Available online at www.isr-publications.com/mns
Math. Nat. Sci., 5 (2019), 20–28
Research Article

Online: ISSN 2600-7665
Mathematics in Natural Science
An International Journal
Journal Homepage: www.isr-publications.com/mns

Permanence and extinction in a periodic ratio-dependent population system with stage structure

Ahmadjan Muhammadhaji

College of Mathematics and Systems Science, Xinjiang University, Urumqi 830046, People's Republic of China.

Abstract

This paper studies a class of nonautonomous two-species ratio-dependent population system with stage structure. Some sufficient conditions on the boundedness, permanence, extinction, and periodic solution of the system are established by using the comparison method.

Keywords: Stage-structured ratio-dependent system, permanence, extinction, periodic solution.

2010 MSC: 34D20, 34D23, 34K20, 34K25.

1. Introduction and preliminaries

As we have well known, in recent years the population dynamical systems are extensively studied [1–16]. Especially, the nonautonomous ratio-dependent population dynamical systems has been extensively studied and excellent results were obtained[3, 4, 8–14]. Some of these studies described the dynamical interactions between species by ratio-dependent terms [1, 4, 7, 12]. For example, May in [7] first suggested the following set of equations:

$$\dot{u}(t) = r_1 u(t)\left[1 - u(t)(a_1 + b_1 v(t))^{-1} - c_1 u(t)\right], \quad \dot{v}(t) = r_2 v(t)\left[1 - v(t)(a_2 + b_2 u(t))^{-1} - c_2 v(t)\right], \tag{1.1}$$

to describe interactions of cooperation, where $u(t)$ and $v(t)$ represent the densities of two cooperative species u and v at time t, respectively.

Meanwhile, population models with stage structure have received much attention in recent years [6, 11, 14, 16]. Most of the authors have investigated population competitive system with stage structure, population predator-prey system with stage structure and population cooperative system with stage structure. Recently, Zhang et al. in [16], have studied the following nonautonomous stage-structured cooperative periodic system without delay

$$\begin{aligned} \dot{x}_1(t) &= \alpha(t)x_2(t) - r_1(t)x_1(t) - \beta(t)x_1(t) - \eta_1(t)x_1^2(t), \\ \dot{x}_2(t) &= \beta(t)x_1(t) - r_2(t)x_2(t) - \eta_2(t)x_2^2(t) + b(t)x_2(t)y(t), \\ \dot{y}(t) &= y(t)[R(t) - a(t)y(t) + c(t)x_2(t)]. \end{aligned} \tag{1.2}$$

Email address: ahmatjanam@aliyun.com (Ahmadjan Muhammadhaji)

doi: 10.22436/mns.05.01.03

Received: 2018-02-18 *Revised:* 2019-01-30 *Accepted:* 2019-02-30

By using the Mawhin's continuation theorem, the sufficient conditions on the existence of positive periodic solutions were established for system (1.1). Based on systems (1.1) and (1.2), the authors in [9] consider the following delayed ratio-dependent cooperative system with stage structure

$$\begin{aligned}
\dot{x}_1(t) &= r_1(t)x_2(t) - d_1(t)x_1(t) - r_1(t-\tau)e^{-\int_{t-\tau}^{t} d_1(s)ds}x_2(t-\tau),\\
\dot{x}_2(t) &= r_1(t-\tau)e^{-\int_{t-\tau}^{t} d_1(s)ds}x_2(t-\tau) - d_2(t)x_2^2(t) - c_1(t)x_2(t)\big(a_1(t)+b_1(t)y(t)\big)^{-1},\\
\dot{y}(t) &= y(t)\big[r_2(t) - d(t)y(t) - c_2(t)\big(a_2(t)+b_2(t)x_2(t)\big)^{-1}\big].
\end{aligned} \tag{1.3}$$

By using the comparison method, the authors in [9] have obtained some sufficient conditions on the permanence and extinction of system (1.3).

It is well known that, the environments of most natural populations undergo temporal variation, causing changes in the growth characteristics of these populations. One of the methods of incorporating temporal nonuniformity of the environments in models is to assume that the parameters are periodic functions of time, [8]. In fact, during the last decade, the dynamics of periodic nonautonomous population dynamical systems with stage-structures have been studied extensively in [2, 5, 13, 15] and the references cited therein. To the best of our knowledge, studies on the periodic ratio-dependent population system with stage structures have not been fully investigated.

