2n+2}y, \{F_{2n+2}y\}_{\alpha_{\mathcal{L}}}, t), \\ \mathcal{M}_{M,N}(f_{2n+1}x, \{F_{2n+2}y\}_{\alpha_{\mathcal{L}}}, t), \mathcal{M}_{M,N}(f_{2n+2}y, \{F_{2n+1}x\}_{\alpha_{\mathcal{L}}}, t) \end{pmatrix}.$$

Proof. As in Theorem 3.2 with ϕ_2. □

Theorem 5.2. *The conclusion of Theorem 4.3 remains valid if the condition (5.1) is satisfied.*

Proof. As in Theorem 4.3 with ϕ_2. □

Theorem 5.3. *The conclusion of Theorems 5.1 and 5.2 remains valid if we have the condition (5.1) as following:*

$$\int_{0_{L^*}}^{\phi(Q)} \varphi(s)ds \geqslant_{L^*} 0_{L^*}, \tag{5.2}$$

where

$$Q = \begin{pmatrix} \int_{0_{L^*}}^{\mathcal{M}_{M,N}(\{F_{2n+1}x\}_{\alpha_\mathcal{L}},\{F_{2n+2}y\}_{\alpha_\mathcal{L}},t)} \psi(s)ds, \int_{0_{L^*}}^{\mathcal{M}_{M,N}(f_{2n+1}x,f_{2n+2}y,t)} \psi(s)ds, \\ \int_{0_{L^*}}^{\mathcal{M}_{M,N}(f_{2n+1}x,\{F_{2n+1}x\}_{\alpha_\mathcal{L}},t)} \psi(s)ds, \int_{0_{L^*}}^{\mathcal{M}_{M,N}(f_{2n+2}y,\{F_{2n+2}y\}_{\alpha_\mathcal{L}},t)} \psi(s)ds, \\ \int_{0_{L^*}}^{\mathcal{M}_{M,N}(f_{2n+1}x,\{F_{2n+2}y\}_{\alpha_\mathcal{L}},t)} \psi(s)ds, \int_{0_{L^*}}^{\mathcal{M}_{M,N}(f_{2n+2}y,\{F_{2n+1}x\}_{\alpha_\mathcal{L}},t)} \psi(s)ds \end{pmatrix}.$$

Proof. As in Theorems 5.1 and 5.2 with ϕ_3. □

Remark 5.4. The conclusion of Corollaries (3.4), (3.5), and (3.6) remains valid if the function Q in conditions (5.1) or (5.2) has the following forms, respectively,

$$Q = \begin{pmatrix} \mathcal{M}_{M,N}(\{F_{2n+1}x\}_{\alpha_\mathcal{L}},\{F_{2n+2}y\}_{\alpha_\mathcal{L}},t), \mathcal{M}_{M,N}(fx,fy,t), \\ \mathcal{M}_{M,N}(fx,\{F_{2n+1}x\}_{\alpha_\mathcal{L}},t), \mathcal{M}_{M,N}(fy,\{F_{2n+2}y\}_{\alpha_\mathcal{L}},t), \\ \mathcal{M}_{M,N}(fx,\{F_{2n+2}y\}_{\alpha_\mathcal{L}},t), \mathcal{M}_{M,N}(fy,\{F_{2n+1}x\}_{\alpha_\mathcal{L}},t) \end{pmatrix}, \tag{5.3}$$

$$Q = \begin{pmatrix} \mathcal{M}_{M,N}(\{F_{2n+1}x\}_{\alpha_\mathcal{L}},\{F_{2n+2}y\}_{\alpha_\mathcal{L}},t), \mathcal{M}_{M,N}(x,y,t), \\ \mathcal{M}_{M,N}(x,\{F_{2n+1}x\}_{\alpha_\mathcal{L}},t), \mathcal{M}_{M,N}(y,\{F_{2n+2}y\}_{\alpha_\mathcal{L}},t), \\ \mathcal{M}_{M,N}(x,\{F_{2n+2}y\}_{\alpha_\mathcal{L}},t), \mathcal{M}_{M,N}(y,\{F_{2n+1}x\}_{\alpha_\mathcal{L}},t) \end{pmatrix}, \tag{5.4}$$

$$Q = \begin{pmatrix} \mathcal{M}_{M,N}(\{Fx\}_{\alpha_\mathcal{L}},\{Fy\}_{\alpha_\mathcal{L}},t), \mathcal{M}_{M,N}(x,y,t), \\ \mathcal{M}_{M,N}(x,\{Fx\}_{\alpha_\mathcal{L}},t), \mathcal{M}_{M,N}(y,\{Fy\}_{\alpha_\mathcal{L}},t), \\ \mathcal{M}_{M,N}(x,\{Fy\}_{\alpha_\mathcal{L}},t), \mathcal{M}_{M,N}(y,\{Fx\}_{\alpha_\mathcal{L}},t) \end{pmatrix}. \tag{5.5}$$

Remark 5.5. The conclusion of Corollaries (4.5), (4.6), and (4.7) remains valid if the function Q in conditions (5.1) or (5.2) has the forms (5.3), (5.4), and (5.5), respectively.

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