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Damping influence on the critical velocity and response characteristics of structurally pre-stressed beam subjected to traveling harmonic load

B. Omolofe[a,*], T. O. Awodola[a], T. O. Adeloye[b]

[a]*Department of Mathematical Sciences, School of Sciences, Federal University of Technology, P. M. B 704, Akure Ondo State, Nigeria.*
[b]*Department of Mathematics, Faculty of Basic Sciences, Nigeria Maritime University, Okerenkoko, Delta State, Nigeria.*

Abstract

In this present study, the response characteristics of a flexible member carrying harmonic moving load are investigated. The beam is assumed to be of uniform cross section and has simple support at both ends. The moving concentrated force is assumed to move with constant velocity type of motion. A versatile mathematical approximation technique often used in structural mechanics called assumed mode method is in first instance used to treat the fourth order partial differential equation governing the motion of the slender member to obtain a sequence of second order ordinary differential equations. Integral transform method is further used to treat this sequence of differential equations describing the motion of the beam-load system. Various results in plotted curves show that, the presence of the vital structural parameters such as the axial force N, rotatory inertia correction factor r^0, the foundation modulus F_0, and the shear modulus G_0, significantly enhances the stability of the beam when under the action of moving load. Dynamic effects of these parameters on the critical speed of the dynamical system are carefully studied. It is found that as the values of these parameters increase, the critical speed also increases. Thereby reducing the risk of resonance and thus the safety of the occupant of this structural member is guaranteed.

Keywords: Response characteristics, flexural member, harmonic load, critical speed, resonance, foundation stiffness, assumed mode, concentrated force.

2010 MSC: 82D40, 74F15, 74H45, 74K20.

1. Introduction

The amplitude of the dynamic deflection and vibration control of beam-like structures carrying moving masses have long been an exciting subject and of great fundamental importance to many researchers. In fact, it is one of the most important subjects in the area of structural dynamics and vibration control. This is due largely to its enormous applications in engineering sciences. In particular, tracks on which vehicle or train travels, fluid-conveying pipe system, beams under the actions of pressure waves and shafts

*Corresponding author
Email addresses: babatope_omolofe@yahoo.com (B. Omolofe), toawodola@yahoo.com (T. O. Awodola), adeyel@yahoo.com (T. O. Adeloye)

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or machining operations which requires axial motions can all be model as continuum systems continuously supporting or carrying moving sub-systems. Bridges, railway bridges, cranes, cable ways, tunnels, and pipes are the typical structural examples of the structure to be designed to support moving masses and loads.

Consequently, there is a large volume of literature devoted to moving load problems in the past few years see for instance [2, 4–6, 9, 10, 14, 19–22, 24, 28, 29, 31, 32, 34–38, 40] and the references therein. The theoretical and experimental studies of the moving load problems have shown that moving loads may be divided into three categories namely: moving oscillator, moving mass, and moving force. Vibrations of beams due to moving oscillators are studied in [14, 17, 23, 33, 39], vibrations of beams due to a moving mass are investigated in [3, 4, 13, 16, 25, 36, 41], and vibration of beams due to moving force have been considered in [1, 8, 15, 18, 27, 40]. Moving load problem with or without elastic foundation has been extensively studied and so many other aspects of the moving load problem have also been considerably explored [7, 11, 12, 21, 26, 30, 34].

In this present study an approximate analytical solution of the transverse response of a simply supported Rayleigh beam resting on variable two-parameter elastic sub-grade and carrying harmonic variable load traveling with constant velocity type of motion is obtained. Effects of internal and external damping on the dynamic characteristics of a beam-like structural member carrying moving load are well studied. Effects of these and some other vital structural parameters on the critical speed of this vibrating system will also be established.

2. Mathematical formulation

Consider a structurally prestressed Rayleigh beam under the actions of traveling load of mass M. The span length of the beam is finite and the mass is assumed to travel along the beam with a constant velocity. Considering damping effects, the transverse displacement in terms of traveling time t and the spatial coordinate is prescribed by the fourth order partial differential equation given by

$$\begin{aligned} EI\frac{\partial^4 W(x,t)}{\partial x^4} - N\frac{\partial^2 W(x,t)}{\partial x^2} + \mu\frac{\partial^2 W(x,t)}{\partial t^2} - \mu r^0\frac{\partial^4 W(x,t)}{\partial x^2\partial t^2} \\ + F(x)W(x,t) + D_e\frac{\partial W(x,t)}{\partial t} + D_i\frac{\partial^5 W(x,t)}{\partial t\partial x^4} - \frac{\partial^4}{\partial}\left[G(x)\frac{\partial W(x,t)}{\partial x}\right] = P(x,t). \end{aligned} \tag{2.1}$$

Where EI is the flexural rigidity of the beam, μ is the mass per unit length of the beam, and $F(x)$ and $G(x)$ are the variable foundation stiffness and shear modulus, respectively. $W(x)$ is the deflection of the beam at point x and time t . $P(x,t)$ denotes the traveling load. A prime denotes differentiation with respect to position coordinate x and an over-dot represents differentiation with respect to time t . The external damping D_e and internal damping D_i of the beam are taken to be proportional to the mass and stiffness of the beam respectively and are given as

$$D_e = \eta_e\mu, \quad D_i = \eta_i EI,$$

where η_e and η_i are the constants of proportionality.