Based on the above works and reasons, in this paper we propose and investigate the following nonautonomous ratio-dependent periodic population system with stage structure

$$\begin{aligned}
\dot{x}_1(t) &= \alpha_1(t)x_2(t) - r_1(t)x_1(t) - \beta_1(t)x_1(t) - \eta_1(t)x_1^2(t),\\
\dot{x}_2(t) &= \beta_1(t)x_1(t) - r_2(t)x_2(t) - \eta_2(t)x_2^2(t) + \frac{b_1(t)x_2(t)}{e_1(t)y_2(t)+\gamma_1(t)},\\
\dot{y}_1(t) &= \alpha_2(t)y_2(t) - c_1(t)y_1(t) - \beta_2(t)y_1(t) - d_1(t)y_1^2(t),\\
\dot{y}_2(t) &= \beta_2(t)y_1(t) - c_2(t)y_2(t) - d_2(t)y_2^2(t) + \frac{b_2(t)y_2(t)}{e_2(t)x_2(t)+\gamma_2(t)}.
\end{aligned} \tag{1.4}$$

Our main purpose is to establish some sufficient conditions on the boundedness, permanence, extinction, and periodic solution of system (1.4) by using the comparison method.

In system (1.4), x_1, x_2 represent immature and mature members of a species X while y_1, y_2 represent immature and mature members of a species Y. $x_1(t)$ and $y_1(t)$ represent the density of immaturity of species X and Y at time t, respectively, $x_2(t)$ and $y_2(t)$ represent the density of maturity of species X and Y at time t, respectively. $r_1(t)$ and $c_1(t)$ represent the death rate of the immature of species X and Y, respectively, and $r_2(t)$ and $c_2(t)$ represent the death rate of the mature of species X and Y, respectively. $\alpha_1(t)$ and $\alpha_2(t)$ represent the birth rate of species X and Y, respectively. $\beta_1(t)$ and $\beta_2(t)$ represent the change rate of species X and Y from the immature to mature, which is directly proportional to the density of the immature. The terms $\frac{b_1(t)x_2(t)}{e_1(t)y_2(t)+\gamma_1(t)}$ and $\frac{b_2(t)y_2(t)}{e_2(t)x_2(t)+\gamma_2(t)}$ characterize the interactions between species X and Y at time t.

In this paper, we always assume that

- (H_1) $r_i(t), \eta_i(t), \alpha_i(t), \beta_i(t), b_i(t), c_i(t), d_i(t)$ $(i = 1, 2)$ are all strictly positive ω-periodic continuous functions with $\omega > 0$;
- (H_2) $r_i(t), \eta_i(t), \alpha_i(t), \beta_i(t), b_i(t), c_i(t), d_i(t)$ $(i = 1, 2)$ are all strictly positive continuous functions.

From the viewpoint of mathematical biology, in this paper for system (1.4) we consider the solution with the following initial condition

$$x_i(t) = \phi_i(t), y_i(t) = \psi_i(t) \quad \text{for all} \quad t \in [0, +\infty),\ i = 1, 2, \tag{1.5}$$

where $\phi_i(t)$ $(i = 1, 2)$, $\psi_i(t)$ $(i = 1, 2)$ are nonnegative continuous functions defined on $[0, +\infty)$ satisfying $\phi_i(0) > 0$ $(i = 1, 2)$, $\psi_i(0) > 0$ $(i = 1, 2)$.

In this paper, for any ω-periodic continuous function $f(t)$ we denote

$$f^L = \min_{t\in[0,\omega]} f(t), \quad f^M = \max_{t\in[0,\omega]} f(t).$$

Now, we present some useful definition and lemmas.

Definition 1.1. System (1.4) is said to be permanent if there exist positive constants m, M, and T, such that each positive solution $(x_1(t), x_2(t), y_1(t), y_2(t))$ of system (1.4) with any positive initial value φ, fulfill $m \leqslant x_i(t) \leqslant M (i = 1, 2)$, $\quad m \leqslant y_i(t) \leqslant M (i = 1, 2)$ for all $t \geqslant T$, where T may depend on φ.

