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Describing ion sound waves in plasma

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Abstract

In this paper, we use the Kudryashov methods to investigate the novel solutions to a nonlinear time fractional model. The 3D and 2D figures are depicted for displaying the physical behavior of travelling solutions for diverse values of uncertain parameters with constraint conditions. Also, via an alternative technique, we investigate the existence and uniqueness of solutions of the governing model and we consider the UHR stability of the obtained solution.

Keywords: NLPDEs, Kudryashov methods.

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1. Introduction

The study of FNPDEs of physical and mathematical models rely on the investigation of the solutions for nonlinear equations. Newly, diverse techniques have been applied to solve FNPDEs, like : SCM [5], VIM [2], BTM [4], and the rest.

The purpose of this paper is to use interesting techniques, namely, Kudryashov methods [3], to obtain the solutions for a nonlinear fractional differential equation with the Jumarie's modified Riemann Liouville derivative of order α, given by

$$D_\tau^\alpha \mathcal{F} = \frac{1}{\Gamma(1-\alpha)}\frac{d}{d\tau}\int_0^\tau (\tau-\rho)^{-\alpha}(\mathcal{F}(\rho)-\mathcal{F}(0))d\rho, \quad 0<\alpha\leqslant 1. \tag{1.1}$$

Also, we show the UHR stability of the obtained solution of the governing models through an alternative technique.

2. The fundamental notion of the Kudryashov methods

In this section, we propose the algorithm of the Kudryashov methods for an NPDE as follows: We assume a common nonlinear PDE of the type:

$$N(u, D_t^\alpha u, D_x^\alpha u, D_x^\beta u, D_x^\alpha D_x^\beta u, D_t^\alpha D_t^\beta u, \cdots) = 0, \quad 0<\beta,\alpha\leqslant 1, \tag{2.1}$$

in which $u = u(x,t)$.

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Convert (2.1) into an ODE by means of

$$\Theta = \frac{dx^{d\beta}}{\Gamma(1+\beta)} + \frac{ct^{c\alpha}}{\Gamma(1+\alpha)}, \quad u(x,t) = u(\Theta), \tag{2.2}$$

in which d and c are constants.

Rewrite (2.1) in the following NODE form:

$$\tilde{N}(u, u', u'', u''', \dots) = 0. \tag{2.3}$$

Suppose the general solution of (2.3) can be written as

$$u(\Theta) = a_0 + a_1\Bbbk(\Theta) + a_2\Bbbk^2(\Theta) + \cdots + a_N\Bbbk^N(\Theta), \tag{2.4}$$

where $\underbrace{a_i}_{1\leqslant i\leqslant N}$, are retrieved later. Note that $N \in \mathbb{N}$ can be computed via the homogeneous balance principle.

In the two subsections below, we express the idea of Kudryashov method I (KM I) and Kudryashov method II (KM II), separately.

2.1. KM I

Here, assume

$$\Bbbk(\Theta) = \frac{1}{1+da^{\Theta}}, \tag{2.5}$$

which satisfies

$$\Bbbk'(\Theta) = \Bbbk(\Theta)(\Bbbk(\Theta) - 1)\ln(a). \tag{2.6}$$

Based on (2.3) and (2.4), a nonlinear system of algebraic type is gained, and by solving it, solitons of (2.4) are obtained.

2.2. KM II

Here, assume

$$\Bbbk(\Theta) = \frac{1}{(B+A)\cosh(\Theta) + (B-A)\sinh(\Theta)}, \tag{2.7}$$

which satisfies

$$(\Bbbk'(\Theta))^2 = \Bbbk^2(\Theta)(1 - 4BA\Bbbk^2(\Theta)). \tag{2.8}$$

According to (2.3) and (2.4), a nonlinear system of algebraic type is gained, and by solving it, solitons of (2.4) are obtained.

3. Application of the Kudryashov methods

Consider the following space-time nonlinear fractional equation which describes ion sound waves in plasma,

$$D_x^{2\alpha}u + D_t^{2\alpha}u + D_t^{\alpha}uD_x^{\alpha}u + uD_t^{\alpha}(D_x^{\alpha}u) + D_t^{2\alpha}(D_x^{2\alpha}u) = 0, \quad \alpha \in (0,1]. \tag{3.1}$$

Consider the following transformation

$$\begin{aligned} \Theta &= \frac{kx^{\alpha}}{\Gamma(1+\alpha)} + \frac{ct^{\alpha}}{\Gamma(1+\alpha)}, \\ u(x,t) &= u(\Theta). \end{aligned} \tag{3.2}$$

in which $k, c \neq 0$ are fixed.

Setting (3.2) in (3.1), we get

$$2k^2c^2u'' + kcu^2 + 2(k^2 + c^2)u = 0. \tag{3.3}$$

Here, we get $N = 2$. Presume the solutions of (3.3) can be gained by

$$\psi(\Theta) = a_0 + a_1\mathbb{k}(\Theta) + a_2(\mathbb{k}(\Theta))^2, \tag{3.4}$$

in which $\underbrace{a_i}_{i=1,2,3}$, are constants to be determined later.

Now, in the two subsections below, we propose the applications of KM I and KM II, separately.

3.1. Application of KM I

Making use of (3.3), (3.4) and (2.6), we get the system of algebraic equations below:

$$\begin{aligned}
&cka_0^2 + 2c^2a_0 + 2k^2a_0 = 0,\\
&2\ln(a)^2c^2k^2a_1 + 2kca_0a_1 + 2c^2a_1 + 2k^2a_1 = 0,\\
&-6\ln(a)^2c^2k^2a_1 + 8\ln(a)^2c^2k^2a_2 + 2kca_0a_2 + kca_1^2 + 2c^2a_2 + 2k^2a_2 = 0,\\
&4\ln(a)^2c^2k^2a_1 - 20\ln(a)^2c^2k^2a_2 + 2cka_1a_2 = 0,\\
&12\ln(a)^2c^2k^2a_2 + kca_2^2 = 0.
\end{aligned}$$

By solving the above system, we have that

$$\begin{aligned}
&c = c, && k = \pm\frac{c}{\sqrt{\ln(a)^2c^2 - 1}},\\
&a_0 = \pm\frac{2c^2\ln(a)^2}{\sqrt{\ln(a)^2c^2 - 1}}, && a_1 = \pm\frac{12c^2\ln(a)^2}{\sqrt{\ln(a)^2c^2 - 1}},\\
&a_2 = \pm\frac{12c^2\ln(a)^2}{\sqrt{\ln(a)^2c^2 - 1}},
\end{aligned}$$

and

$$\begin{aligned}
&c = c, && k = \pm\frac{c}{\sqrt{\ln(a)^2c^2 - 1}},\\
&a_0 = 0, && a_1 = \mp\frac{12c^2\ln(a)^2}{\sqrt{-\ln(a)^2c^2 - 1}},\\
&a_2 = \mp\frac{12c^2\ln(a)^2}{\sqrt{-\ln(a)^2c^2 - 1}}.
\end{aligned}$$