In this study, it is assumed that the load function $P(x,t)$ is given in the form

$$P(x,t) = h(x)\cos wt,$$

where $h(x)$ is an arbitrary deterministic distributed load.

Now, considering the action of a concentrated harmonic force $H(t)$ at a position, $x = ut$. The load $P(x,t)$ can be written as

$$P(x,t) = H_0\cos wt\delta(x-ut),$$

$\delta(\cdot)$ is the well-known Dirac delta function with the property

$$\int_a^b \delta(x-k)f(x)dx = \begin{cases} 0, & \text{for } k<a<b, \\ f(k), & \text{for } a<k<b, \\ 0, & \text{for } a<b<k. \end{cases} \tag{2.2}$$

The variable foundation stiffness and shear modulus are respectively taken to be

$$F(x) = F_0(4x - 3x^2 + x^3)$$

and

$$G(x) = G_0(12 - 13x + 6x^2 - x^3).$$

It is remarked here that the beam under consideration is assumed to have simple support at both ends $x = 0$ and $x = L$. Thus boundary conditions are given as

$$W(0,t) = 0 = W(L,t), \frac{\partial W(0,t)}{\partial x} = 0 = \frac{\partial^2 W(L,t)}{\partial x^2}$$

and the initial conditions is given as

$$W(0,t) = 0 = \frac{\partial W(x,0)}{\partial t}. \tag{2.3}$$

Substituting equation (2.2) into (2.1) we have

$$\begin{aligned} EI\frac{\partial^4 W(x,t)}{\partial x^4} - N\frac{\partial^2 W(x,t)}{\partial x^2} + \mu\frac{\partial^2 W(x,t)}{\partial t^2} - \mu r^0\frac{\partial^4 W(x,t)}{\partial x^2 \partial t^2} + F(x)W(x,t) \\ + D_e\frac{\partial W(x,t)}{\partial t} + D_i\frac{\partial^5 W(x,t)}{\partial t \partial x^4} - \frac{\partial}{\partial x}\left[G(x)\frac{\partial W(x,t)}{\partial x}\right] = H_0 \cos wt\delta(x-ut). \end{aligned} \tag{2.4}$$

Equation (2.4) is the fourth order partial differential equation governing the flexural motion of the Structurally prestressed Rayleigh beam.

3. Solution procedures

To find an approximate solution of the boundary-initial-value problem (2.1), an assumed mode method is employed. By this method, the j^{th} term approximate solution of (2.4) is sought in the form.

$$W_j(x,t) = \sum_{m=1}^{\infty} Z_m(t)U_m(x), \tag{3.1}$$

where $Z_m(t)$ are coordinates in modal space and $U_m(x)$ are the normal modes of vibration written as

$$U_m(x) = \sin\frac{\lambda_m x}{L} + A_m \cos\frac{\lambda_m x}{L} + B_m \sinh\frac{\lambda_m x}{L} + C_m \cosh\frac{\lambda_m x}{L}. \tag{3.2}$$

It can be shown that, for a structural member having simple supports at ends $x = 0$ and $x = L$ equation (3.1) in view of equation (3.2) can be written as

$$W_j(x,t) = \sum_{m=1}^{\infty} Z_m(t) \sin\frac{m\pi x}{L}. \tag{3.3}$$

Substituting equation (3.3) into the governing equation (2.4), one obtains

$$EI\left(\frac{m\pi}{L}\right)^4 \sum_{m=1}^{\infty} Z_m(t)\sin\frac{m\pi x}{L} + N\left(\frac{m\pi}{L}\right)^2 \sum_{m=1}^{\infty} Z_m(t)\sin\frac{m\pi x}{L} + \mu \sum_{m=1}^{\infty} \ddot{Z}_m(t)\sin\frac{m\pi x}{L}$$

$$+\mu r^0\left(\frac{m\pi}{L}\right)^2\sum_{m=1}^{\infty}\ddot{Z}_m(t)\sin\frac{m\pi x}{L}+F_0(4x-3x^2+x^3)\sum_{m=1}^{\infty}Z_m(t)\sin\frac{m\pi x}{L}+D_e\sum_{m=1}^{\infty}\dot{Z}_m(t)\sin\frac{m\pi x}{L}$$
$$+D_i\left(\frac{m\pi}{L}\right)^4\sum_{m=1}^{\infty}\dot{Z}_m(t)\sin\frac{m\pi x}{L}+G_0(12-13x+6x^2)\left(\frac{m\pi}{L}\right)^2\sum_{m=1}^{\infty}Z_m(t)\sin\frac{m\pi x}{L}$$
$$+G_0(13-12x+3x^2)\frac{m\pi}{L}\sum_{m=1}^{\infty}Z_m(t)\cos\frac{m\pi x}{L}=H_0\cos wt\delta(x-ut),$$