Lemma 1.2 ([3])**.** *If $a(t), b(t), c(t), d(t)$, and $f(t)$ are all ω-periodic, then system*

$$\dot{x}_1(t) = a(t)x_2(t) - b(t)x_1(t) - d(t)x_1^2(t), \quad \dot{x}_2(t) = c(t)x_1(t) - f(t)x_2^2(t),$$

has a positive ω-periodic solution $(x_1^*(t), x_2^*(t))$ which is globally asymptotically stable in $R_+^2 = \{(x_1, x_2) : x_1 > 0, x_2 > 0\}$.

Lemma 1.3 ([10])**.** *If there exist positive constants m and M for any $\Phi \in C_+^n[-\tau, 0]$ such that*

$$m < \liminf_{t\to\infty} x_i(t, 0, \Phi) \leqslant \limsup_{t\to\infty} x_i(t, 0, \Phi) < M, \quad i = 1, 2, \dots, n,$$

then the following general functional differential equation

$$\frac{dx}{dt} = F(t, x_t)$$

admits at least one positive ω-periodic solution. Where $x(t) \in R^n$ and $F(t, x_t)$ is a n-dimensional continuous functional, $x(t, 0, \Phi) = (x_1(t, 0, \Phi), x_2(t, 0, \Phi), \dots, x_n(t, 0, \Phi))$ is a solution of the functional differential equation with initial condition $x_0 = \Phi$.

2. Main results

In this section, we will obtain some sufficient conditions for the ultimately boundedness, permanence, extinction, and existence of periodic solution of system (1.4).

Theorem 2.1. *Assume that (H_1) holds, then solutions of system (1.4) with initial condition (1.5) are ultimately bounded from above.*

Proof. Suppose that $(x_1(t), x_2(t), y_1(t), y_2(t))$ is any solution of (1.4) with initial condition (1.5). Defining the function

$$W(t) = x_1(t) + x_2(t) + y_1(t) + y_2(t),$$

and calculating the derivative of $W(t)$ along the positive solutions of system (1.2), we have

$$\begin{aligned}\dot{W}(t) \leqslant\ & \alpha_1^M x_2(t) - r_1^L x_1(t) - \eta_1^L x_1^2(t) - r_2^L(t)x_2(t) - \eta_2(t)x_2^2(t) + \frac{b_1^M}{\gamma_1^L}x_2(t)\\ & + \alpha_2^M y_2(t) - c_1^L y_1(t) - d^L y_1^2(t) - c_2^L(t)y_2(t) - d_2(t)y_2^2(t) + \frac{b_2^M}{\gamma_2^L}y_2(t).\end{aligned}$$

Then

$$\dot{W}(t) + A_1W(t) \leqslant (\alpha_1^M + \frac{b_1^M}{\gamma_1^L})x_2(t) - \eta_2^L x_2^2(t) + (\alpha_2^M + \frac{b_2^M}{\gamma_2^L})y_2(t) - d_2^L y_2^2(t),$$

where $A_1 = \min\{r_1^L, r_2^L, c_1^L, c_2^L\}$. Then there exists a positive number A_2 such that

$$\dot{W}(t) + A_1W(t) \leqslant A_2,$$

where $A_2 = \frac{1}{4}\left[\left(\frac{\alpha_1^M + \frac{b_1^M}{\gamma_1^L}}{\eta_2^L}\right)^2 + \left(\frac{\alpha_2^M + \frac{b_2^M}{\gamma_2^L}}{d_2^L}\right)^2\right]$, which yields

$$W(t) \leqslant \frac{A_2}{A_1} + (W(0) - \frac{A_2}{A_1})e^{-A_1 t}.$$

Hence, there exist positive constant T_0 and $M = \frac{A_2}{A_1}$ such that $x_i(t) \leqslant M(i = 1, 2), y_i(t) \leqslant M(i = 1, 2)$ for $t \geqslant T_0$. This implies that any positive solutions of system (1.4) is ultimately bounded. This completes the proof. □

Theorem 2.2. *Assume that* (H_1) *holds and* $B_i > 0$ $(i = 1, 2)$, *then system* (1.4) *is permanent, where* $B_1 = b_1^L - r_2^M(e_1^M M + \gamma_1^M)$ *and* $B_2 = b_2^L - c_2^M(e_2^M M + \gamma_2^M)$.