According to the obtained results, the following solitons to (3.1) are obtained

$$\begin{aligned}
u(x,t) = &\pm\frac{2c^2\ln(a)^2}{\sqrt{\ln(a)^2c^2 - 1}}\\
&\pm\frac{12c^2\ln(a)^2}{\sqrt{\ln(a)^2c^2 - 1}}\,\frac{1}{1 + da^{\frac{\pm\frac{c}{\sqrt{\ln(a)^2c^2-1}}x^\alpha}{\Gamma(1+\alpha)} + \frac{ct^\alpha}{\Gamma(1+\alpha)}}}\\
&\pm\frac{12c^2\ln(a)^2}{\sqrt{\ln(a)^2c^2 - 1}}\left(\frac{1}{1 + da^{\frac{\pm\frac{c}{\sqrt{\ln(a)^2c^2-1}}x^\alpha}{\Gamma(1+\alpha)} + \frac{ct^\alpha}{\Gamma(1+\alpha)}}}\right)^2,
\end{aligned} \tag{3.5}$$

and

$$\begin{aligned} u(x,t) =& \mp \frac{12c^2\ln(a)^2}{\sqrt{-\ln(a)^2c^2-1}} \frac{1}{1+da^{\frac{\pm\frac{c}{\sqrt{\ln(a)^2c^2-1}}x^{\alpha}}{\Gamma(1+\alpha)}+\frac{ct^{\alpha}}{\Gamma(1+\alpha)}}} \\ &\mp \frac{12c^2\ln(a)^2}{\sqrt{-\ln(a)^2c^2-1}} \left(\frac{1}{1+da^{\frac{\pm\frac{c}{\sqrt{\ln(a)^2c^2-1}}x^{\alpha}}{\Gamma(1+\alpha)}+\frac{ct^{\alpha}}{\Gamma(1+\alpha)}}}\right)^2. \end{aligned} \tag{3.6}$$

The Figures 1 and 2 display the plots of (3.5) for specific values and $\alpha = 0.1, 0.2, 0.3$, and also the Figures 3 and 4 display the plots of (3.6) for specific values and $a = 2, 4, 6$.

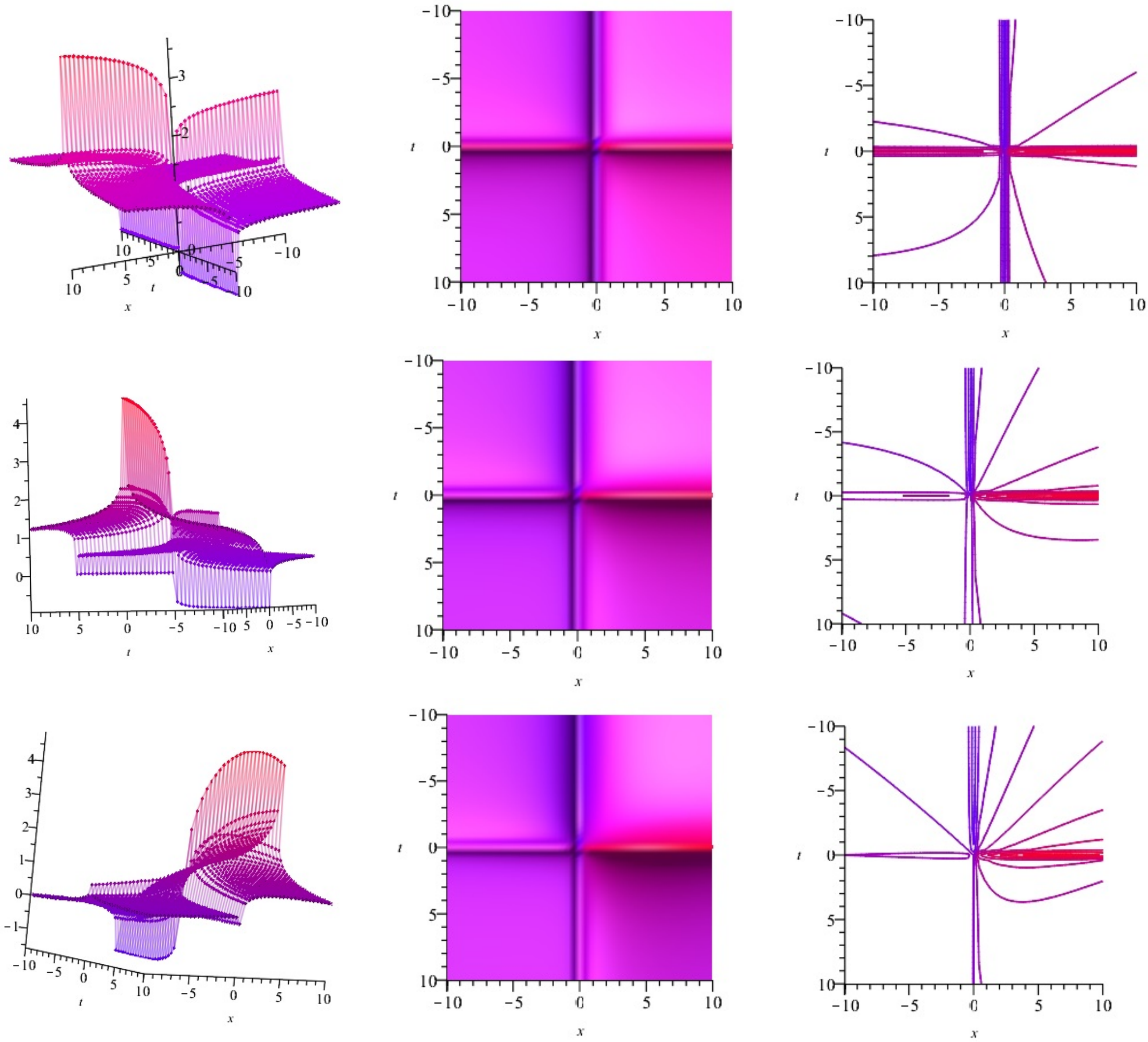

Figure 1: The plots of the real part of (3.5) for $\alpha = 0.1, 0.2, 0.3$.