which after some re-arrangements gives

$$\begin{aligned}\sum_{m=1}^{\infty}&\left\{\left(\mu\sin\frac{m\pi x}{L}+\mu r^0\left(\frac{m\pi}{L}\right)^2\sin\frac{m\pi x}{L}\right)\ddot{Z}_m(t)+\left(D_e\sin\frac{m\pi x}{L}+D_i\left(\frac{m\pi}{L}\right)^4\sin\frac{m\pi x}{L}\right)\dot{Z}_m(t)\right.\\&+\left(EI\left(\frac{m\pi}{L}\right)^4\sin\frac{m\pi x}{L}+N\left(\frac{m\pi}{L}\right)^2\sin\frac{m\pi x}{L}+F_0(4x-3x^2+x^3)\sin\frac{m\pi x}{L}\right.\\&\left.\left.+G_0(12-13x+6x^2-x^3)\left(\frac{m\pi}{L}\right)^2\sin\frac{m\pi x}{L}+G_0(12-13x+6x^2-x^3)\frac{m\pi x}{L}\sin\frac{m\pi x}{L}\right)Z_m(t)\right\}\\&-H_0\cos wt\delta(x-ut)=0.\end{aligned}\tag{3.4}$$

In order to determine the expression for $Z_m(t)$, it is required that the expression on the LHS of equation (3.4) be orthogonal to the function $\sin\frac{k\pi x}{L}$. Thus, multiplying equation (3.4) by $\sin\frac{k\pi x}{L}$ and integrating from $x=0$ to $x=L$ leads to

$$\begin{aligned}\int_0^L&\left[\sum_{m=1}^{\infty}\left\{\left(\mu\sin\frac{m\pi x}{L}+\mu r^0\left(\frac{m\pi}{L}\right)^2\sin\frac{m\pi x}{L}\right)\ddot{Z}_m(t)+\left(D_e\sin\frac{m\pi x}{L}+D_i\left(\frac{m\pi}{L}\right)^4\sin\frac{m\pi x}{L}\right)\dot{Z}_m(t)\right.\right.\\&+\left(EI\left(\frac{m\pi}{L}\right)^4\sin\frac{m\pi x}{L}+N\left(\frac{m\pi}{L}\right)^2\sin\frac{m\pi x}{L}+F_0(4x-3x^2+x^3)\sin\frac{m\pi x}{L}\right.\\&\left.\left.+G_0(12-13x+6x^2-x^3)\left(\frac{m\pi}{L}\right)^2\sin\frac{m\pi x}{L}+G_0(12-13x+6x^2-x^3)\frac{m\pi x}{L}\sin\frac{m\pi x}{L}\right)Z_m(t)\right\}\\&\left.-H_0\cos wt\delta(x-ut)\right]\sin\frac{k\pi x}{L}dx=0,\end{aligned}$$

which after some simplifications yields

$$\begin{aligned}\sum_{m=1}^{\infty}&\left\{(\vartheta_1+\vartheta_2)\ddot{Z}_m(t)+(\vartheta_3+\vartheta_4)\dot{Z}_m(t)+(\vartheta_5+\vartheta_6+\vartheta_7+\vartheta_8+\vartheta_9)Z_m(t)\right\}\\&=\int_0^L H_0\cos wt\delta(x-ut)\sin\frac{k\pi x}{L}dx,\end{aligned}\tag{3.5}$$

where

$$\vartheta_1=\mu\int_0^L\sin\frac{m\pi x}{L}\sin\frac{k\pi x}{L}dx,\qquad \vartheta_2=\mu r^0\left(\frac{m\pi}{L}\right)^2\int_0^L\sin\frac{m\pi x}{L}\sin\frac{k\pi x}{L}dx,$$
$$\vartheta_3=D_1\int_0^L\sin\frac{m\pi x}{L}\sin\frac{k\pi x}{L}dx,\qquad \vartheta_4=D_2\left(\frac{m\pi}{L}\right)^4\int_0^L\sin\frac{m\pi x}{L}\sin\frac{k\pi x}{L}dx,$$
$$\vartheta_5=EI\left(\frac{m\pi}{L}\right)^4\int_0^L\sin\frac{m\pi x}{L}\sin\frac{k\pi x}{L}dx,\qquad \vartheta_6=N\left(\frac{m\pi}{L}\right)^2\int_0^L\sin\frac{m\pi x}{L}\sin\frac{k\pi x}{L}dx,$$