Proof. Suppose $z(t) = (x_1(t), x_2(t), y_1(t), y_2(t))$ is any positive solution of system (1.4) with initial condition (1.5). Firstly, it follows from the first and second equation of system (1.4) and the condition of positivity of B_1 that for $t \geqslant T_0$, we have

$$\begin{aligned}
&\dot{x}_1(t) = \alpha_1(t)x_2(t) - (r_1(t) + \beta_1(t))x_1(t) - \eta_1(t)x_1^2(t),\\
&\dot{x}_2(t) \geqslant \beta_1(t)x_1(t) - r_2^M x_2(t) - \eta_2(t)x_2^2(t) + \frac{b_1^L x_2(t)}{e_1^M M + \gamma_1^M} \geqslant \beta_1(t)x_1(t) - \eta_2(t)x_2^2(t).
\end{aligned}$$

Now, we consider the following auxiliary equation

$$\begin{aligned}
&\dot{u}_1(t) = \alpha_1(t)u_2(t) - (r_1(t) + \beta_1(t))u_1(t) - \eta_1(t)u_1^2(t),\\
&\dot{u}_2(t) = \beta_1(t)u_1(t) - \eta_2(t)u_2^2(t).
\end{aligned} \tag{2.1}$$

By Lemma 1.2, we have that system (2.1) has a unique globally attractive positive ω-periodic solution $(\bar{x}_1(t), \bar{x}_2(t))$. Let $(u_1(t), u_2(t))$ be the solution of (2.1) with $(u_1(T_1), u_2(T_1)) = ((x_1(T_1), x_2(T_1))$, by comparison theorem, we have

$$x_i(t) \geqslant u_i(t)(i = 1, 2), \quad t \geqslant T_1. \tag{2.2}$$

Also from the global attractivity of $(\bar{x}_1(t), \bar{x}_2(t))$, there exists a constant $T_1 > 0$, such that

$$|u_i(t) - \bar{x}_i(t)| < \frac{\bar{x}_i(t)}{2}, \quad t \geqslant T_1. \tag{2.3}$$

Inequality (2.3) combine with (2.2) leads to

$$x_i(t) > \min_{0 \leqslant t \leqslant \omega}\left\{\frac{\bar{x}_i(t)}{2}\right\} =: m_i, \ i = 1, 2, \quad t > T_1.$$

Therefore,

$$\lim_{t \to +\infty} \inf x_i(t) \geqslant m_i, \ \ i = 1, 2.$$

Next, from the third and fourth equation of system (1.4) that for $t \geqslant T_0$, we have

$$\begin{aligned}
&\dot{y}_1(t) = \alpha_2(t)y_2(t) - (c_1(t) + \beta_2(t))y_1(t) - d_1(t)y_1^2(t),\\
&\dot{y}_2(t) \geqslant \beta_2(t)y_1(t) - c_2^M y_2(t) - d_2(t)y_2^2(t) + \frac{b_2^L y_2(t)}{e_2^M M + \gamma_2^M} \geqslant \beta_2(t)y_1(t) - \eta_2(t)y_2^2(t).
\end{aligned}$$

We consider the following auxiliary equation

$$\dot{u}_1(t) = \alpha_2(t)u_2(t) - (c_1(t) + \beta_2(t))u_1(t) - d_1(t)u_1^2(t), \ \ \dot{u}_2(t) = \beta_2(t)u_1(t) - d_2(t)u_2^2(t). \tag{2.4}$$

By Lemma 1.2, we have that system (2.4) has a unique globally attractive positive ω periodic solution $(\bar{y}_1(t), \bar{y}_2(t))$. The rest of the proof is similar to the above discussion, we can obtain there exists a constant $T_2 > 0$, such that

$$y_i(t) > \min_{0\leqslant t\leqslant\omega} \left\{\frac{\bar{y}_i(t)}{2}\right\} =: m_{i+2}, \ i = 1, 2, \quad t > T_2.$$