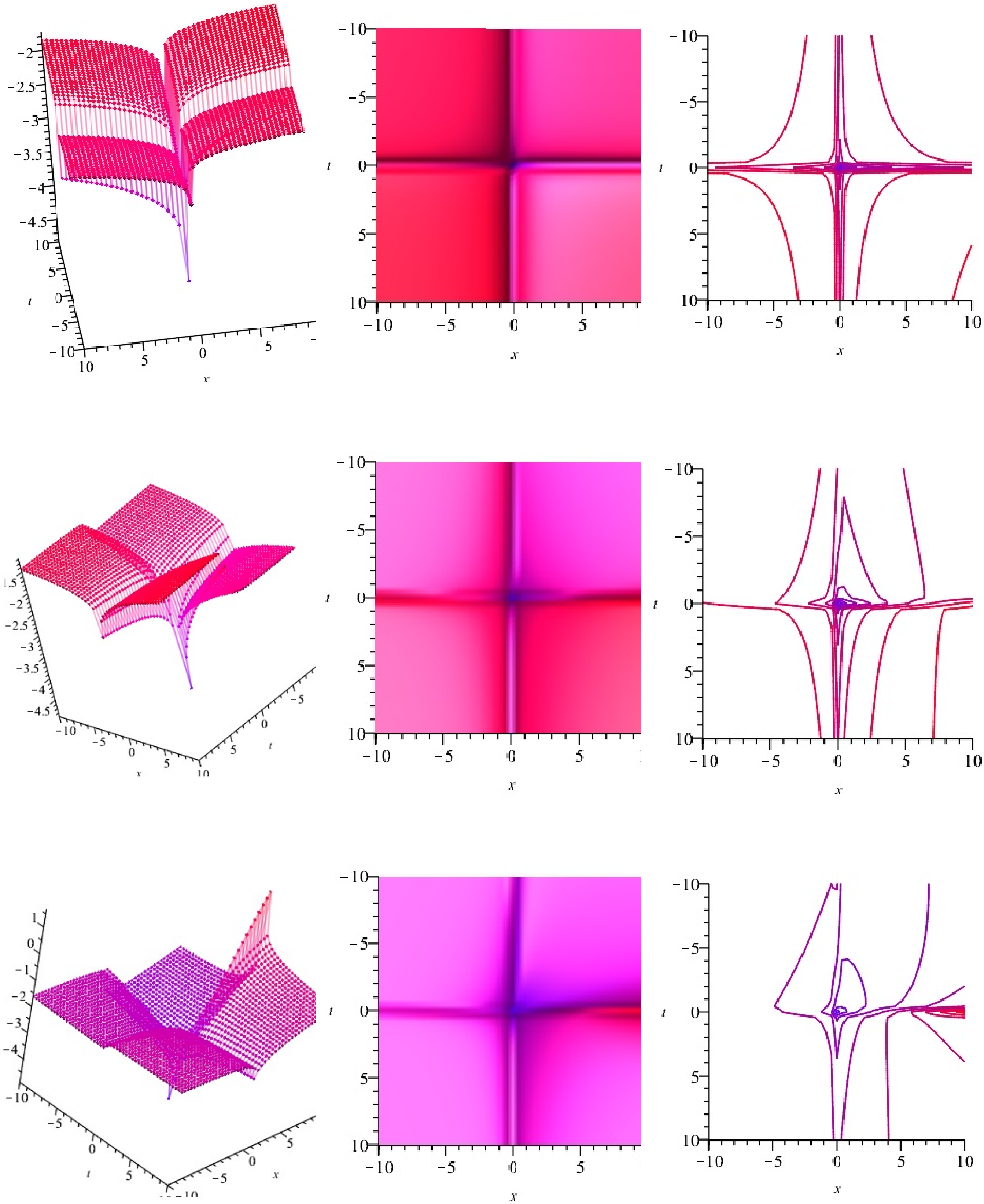

Figure 2: The plots of the imaginary part of (3.5) for $\alpha = 0.1, 0.2, 0.3$.

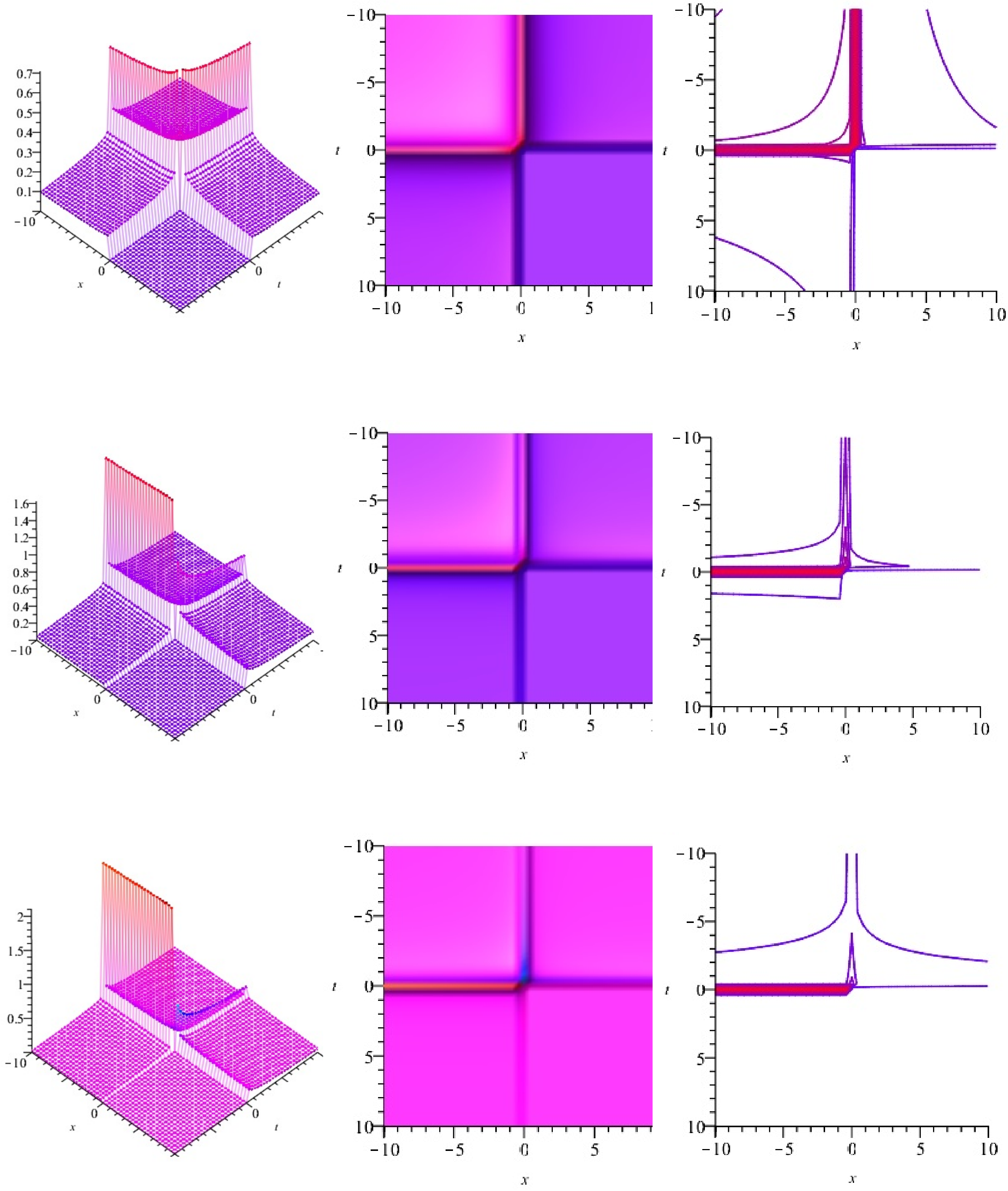

Figure 3: The plots of the real part of (3.5) for $\alpha = 2, 4, 6$.