$$\vartheta_7 = \int_0^L F_0(4x - 3x^2 + x^3) \sin \frac{m\pi x}{L} \sin \frac{k\pi x}{L} dx, \qquad \vartheta_8 = \left(\frac{m\pi}{L}\right)^2 \int_0^L G_0(12 - 13x + 6x^2) \sin \frac{m\pi x}{L} \sin \frac{k\pi x}{L} dx,$$

$$\vartheta_9 = \frac{m\pi}{L} \int_0^L G_0(13 - 12x + 3x^2) \cos \frac{m\pi x}{L} \sin \frac{k\pi x}{L} dx.$$

Noting property (2.2) and considering the m^{th} particle of the vibrating system, equation (3.5) can then be written after some rearrangements and simplifications as

$$\ddot{Z}_m(t) + Q_1 Z_m(t) + Q_2 Z_m(t) = \frac{Q_4}{2}[\sin \alpha t - \sin \beta t], \tag{3.6}$$

where,

$$Q_1 = \frac{\vartheta_3 + \vartheta_4}{\vartheta_1 + \vartheta_2},\ Q_2 = \frac{\vartheta_5 + \vartheta_6\vartheta_7 + \vartheta_8\vartheta_9}{\vartheta_1 + \vartheta_2},\ Q_3 = \vartheta_1 + \vartheta_2,\ Q_4 = \frac{H_0}{Q_3},\ \alpha = \omega + \frac{k\pi u}{L},\ \beta = \omega - \frac{k\pi u}{L}.$$

To obtain solution of the second order ordinary differential equation (3.6) above, we subjected it to a Laplace transform defined as

$$\dot{\sim} = \int_0^\infty (\cdot) e^{-st} dt,$$

where s is the Laplace parameter. Invoking the initial condition (2.3), one obtains the simple algebraic expression given as

$$Z_m(s) = \frac{\frac{Q_4}{2}\left[\frac{\alpha}{s^2+\alpha^2} - \frac{\beta}{s^2+\beta^2}\right]}{s^2 + sQ_1 + Q_2},$$

which is further simplified to give

$$Z_m(s) = \frac{Q_4}{2(b_1 - b_2)} \left\{ \frac{\alpha}{s^2 + \alpha^2} . \frac{1}{s - b_1} - \frac{\alpha}{s^2 + \alpha^2} . \frac{1}{s - b_2} - \frac{\beta}{s^2 + \beta^2} . \frac{1}{s - b_1} + \frac{\beta}{s^2 + \beta^2} . \frac{1}{s - b_2} \right\}, \tag{3.7}$$

where

$$b_1 = \frac{-Q_1 + \sqrt{Q_1^2 - 4Q_2}}{2},\ b_2 = \frac{-Q_1 - \sqrt{Q_1^2 - 4Q_2}}{2}.$$

In order to obtain the Laplace inversion of (3.7), the following representations are made

$$f_1(t) = \sin \alpha t,\ f_2(t) = \sin \beta t, \qquad g_1(t) = e^{b_1 t},\ g_2(t) = e^{b_2 t}.$$

So that the Laplace inversion of (3.7) is the convolution of f_i's and g_i's defined by

$$f_i * g_i = \int_0^t f_i(t-u) g_i(u) du, \quad i = 1, 2, 3, \ldots.$$

Thus, Laplace inversion of (3.7) is given as

$$Z_m(t) = \frac{Q_4}{2(b_1 - b_2)} \{I_A - I_B - I_C + I_D\}, \tag{3.8}$$

where

$$\begin{aligned} I_A &= \int_0^t (\sin \alpha t \cos \alpha u - \cos \alpha t \sin \alpha u) e^{b_1 u} du, \quad I_B = \int_0^t (\sin \alpha t \cos \alpha u - \cos \alpha t \sin \alpha u) e^{b_2 u} du, \\ I_C &= \int_0^t (\sin \beta t \cos \beta u - \cos \beta t \sin \beta u) e^{b_1 u} du, \quad I_D = \int_0^t (\sin \beta t \cos \beta u - \cos \beta t \sin \beta u) e^{b_2 u} du. \end{aligned} \tag{3.9}$$

Evaluating the integrals (3.9), yields

$$I_A = \frac{1}{\alpha^2 + b_1^2}\{\alpha e^{b_1 t} - \alpha \cos \alpha t - b_1 \sin \alpha t\}, \quad I_B = \frac{1}{\alpha^2 + b_2^2}\{\alpha e^{b_2 t} - \alpha \cos \alpha t - b_2 \sin \alpha t\},$$
$$I_C = \frac{1}{\beta^2 + b_1^2}\{\beta e^{b_1 t} - \beta \cos \beta t - b_1 \sin \beta t\}, \quad I_D = \frac{1}{\beta^2 + b_2^2}\{\beta e^{b_2 t} - \beta \cos \beta t - b_2 \sin \beta t\}.$$