Therefore,

$$\lim_{t\to+\infty} \inf y_i(t) \geqslant m_{i+2}, \ \ i = 1, 2.$$

Finally, there exists a constant $T > \max\{T_0, T_1, T_2\}$ such that $x_i(t) \geqslant m(i = 1, 2)$ and $y_i(t) \geqslant m(i = 1, 2)$, where $m = \min\{m_1, m_2, m_3, m_4\}$ for $t \geqslant T$. This completes the proof of Theorem 2.2. □

On the existence of positive periodic solutions of system (1.4) we have the following result. As a direct result of Lemma 2, from Theorem 2, we have

Corollary 2.3. *If the assumptions of Theorem 2.2 hold, then system (1.4) has at least one positive ω-periodic solution.*

Theorem 2.4. *Immature species x_1, y_1 and mature species x_2, y_2 of system (1.1) become extinct if (H_2) holds and $C_i > 0 (i = 1, 2)$, where $C_1 = r_2^L - \alpha_1^M - \frac{b_1^M}{\gamma_1^L}$ and $C_2 = c_2^L - \alpha_2^M - \frac{b_2^M}{\gamma_2^L}$.*

Proof. Suppose $(x_1(t), x_2(t), y_1(t), y_2(t))$ is a positive solution of of system (1.4) with initial conditions (1.5). Defining the function

$$V(t) = x_1(t) + x_2(t) + y_1(t) + y_2(t),$$

and calculating the derivative of V(t) along the positive solutions of system (1.4), we have

$$\begin{aligned}\dot{V}(t) \leqslant\ & \alpha_1(t)x_2(t) - r_1(t)x_1(t) - r_2(t)x_2(t) + \frac{b_1(t)}{\gamma_1(t)}x_2(t)\\ &+ \alpha_2(t)y_2(t) - c_1(t)y_1(t) - c_2(t)y_2(t) + \frac{b_2(t)}{\gamma_2(t)}y_2(t),\end{aligned} \tag{2.5}$$

Then, it follows from (2.5) for $t > T_0$

$$\dot{V}(t) \leqslant -r_1^L x_1(t) - (r_2^L - \alpha_1^M - \frac{b_1^M}{\gamma_1^L})x_2(t) - c_1^L y_1(t) - (c_2^L - \alpha_2^M - \frac{b_2^M}{\gamma_2^L})y_2(t) \leqslant -kV(t),$$

where $k = \min\{r_1^L, c_1^L, C_1, C_2\}$, which yields

$$V(t) \leqslant V(0)e^{-kt}.$$

Then we have

$$\lim_{t\to+\infty} V(t) = \lim_{t\to+\infty}(x_1(t) + x_2(t) + y_1(t) + y_2(t)) = 0. \tag{2.6}$$

From (2.6) there exists a constant $T^* > 0$ such that $x_1(t) \to 0$, $x_2(t) \to 0$, $y_1(t) \to 0$ and $y_2(t) \to 0$ for $t > T^*$. This completes the proof. □

3. Examples

Example 3.1. First, we consider the following system

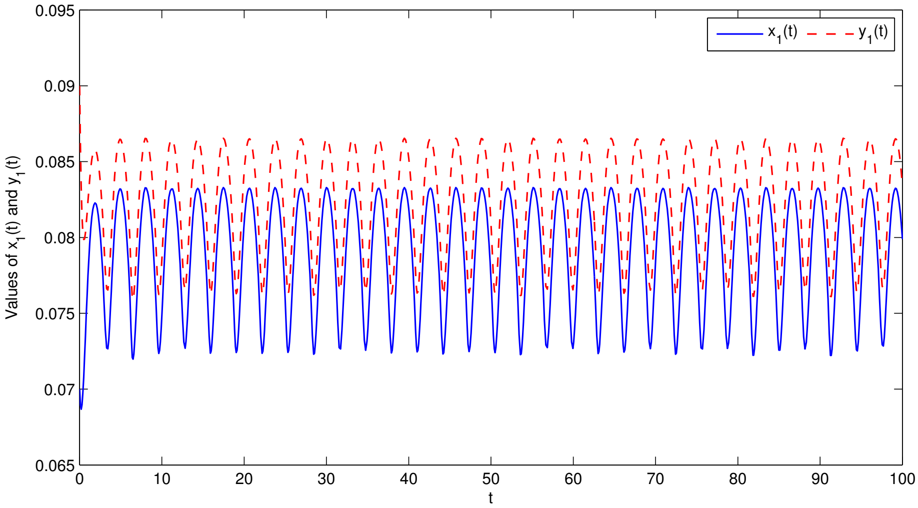
$$\begin{aligned}\dot{x}_1(t) =\ & (0.15 + 0.15|\sin(t)|)x_2(t) - (0.85 + 0.5|\sin(t)|)x_1(t) - (1 + 0.5|\sin(t)|)x_1(t)\\ &- (1 + 0.5|\sin(t)|)x_1^2(t),\end{aligned}$$