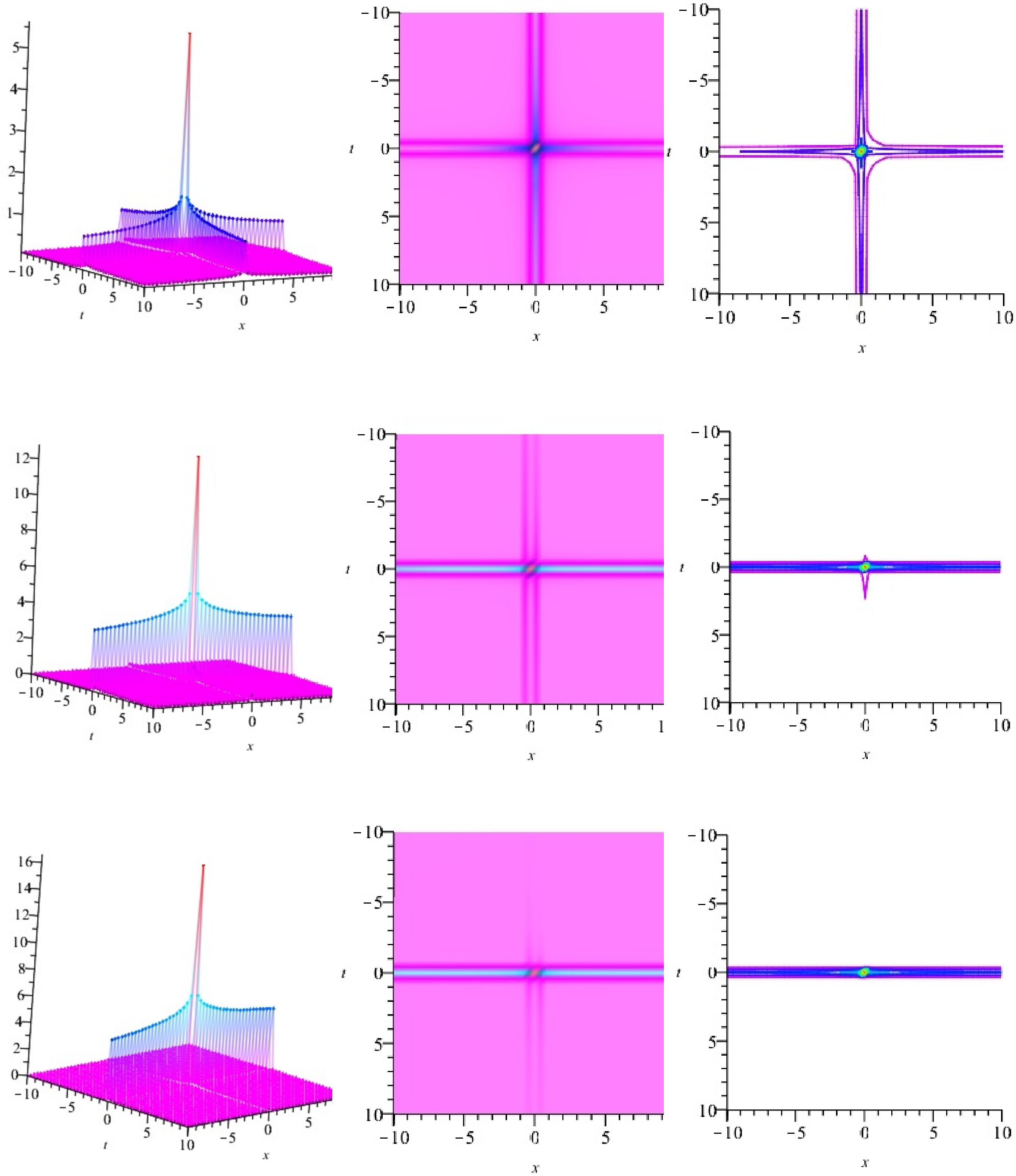

Figure 4: The plots of the imaginary part of (3.5) for $\alpha = 2, 4, 6$.

3.2. Application of KM II

Making use of (3.3), (3.4) and (2.8), we obtain the system of algebraic equations below

$$
\begin{aligned}
& cka_0^2 + 2c^2 a_0 + 2k^2 a_0 = 0, \\
& 2c^2k^2 a_1 + 2cka_0 a_1 + 2c^2 a_1 + 2k^2 a_1 = 0, \\
& 8c^2k^2 a_2 + 2cka_0 a_2 + cka_1^2 + 2c^2 a_2 + 2k^2 a_2 = 0, \\
& -16ABc^2k^2 a_1 + 2cka_1 a_2 = 0, \\
& -48ABc^2k^2 a_2 + cka_2^2 = 0.
\end{aligned}
$$

By solving the obtained system, we get

$$
\begin{aligned}
& c = c, && k = \pm\frac{c}{\sqrt{4c^2-1}}, \\
& a_0 = \pm\frac{8c^2}{\sqrt{4c^2-1}}, && a_1 = 0, \\
& a_2 = \pm\frac{48c^2AB}{\sqrt{4c^2-1}},
\end{aligned}
$$

and

$$
\begin{aligned}
& c = c, && k = \pm\frac{c}{\sqrt{-4c^2-1}}, \\
& a_0 = 0, && a_1 = 0, \\
& a_2 = \pm\frac{48c^2AB}{\sqrt{-4c^2-1}}.
\end{aligned}
$$

Through the obtained results, the following solitons to (3.1) are obtained

$$
\begin{aligned}
u(x,t) = & \pm\frac{8c^2}{\sqrt{4c^2-1}} \\
& \pm\frac{48c^2AB}{\sqrt{4c^2-1}}\left(\frac{1}{(B+A)\cosh(\frac{\pm\frac{c}{\sqrt{4c^2-1}}x^\alpha}{\Gamma(1+\alpha)}+\frac{ct^\alpha}{\Gamma(1+\alpha)})+(B-A)\sinh(\frac{\pm\frac{c}{\sqrt{4c^2-1}}x^\alpha}{\Gamma(1+\alpha)}+\frac{ct^\alpha}{\Gamma(1+\alpha)})}\right)^2,
\end{aligned}
\tag{3.7}
$$

and

$$
u(x,t) = \pm\frac{48c^2AB}{\sqrt{-4c^2-1}}\left(\frac{1}{(B+A)\cosh(\frac{kx^\alpha}{\Gamma(1+\alpha)}+\frac{ct^\alpha}{\Gamma(1+\alpha)})+(B-A)\sinh(\frac{\pm\frac{c}{\sqrt{-4c^2-1}}x^\alpha}{\Gamma(1+\alpha)}+\frac{ct^\alpha}{\Gamma(1+\alpha)})}\right)^2. \tag{3.8}
$$

The Figures 5 and 6 display the plots of (3.7) for specific values and $\alpha = 0.1, 0.2, 0.3$, and also the Figures 7 and 8 display the plots of (3.8) for specific values and $a = 2, 4, 6$.