Thus, equation (3.8) in view of (3.9) leads to

$$\begin{aligned} Z_m(t) = \frac{Q_4}{2(b_1 - b_2)} \Bigg\{ & \frac{\alpha e^{b_1 t} - \alpha \cos \alpha t - b_1 \sin \alpha t}{\alpha^2 + b_1^2} - \frac{\alpha e^{b_2 t} - \alpha \cos \alpha t - b_2 \sin \alpha t}{\alpha^2 + b_2^2} \\ & - \frac{\beta e^{b_1 t} - \beta \cos \beta t - b_1 \sin \beta t}{\beta^2 + b_1^2} + \frac{\beta e^{b_2 t} - \beta \cos \beta t - b_2 \sin \beta t}{\beta^2 + b_2^2} \Bigg\}. \end{aligned} \tag{3.10}$$

Thus, in view of (3.3), taking into account (3.10) one obtains

$$\begin{aligned} W(x,t) = \sum_{m=1}^{j} \frac{Q_4}{2(b_1 - b_2)} \Bigg\{ & \frac{\alpha e^{b_1 t} - \alpha \cos \alpha t - b_1 \sin \alpha t}{\alpha^2 + b_1^2} - \frac{\alpha e^{b_2 t} - \alpha \cos \alpha t - b_2 \sin \alpha t}{\alpha^2 + b_2^2} \\ & - \frac{\beta e^{b_1 t} - \beta \cos \beta t - b_1 \sin \beta t}{\beta^2 + b_1^2} + \frac{\beta e^{b_2 t} - \beta \cos \beta t - b_2 \sin \beta t}{\beta^2 + b_2^2} \Bigg\} . \sin \frac{m\pi x}{L}, \end{aligned} \tag{3.11}$$

which represents the transverse response of structurally prestressed slender member to harmonic moving loads.

4. Comments on the closed-form solution

Occurrence of resonance in a dynamical system is of a great concern in design engineering and engineering analysis as the amplitude of vibration of a structural member carrying travelling loads may grow without bound. Thus, this section seeks to examine and establish the conditions under which this unpleasant phenomenon may occur. It is clearly seen from equation (3.11) that a structurally damped system considered in this study will experience a state of resonance under any of the following stated conditions:

$$\begin{aligned} &\text{i.} \quad b_1\alpha^2 + b_1^3 = b_2\alpha^2 + b_2 b_1^2, \\ &\text{ii.} \quad b_2\alpha^2 + b_2^3 = b_1\alpha^2 + b_1 b_2^2, \\ &\text{iii.} \quad b_1\beta^2 + b_1^3 = b_2\beta^2 + b_2 b_1^2, \\ &\text{iv.} \quad b_2\beta^2 + b_2^3 = b_1\beta^2 + b_1 b_2^2 \end{aligned} \tag{4.1}$$

and the velocity, at which this phenomenon may occur termed the critical velocity associated with the conditions listed in (4.1) respectively are given as

$$u_{cr}^1 = \frac{L}{2\pi k}\sqrt{8Q_2 + 2\sqrt{Q_1^2 - 4Q_2 - 3Q_1^2}} - \frac{\omega L}{k\pi}, \quad u_{cr}^2 = \frac{L}{2\pi k}\sqrt{8Q_2 - 2\sqrt{Q_1^2 - 4Q_2 - 3Q_1^2}} - \frac{\omega L}{k\pi},$$
$$u_{cr}^3 = \frac{\omega L}{k\pi} - \frac{L}{2\pi k}\sqrt{8Q_2 + 2\sqrt{Q_1^2 - 4Q_2 - 3Q_1^2}}, \quad u_{cr}^4 = \frac{\omega L}{k\pi} - \frac{L}{2\pi k}\sqrt{8Q_2 - 2\sqrt{Q_1^2 - 4Q_2 - 3Q_1^2}},$$

where all the parameters are as previously defined.

5. Numerical Result and discussion

In this section, the analysis proposed in the previous sections are illustrated by considering a homogeneous beam of modulus of elasticity $E = 2.9012 \times 10^9 N/M^2$, the moment of inertial $I = 2.87698 \times 10^{-3} kgm^2$, the beam span $L = 12.192m$ and the mass per unit length of the beam $\mu = 2758.291 kg/m$. The load is also assumed to travel along the beam with constant velocity $V = 3.128 m/s$, the values of foundation moduli K_0 are varied between $0N/m^3$ and $4 \times 10^8 N/m^3$, the values of axial force N varied between 0N and $2.0 \times 10^{11} N$. The values of the shear modulus G_0 are varied between $0N/m^3$ and $4 \times 10^8 N/m^3$ and the values of the rotatory inertia correction factor r^0 varied between $0N/m^3$ and $5.5 \times 10^5 N/m^3$.