$$\begin{aligned}
\dot{x}_2(t) &= (1+|\sin(t)|)x_1(t) - (0.15+0.01|\sin(t)|)x_2(t) - (1+0.5|\sin(t)|)x_2^2(t) \\
&\quad + \frac{(2+0.5|\sin(t)|)x_2(t)}{(0.1+0.01|\sin(t)|)y_2(t)+2+0.01|\sin(t)|}, \\
\dot{y}_1(t) &= (0.15+0.15|\sin(t)|)y_2(t) - (0.75+0.5|\sin(t)|)y_1(t) - (1+0.5|\sin(t)|)y_1(t) \\
&\quad - (1+0.5|\sin(t)|)y_1^2(t), \\
\dot{y}_2(t) &= (1+|\sin(t)|)y_1(t) - (0.15+0.01|\sin(t)|)y_2(t) - (1+0.5|\sin(t)|)y_2^2(t) \\
&\quad + \frac{(2+0.5|\sin(t)|)y_2(t)}{(0.1+0.01|\sin(t)|)x_2(t)+2+0.01|\sin(t)|}.
\end{aligned} \tag{3.1}$$

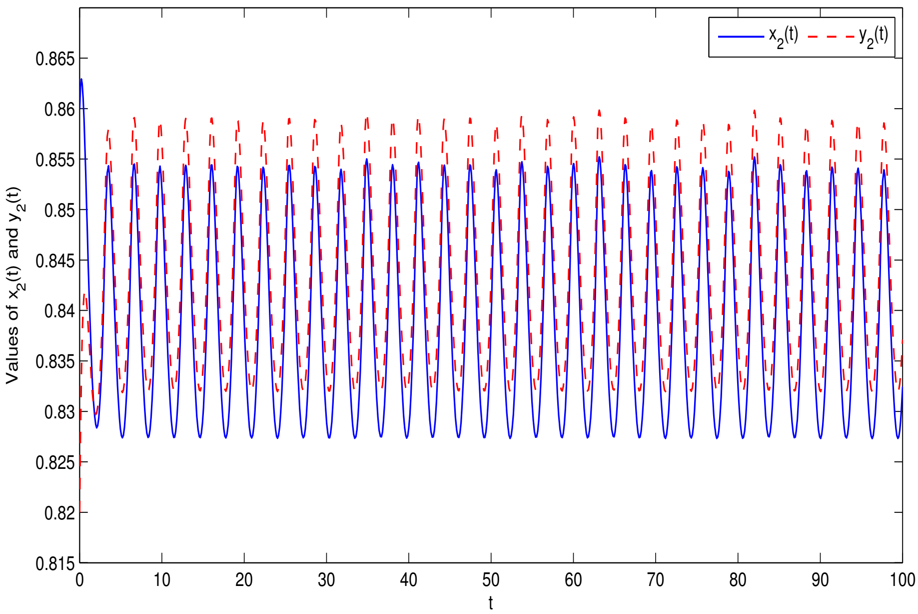
By directly calculation we can get

$$A_1 = 0.15, \quad A_2 \approx 1.21, \quad M \approx 8.01, \quad B_1 = B_2 \approx 1.54 > 0.$$

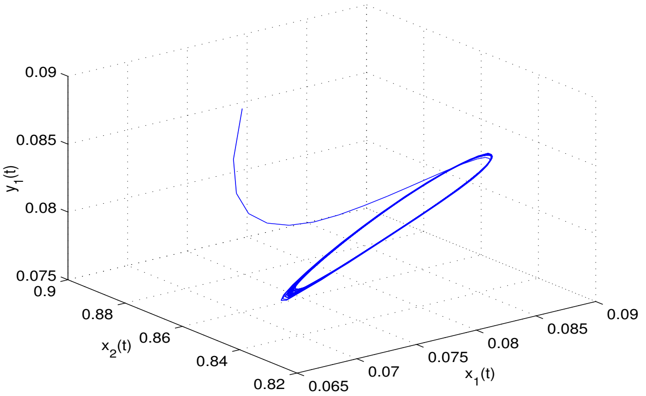
It is clear that the conditions of Theorem 2.2 and Corollary 2.3 hold.