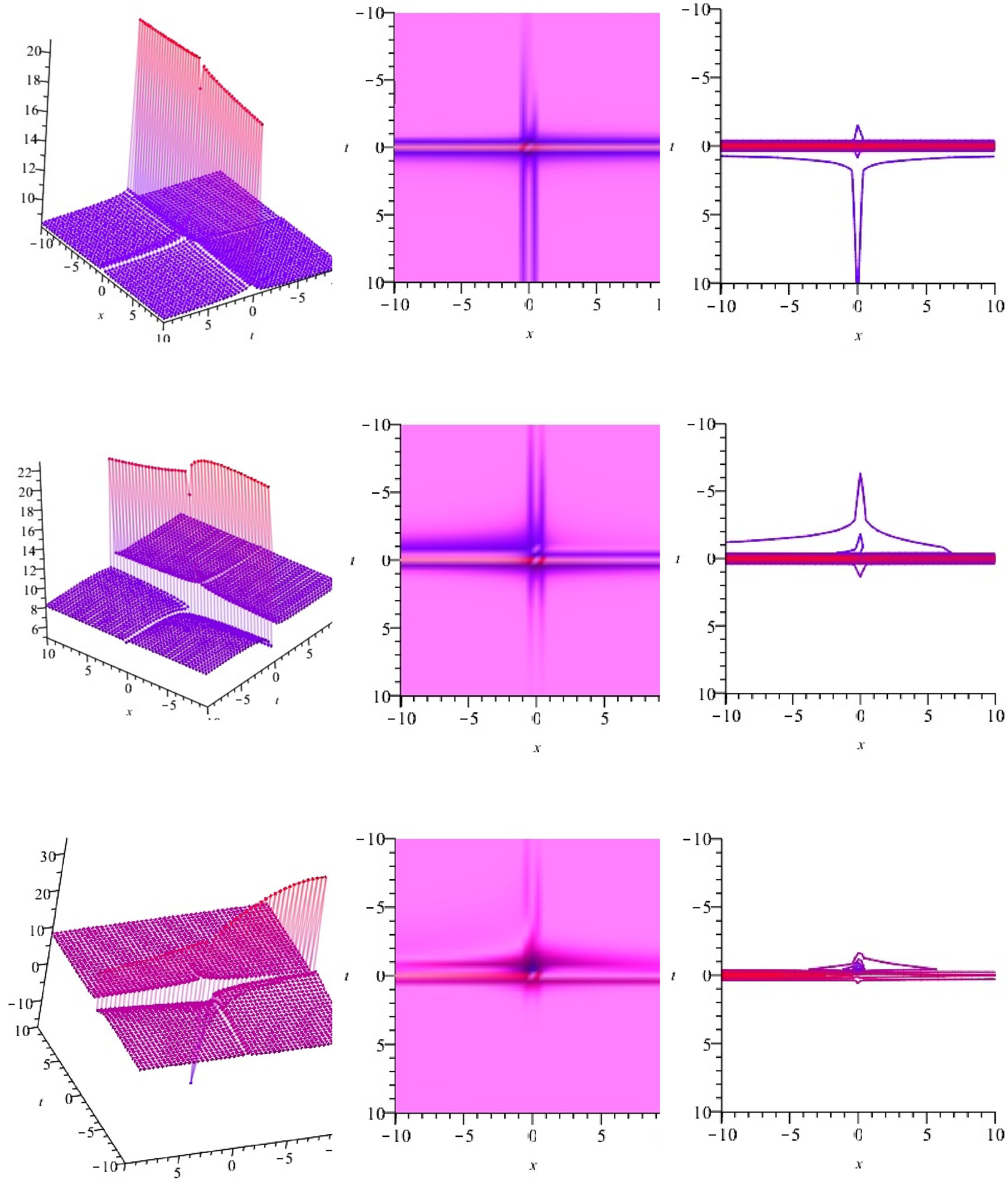

Figure 5: The real parts of (3.7) for $\alpha = 0.1, 0.2, 0.3$.

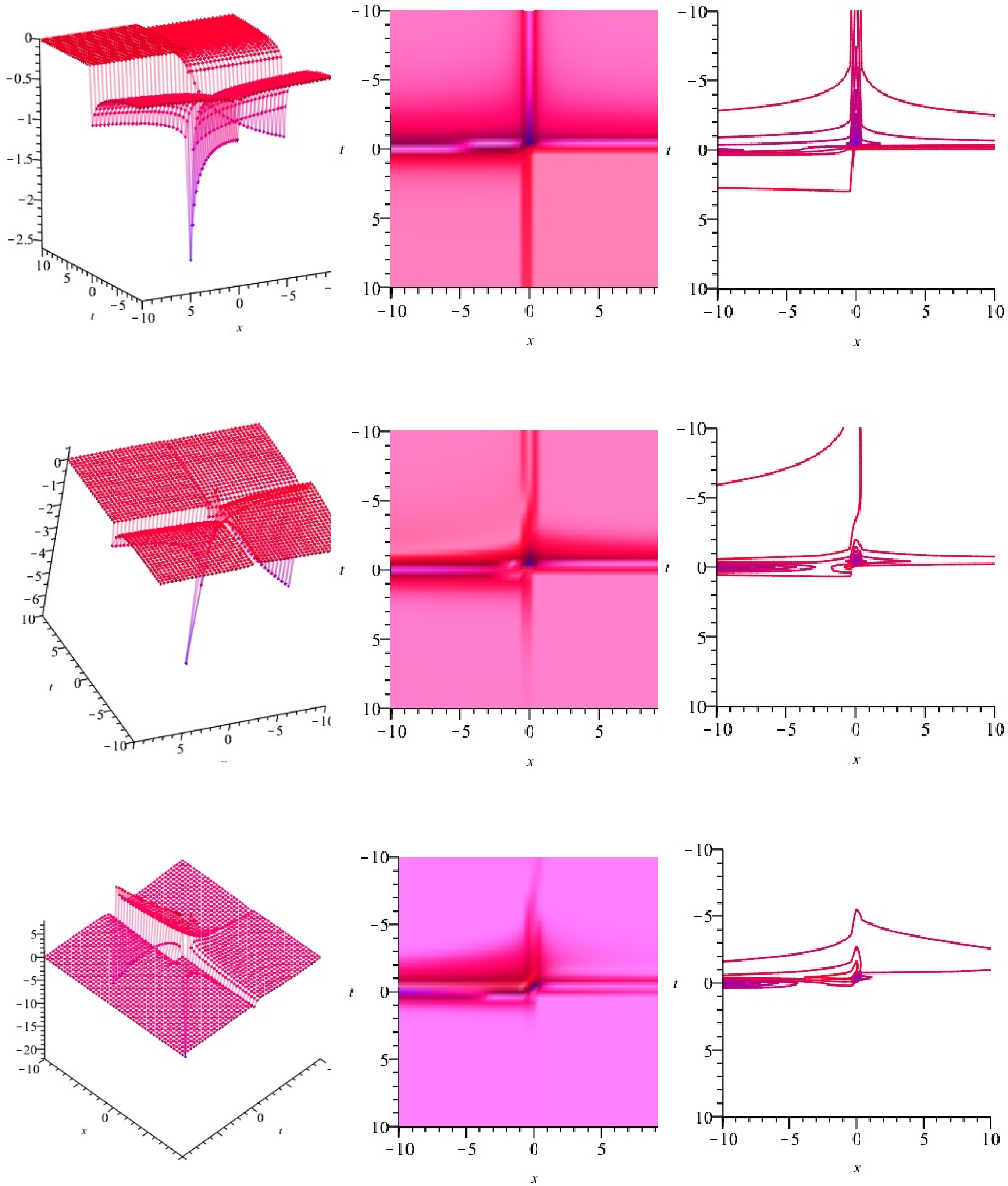

Figure 6: The imaginary parts of (3.7) for $\alpha = 0.1, 0.2, 0.3$.