Figure 1 displays the transverse displacement response of a simply supported uniform beam under the action of harmonic forces traveling at constant velocity for the various values of axial force N and for fixed values of subgrade moduli $F_0 = 40000$ and shear modulus $G_0 = 30000$ and rotatory inertia $r^0 = 0.5$. The figures show that as N increases, the response amplitude of the uniform beam decreases. For various traveling time t, the displacement response of the beam for various values of subgrade moduli F_0 and for fixed values of axial force $N = 20000$, shear modulus $G_0 = 30000$ and $r^0 = 0.5$ are shown in Figure 2. It is observed that the higher the values of subgrade moduli F_0 the smaller the response amplitude of the vibrating beam. Figure 3 displays the deflection profile of the simply supported uniform beam under harmonic forces traveling at constant velocity for various values of shear modulus G_0 and fixed values of axial force $N = 20000$, subgrade moduli $F_0 = 40000$ and $r^0 = 0.5$. It is seen from the figure that as the values of the shear modulus increases the deflection of the beam decreases significantly. The response of the elastic beam to the traveling harmonic forces for various values of the load position coordinate x and for fixed values of other parameters is displayed in Figure 4. It is deduced from the figure that the dynamic deflection at the mid-span of the beam is very large compare to other load positions. Figure 5 displays the deflection profile of the simply supported uniform beams subjected to forces traveling at constant velocity for various values of rotatory inertial correction factor r^0 and fixed values of axial force $N = 20000$, subgrade moduli $F_0 = 40000$ and shear modulus $G_0 = 30000$. The figure clearly shows that as the value of the rotatory inertia r^0 increases the deflection of the simply supported uniform beam under the action of moving forces traveling at constant velocity decreases.

Figures 6 and 7 depict the response amplitude of a simply supported uniform beam under the action of harmonic forces traveling at constant velocity for various values of external and internal damping D_d and D_i and for fixed values of axial force $N = 20000$, subgrade moduli $F_0 = 40000$ and shear modulus $G_0 = 40000$. The figures show that higher values of these parameters reduce the deflection of the beam considerably. For various values of circular frequency ω and for fixed values of other parameters, Figure 8 depicts the deflection profile of the vibrating beam. The figure clearly shows that the higher the value of the circular frequency the lower the deflection of the beam. The transverse response of the elastic uniform beam to the traveling load for various load velocities is presented in Figure 9. It is shown from the figure that the higher the speed of the traveling load the larger the deflection of the structural member.

6. Concluding remarks

The problem of the response of elastic beam carrying traveling variable magnitude load is investigated in this study. Solution procedure, involving assumed mode method and integral transform method is developed to obtain exact solution to the fourth order partial differential equation describing the motion of the beam-load system. Various results in plotted curves show that, the presence of some vital structural parameters such as the axial force N, rotatory inertia correction factor r^0, the foundation modulus F_0 and the shear modulus G_0 significantly enhances the stability of the beam when under the actions of the fast traveling load. Conditions under which the beam-load system will experience resonance phenomenon are also established. The speeds at which this may occur are also established.

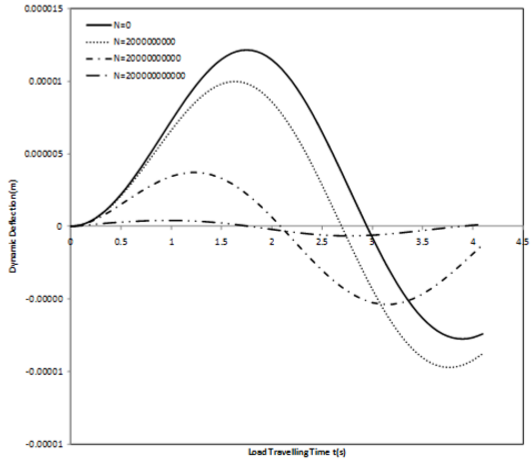

Figure 1: Transverse displacement response of a simply supported structural members resting on elastic foundation and under the actions of uniform partially distributed forces for various values of axial force N and for fixed values of $F_0 = 40000, G_0 = 30000$.

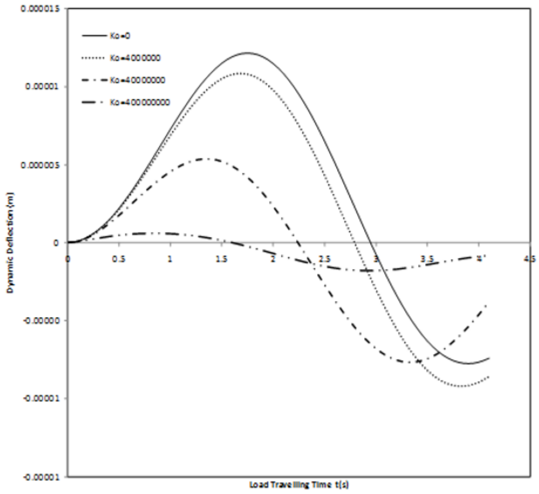

Figure 2: Transverse displacement response of a simply supported structural members resting on elastic foundation and under the actions of uniform partially distributed forces for various values of axial force K_0 and for fixed values of $F_0 = 40000, G_0 = 30000$.