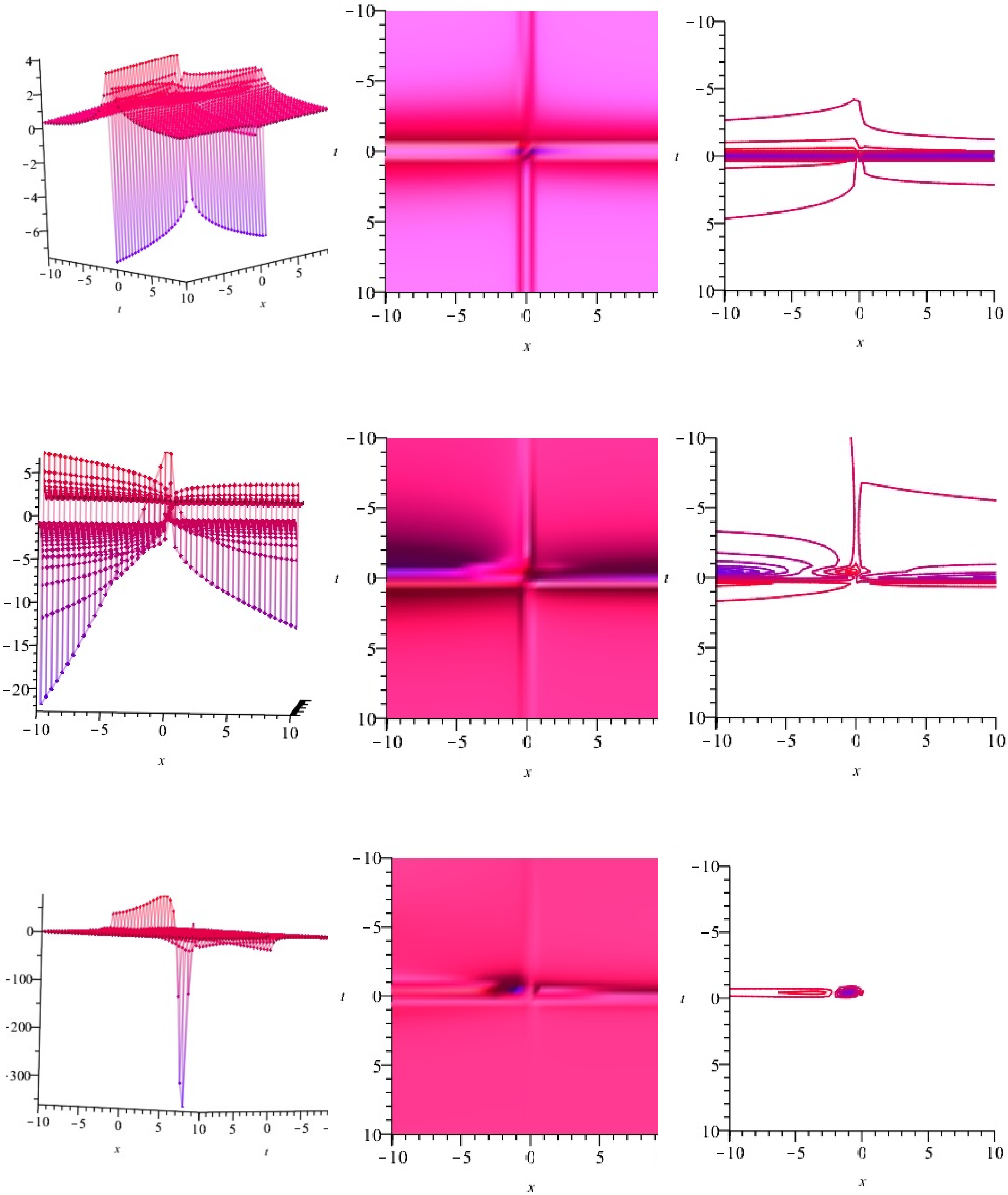

Figure 7: The real parts of (3.7) for $a = 2, 4, 6$.

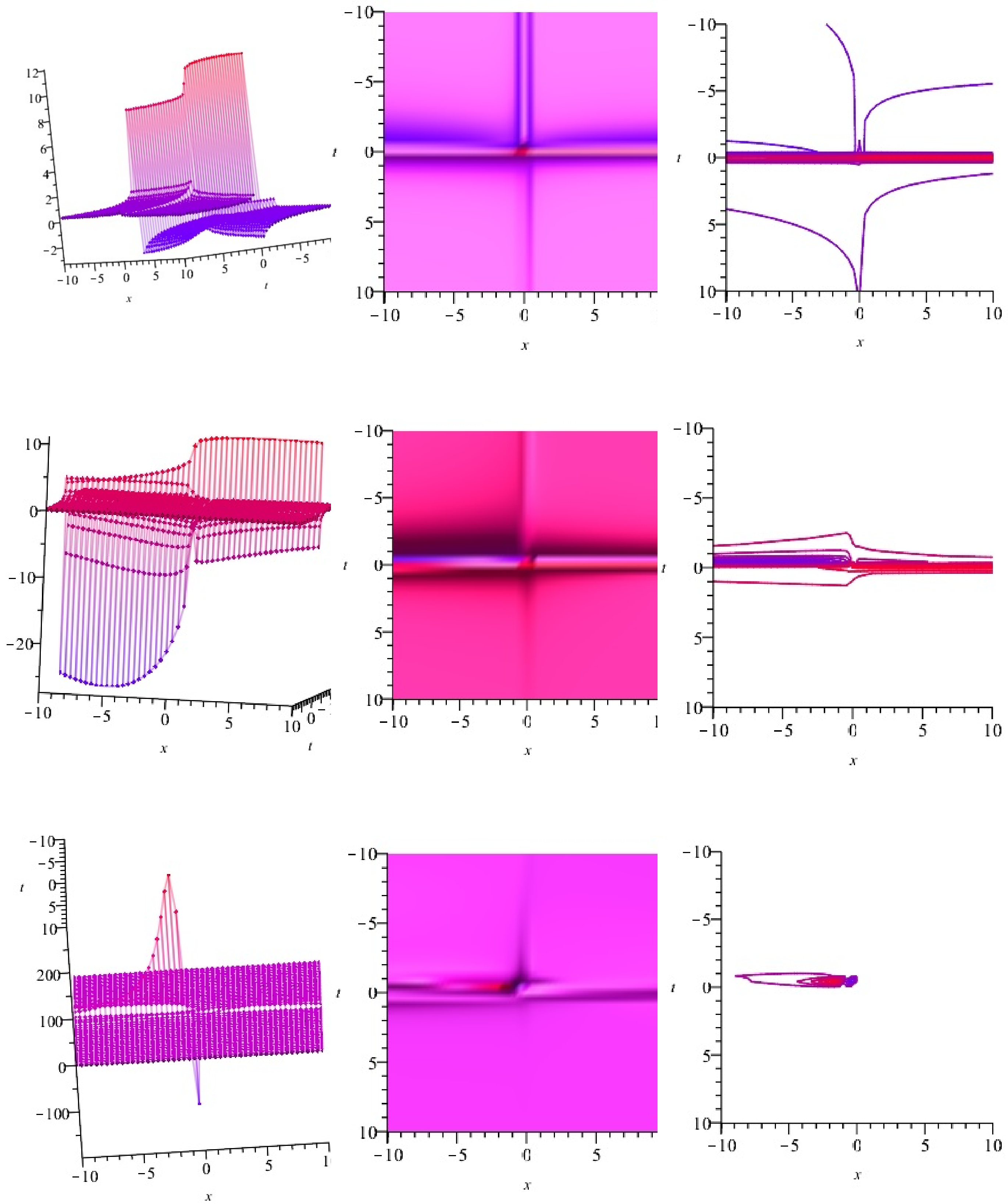

Figure 8: The imaginary parts of (3.7) for $a = 2, 4, 6$.

4. Discussion

Table 1, present the absolute value of the solutions (3.5), (3.6), (3.7), and (3.8) obtained through Kudryashov methods, with diverse point sources through arbitrary, and $\alpha = 0.2$.

Figure 9, shows the plots of the differences between the absolute value of the solutions (3.5) and (3.6), and also the differences between the absolute value of the solutions (3.7) and (3.8), respectively.

Based on Table 1, in Table 2, separately, we propose the differences between solutions (3.5), (3.6), (3.7), and (3.8), represented by $|\Delta u(x,t)|$, for fixed x, diverse values of t, and $\alpha = 0.20$. It is clear that these differences gained by the KM I are less than those of the KM II. In other words, for fixed x, by changing t, the KM I results in more minor changes than the KM II. Therefore, the obtained solution by KM I are stable against small perturbations, and we can conclude the results gained by the KM I has higher accuracy than the KM II.