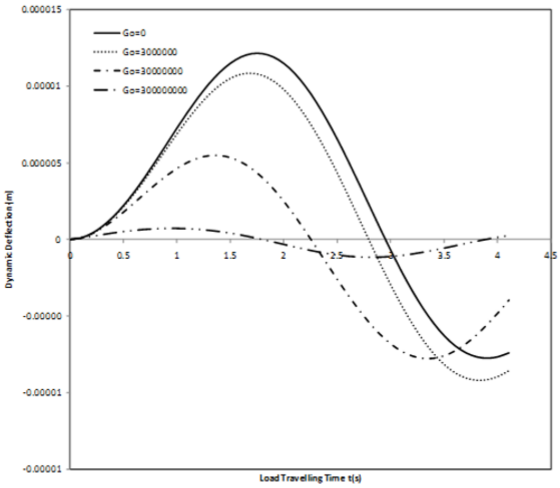

Figure 3: Transverse displacement response of a simply supported structural members resting on elastic foundation and under the actions of uniform partially distributed forces for various values of axial force G_0 and for fixed values of $F_0 = 40000, K_0 = 30000$.

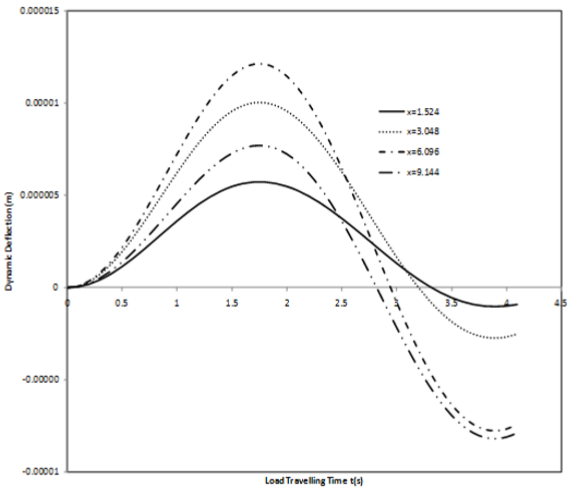

Figure 4: Response Amplitude of a simply supported structural members resting on elastic foundation and under the actions of uniform partially distributed forces for various values of the load position and for fixed values of $G_0 = 30000, F_0 = 40000$ and $N = 20000$.

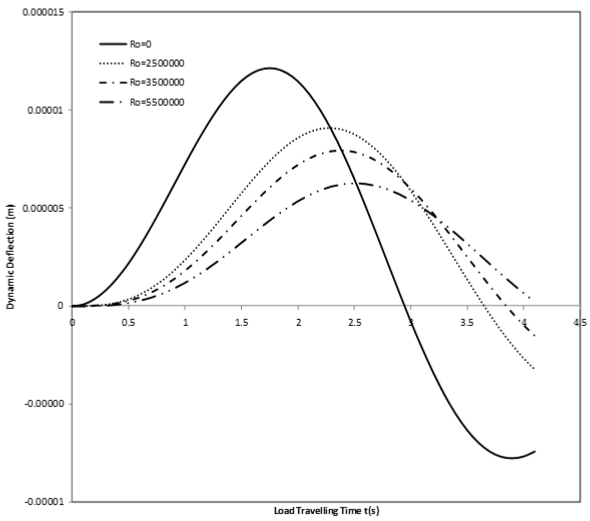

Figure 5: Response Amplitude of a simply supported structural members resting on elastic foundation and under the actions of uniform partially distributed forces for various values of the rotatory inertial r0 and for fixed values of $G_0 = 30000, F_0 = 40000$ and $N = 20000$.

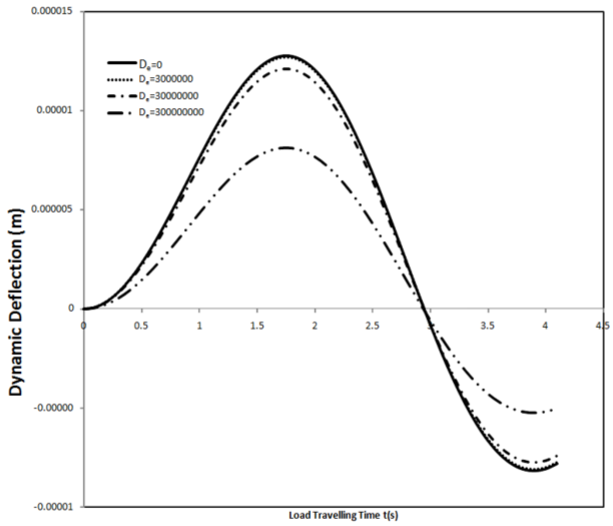

Figure 6: Response of a simply supported structural members resting on elastic foundation to uniform partially distributed forces for various values of external damping D_e and for fixed values of $G_0 = 30000, F_0 = 40000$ and $N = 20000$.