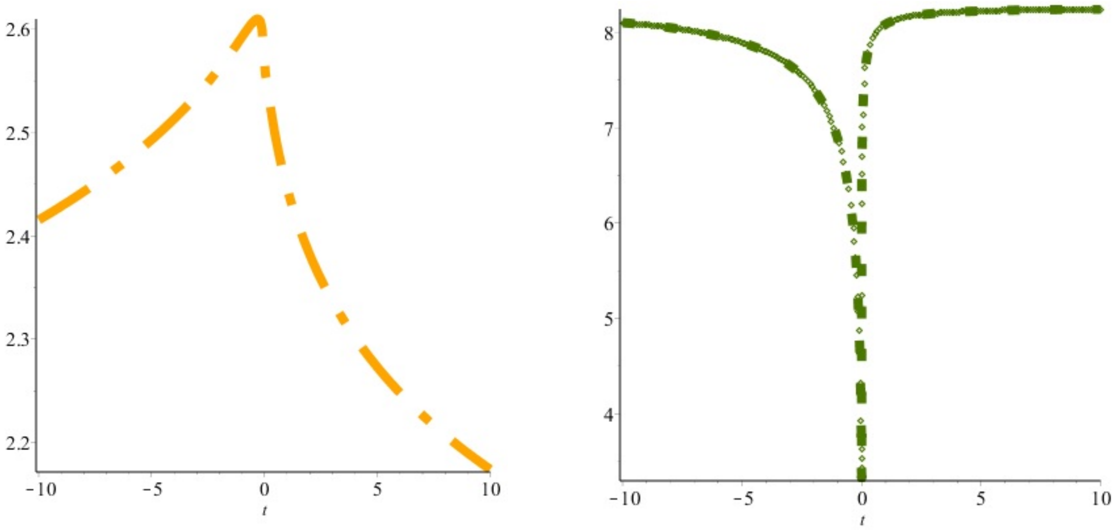

Figure 9: The 2D with the differences between the absolute value of the solutions (3.5) and (3.6), and also the differences between the absolute value of the solutions (3.7) and (3.8).

Table 1: The absolute value of the solutions (3.5), (3.6), (3.7), and (3.8).

		KM I		KM II	
x	t	$u_1(x,t)$	$u_2(x,t)$	$u_1(x,t)$	$u_2(x,t)$
0.012	0.012	1.860	4.022	4.238	11.268
	0.037	1.595	3.832	2.881	10.264
	0.062	1.468	3.735	2.315	9.858
0.037	0.012	1.585	4.005	4.279	10.974
	0.037	1.358	3.812	2.900	10.059
	0.062	1.249	3.713	2.327	9.692
0.062	0.012	1.453	3.994	4.304	10.826
	0.037	1.244	3.799	2.912	9.957
	0.062	1.144	3.699	2.335	9.609

Table 2: The differences between the absolute value of solutions (3.9) and (3.10), represented by $|\Delta u(x,t)|$, for fixed x and diverse values of t.

		KM I		KM II	
x	t	$\|\Delta u_1(x,t)\|$	$\|\Delta u_2(x,t)\|$	$\|\Delta u_1(x,t)\|$	$\|\Delta u_2(x,t)\|$
0.012	0.012-0.037	0.265	0.190	1.357	1.004
	0.037-0.062	0.127	0.097	0.566	0.679
	0.012-0.062	0.392	0.287	1.923	1.410
0.037	0.012-0.037	0.227	0.193	1.379	0.915
	0.037-0.062	0.109	0.099	0.573	0.367
	0.012-0.062	0.336	0.292	1.952	1.282
0.062	0.012-0.037	0.209	0.195	1.392	0.869
	0.037-0.062	0.100	0.100	0.577	0.348
	0.012-0.062	0.309	0.295	1.969	1.217

5. Exploiting the Cadariu–Radu method

Now we propose the UHR stability for (3.1) using an alternative technique from the literature [1]:

Theorem 5.1. *Consider the Banach space Δ, $\chi_1 \in \Delta$, $\varepsilon : \Delta^2 \longrightarrow [0,\infty]$ and the contractive function $\mathfrak{S} : \Delta \longrightarrow \Delta$ with $\varepsilon(\mathfrak{S}\chi_1, \mathfrak{S}\chi_2) \leqslant \iota\varepsilon(\chi_1,\chi_2)$, where $\iota < 1$. If we get a $s_0 \in \mathbb{N}$ s.t. $\varepsilon(\mathfrak{S}^s\chi_1, \mathfrak{S}^{s+1}\chi_1) < \infty$, for every $s \geqslant s_0$, then, we have that*

- *the fixed point χ_2^* of $\mathfrak{S}$ is the convergence point of $\{\mathfrak{S}^s\chi_1\}$;*
- *χ_2^* is the unique fixed point of $\mathfrak{S}$ in $\{\chi_2 \in \Delta \mid \varepsilon(\mathfrak{S}^{\rho_0}\chi_1, \chi_2) < \infty\}$;*
- *$(1-\iota)\varepsilon(\chi_2, \chi_2^*) \leqslant \varepsilon(\chi_2, \mathfrak{S}\chi_2)$ for every $\chi_2 \in \Delta$.*

Now, consider (3.3) as defined by

$$u'' + \frac{1}{2kc}u^2 + \frac{k^2+c^2}{k^2c^2}u = 0. \tag{5.1}$$

Theorem 5.2. *Every solution $\psi : [0,+\infty) \to [0,+\infty)$ of (5.1) is bounded.*

Proof. Multiplying (5.1) by $u'(\Theta)$ and integrating it from 0 to Θ, we get

$$\frac{1}{2}(u')^2 + \frac{1}{6kc}u^3 + \frac{k^2+c^2}{2k^2c^2}u^2 = b, \tag{5.2}$$

where b is the integral constant. Now, we get

$$\begin{aligned} \frac{k^2+c^2}{2k^2c^2}u^2 &\leqslant \frac{k^2+c^2}{2k^2c^2}u^2 + \frac{1}{2}(u')^2 \\ &= -\frac{1}{6kc}u^3 + b. \end{aligned}$$

Assume $b := \frac{1}{6kc}a^3 + \frac{k^2-c^2}{2k^2c^2}a^2$, where $a \in \mathbb{R}$.

Therefore, we have

$$\begin{aligned}
&\frac{k^2+c^2}{2k^2c^2}u^2+\frac{1}{6kc}u^3-\frac{1}{6kc}a^3-\frac{k^2-c^2}{2k^2c^2}a^2\\
&\quad=\frac{k^2+c^2}{2k^2c^2}(u^2-a^2)+\frac{1}{6kc}(u^3-a^3)\\
&\quad=\frac{k^2+c^2}{2k^2c^2}(u-a)(u+a)+\frac{1}{6kc}(u-a)(u^2+ua+a^2)\\
&\quad=(u-a)[\frac{k^2+c^2}{2k^2c^2}(u+a)+\frac{1}{6kc}(u^2+ua+a^2)]\\
&\quad=(u-a)[\frac{1}{6kc}u^2+(\frac{k^2+c^2}{2k^2c^2}+\frac{a}{6kc})u+\frac{k^2+c^2}{2k^2c^2}a+\frac{a^2}{6kc}]\\
&\quad=(u-a)\cdot\left(u+\frac{1}{3kc}\left[(\frac{k^2+c^2}{2k^2c^2})+\frac{a}{6kc}+\sqrt{(\frac{k^2+c^2}{2k^2c^2}+\frac{a}{6kc})^2-\frac{2}{3kc}(\frac{k^2+c^2}{2k^2c^2}a+\frac{a^2}{6kc})}\right]\right)\\
&\qquad\cdot\left(u+\frac{1}{3kc}\left[(\frac{k^2+c^2}{2k^2c^2})+\frac{a}{6kc}-\sqrt{(\frac{k^2+c^2}{2k^2c^2}+\frac{a}{6kc})^2-\frac{2}{3kc}(\frac{k^2+c^2}{2k^2c^2}a+\frac{a^2}{6kc})}\right]\right).
\end{aligned}$$

Thus,

$$(u-a)\cdot\left(u^2+\frac{1}{3kc}\left[(\frac{k^2+c^2}{2k^2c^2})+\frac{a}{6kc}\pm\sqrt{\frac{1}{18}(3-2a)(3k^2+3c^2+ak^2)}\right]\right)\leqslant 0.$$

Therefore, there is a constant $M>0$, s.t. $|u|<M$. □

We now have the following theorem.