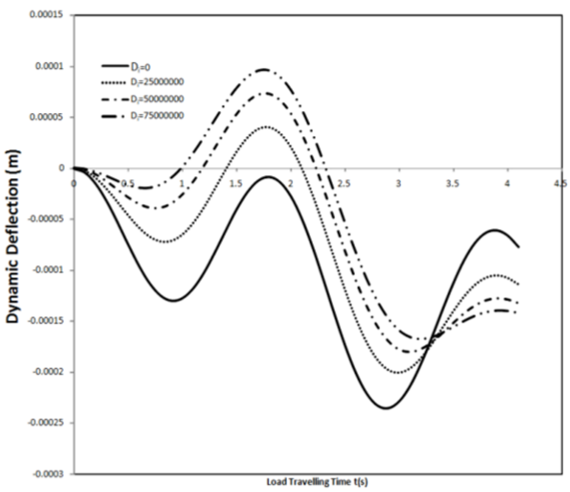

Figure 7: Response of a simply supported structural members resting on elastic foundation to uniform partially distributed forces for various values of internal damping D_i and for fixed values of $G_0 = 30000, F_0 = 40000$ and $N = 20000$.

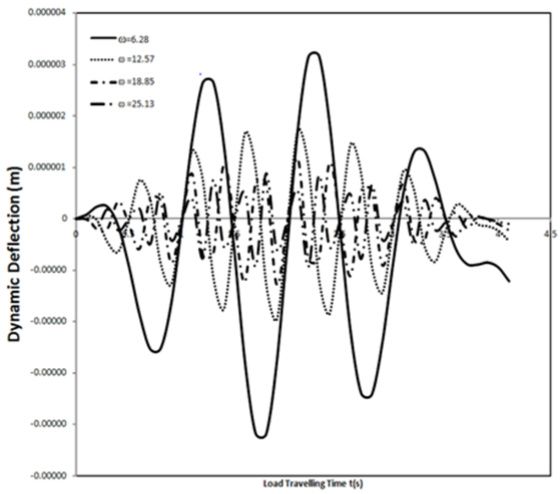

Figure 8: Response Amplitude of a simply supported structural members resting on elastic foundation and under the actions of uniform partially distributed forces for various values of the load circular fequency ω and for fixed values of $G_0 = 30000, F_0 = 40000$ and $N = 20000$.

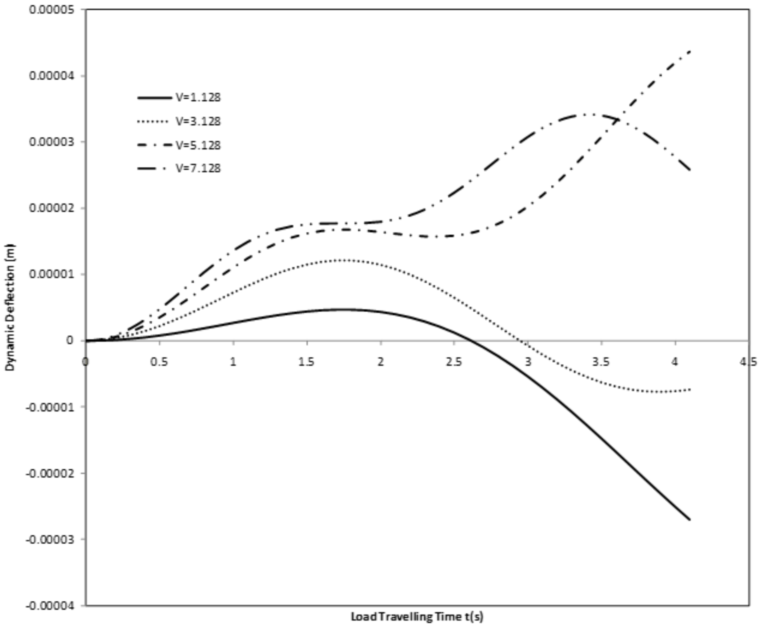

Figure 9: Response Amplitude of a simply supported structural members resting on elastic foundation and under the actions of uniform partially distributed forces for various values of the load velocity and for fixed values of $G_0 = 30000, F_0 = 40000$ and $N = 20000$.

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