Theorem 5.3. *Consider $\psi\in C[0,\infty)$ which satisfies the following inequality*

$$\left\|u''+\frac{1}{2kc}u^2+\frac{k^2+c^2}{k^2c^2}u\right\|\leqslant\Psi(\Theta),\qquad \Theta\in[0,\infty),\tag{5.3}$$

and also, assume

$$\int_0^\Theta\Psi(s)ds\leqslant\aleph\Psi(\Theta)\ \ \textit{for some}\ \ 0<\aleph<1,\tag{5.4}$$

where $\Theta\in[0,\infty)$, $\Psi:[0,\infty)\to(0,\infty)$ is continuous.

Assume $\aleph^2\left[\frac{k^2+c^2}{k^2c^2}+|\frac{M}{kc}|\right]<1$, in which $0<\aleph<1$, and $kc<0$. Therefore, we can obtain a $u_\circ\in C[0,\infty)$, s.t.

$$u_\circ(\Theta)=-\int_0^\Theta\int_0^\tau\left(\frac{1}{2kc}u^2(s)+\frac{k^2+c^2}{k^2c^2}u(s)\right)dsd\tau,\tag{5.5}$$

and

$$\|u(\Theta)-u_\circ(\Theta)\|\leqslant\frac{1}{1-\aleph^2\left[\frac{k^2+c^2}{k^2c^2}+|\frac{M}{kc}|\right]}\Psi(\Theta),\tag{5.6}$$

where $\Theta\in[0,\infty)$, $0<\aleph<1$.

Proof. Assume $\Delta := C[0,\infty)$, and consider a mapping $\varepsilon : \Delta \longrightarrow [0,\infty]$, defined by

$$\varepsilon(u(\Theta),\widehat{u}(\Theta)) = \inf\Big\{\Omega \geqslant 0 : \|u(\Theta)-\widehat{u}(\Theta)\| \leqslant \Omega\Psi(\Theta),\ \Theta \in [0,\infty)\Big\}. \tag{5.7}$$

Now, define $\mathfrak{S} : \Delta \longrightarrow \Delta$ as

$$\mathfrak{S}u(\Theta) = -\int_0^{\Theta}\int_0^{\tau}\left(\frac{1}{2kc}u^2(s) + \frac{k^2+c^2}{k^2c^2}u(s)\right)dsd\tau. \tag{5.8}$$

We prove $\mathfrak{S}$ is contractive on Δ. Assume $u,\widehat{u} \in \Delta, \Omega \geqslant 0$, and $\varepsilon(u(\Theta),\widehat{u}(\Theta)) \leqslant \Omega$.

For each $\Theta \in [0,\infty)$,

$$\begin{aligned}
\|\mathfrak{S}u(\Theta)-\mathfrak{S}\widehat{u}(\Theta)\| &\leqslant \int_0^{\Theta}\int_0^{\tau}\left(\frac{k^2+c^2}{k^2c^2}\Big\|u(s)-\widehat{u}(s)\Big\| + |\frac{1}{2kc}|\Big\|u^2(s)-\widehat{u}^2(s)\Big\|\right)dsd\tau\\
&\leqslant \int_0^{\Theta}\int_0^{\tau}\left(\frac{k^2+c^2}{k^2c^2}\Big\|u(s)-\widehat{u}(s)\Big\| + |\frac{1}{2kc}|\Big\|(u(s)-\widehat{u}(s))(u(s)+\widehat{u}(s))\Big\|\right)dsd\tau\\
&\leqslant \int_0^{\Theta}\int_0^{\tau}\left(\Big\|u(s)-\widehat{u}(s)\Big\|\left[\frac{k^2+c^2}{k^2c^2} + |\frac{1}{2kc}|(\|u(s)\|+\|\widehat{u}(s)\|)\right]\right)dsd\tau\\
&\leqslant \int_0^{\Theta}\int_0^{\tau}\left(\Omega\left[\frac{k^2+c^2}{k^2c^2} + |\frac{1}{kc}|M\right]\Psi(s)\right)dsd\tau\\
&\leqslant \aleph^2\Omega\left[\frac{k^2+c^2}{k^2c^2} + |\frac{M}{kc}|\right]\Psi(\Theta).
\end{aligned}$$

in which $0 < \aleph < 1$, and $M > 0$.

Thus, we get

$$\varepsilon(\mathfrak{S}\psi(\Theta),\mathfrak{S}\widehat{\psi}(\Theta)) \leqslant \aleph^2\left[\frac{k^2+c^2}{k^2c^2} + |\frac{M}{kc}|\right]\ \varepsilon(\psi(\Theta),\widehat{\psi}(\Theta)),$$

in which $0 < \aleph < 1$ and $\Theta \in [0,\infty)$. Therefore, we can conclude the contractively property of $\mathfrak{S}$, since $\aleph^2\left[\frac{k^2+c^2}{k^2c^2} + |\frac{M}{kc}|\right] < 1$.

By considering (5.3) and integrating it twice, we have

$$\left\|\psi(\Theta) + \int_0^{\Theta}\int_0^{\tau}\left(\frac{1}{2kc}u^2(s) + \frac{k^2+c^2}{k^2c^2}u(s)\right)dsd\tau\right\| \leqslant \aleph^2\Psi(\Theta), \qquad \Theta \in [0,\infty), \tag{5.9}$$

where $0 < \aleph < 1$. Therefore, we get

$$\varepsilon(\mathfrak{S}\psi(\Theta),\psi(\Theta)) < 1,$$

in which $\Theta \in [0,\infty)$. Then, the conditions of Theorem 5.1 are satisfied, and we have that

$$|\psi(\Theta)-\psi_\circ(\Theta)| \leqslant \frac{1}{1-\aleph^2\left[\frac{k^2+c^2}{k^2c^2} + |\frac{M}{kc}|\right]}\Psi(\Theta), \qquad \Theta \in [0,\infty), \tag{5.10}$$

where $\psi_\circ(\Theta) = -\int_0^{\Theta}\int_0^{\tau}\left(\frac{1}{2kc}u^2(s) + \frac{k^2+c^2}{k^2c^2}u(s)\right)dsd\tau$ is a unique solution in $\Big\{\upsilon \in \Delta : \varepsilon(\mathfrak{S}\psi_\circ,\upsilon) < \infty\Big\}$. $\square$

6. Conclusion

In this paper, we applied the Kudryashov methods to investigate the novel solutions to a nonlinear time fractional model. The 3D and 2D figures were depicted for displaying the physical behavior of travelling solutions for diverse values of uncertain parameters with constraint conditions. Also, via an alternative technique, we investigated the UHR stability of the obtained solutions.

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