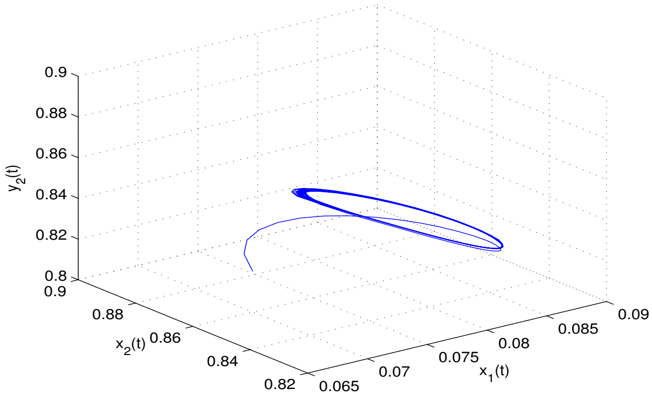
(a)



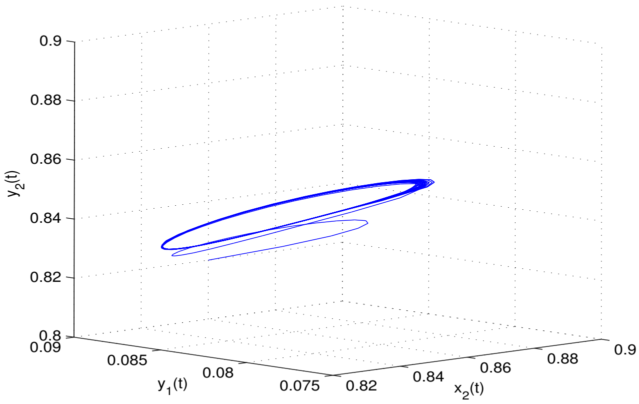
(b)



(c)



(d)



(e)

Figure 1: Dynamics of system (3.1).

From the Fig. 1. we can see, system (3.1) is permanent and has a periodic solution.

Example 3.2. Next, we consider the following system

$$\begin{aligned}
\dot{x}_1(t) &= (0.15+0.2|\sin(t)|)x_2(t)-(0.85+0.5|\sin(t)|)x_1(t)-(1+0.5|\sin(t)|)x_1(t)\\
&\quad -(1+0.5|\sin(t)|)x_1^2(t),\\
\dot{x}_2(t) &= (1+|\sin(t)|)x_1(t)-(2.25+0.01|\sin(t)|)x_2(t)-(1+0.5|\sin(t)|)x_2^2(t)\\
&\quad +\frac{(2+0.5|\sin(t)|)x_2(t)}{(0.1+0.01|\sin(t)|)y_2(t)+2+0.01|\sin(t)|},\\
\dot{y}_1(t) &= (0.15+0.25|\sin(t)|)y_2(t)-(0.75+0.5|\sin(t)|)y_1(t)-(1+0.5|\sin(t)|)y_1(t)\\
&\quad -(1+0.5|\sin(t)|)y_1^2(t),\\
\dot{y}_2(t) &= (1+|\sin(t)|)y_1(t)-(2.35+0.01|\sin(t)|)y_2(t)-(1+0.5|\sin(t)|)y_2^2(t)\\
&\quad +\frac{(2+0.5|\sin(t)|)y_2(t)}{(0.1+0.01|\sin(t)|)x_2(t)+2+0.01|\sin(t)|}.
\end{aligned} \tag{3.2}$$

By directly calculation we can get

$$C_1 \approx 0.65 > 0, \quad C_1 \approx 0.7 > 0.$$

It is clear that the conditions of Theorem 2.4 hold.

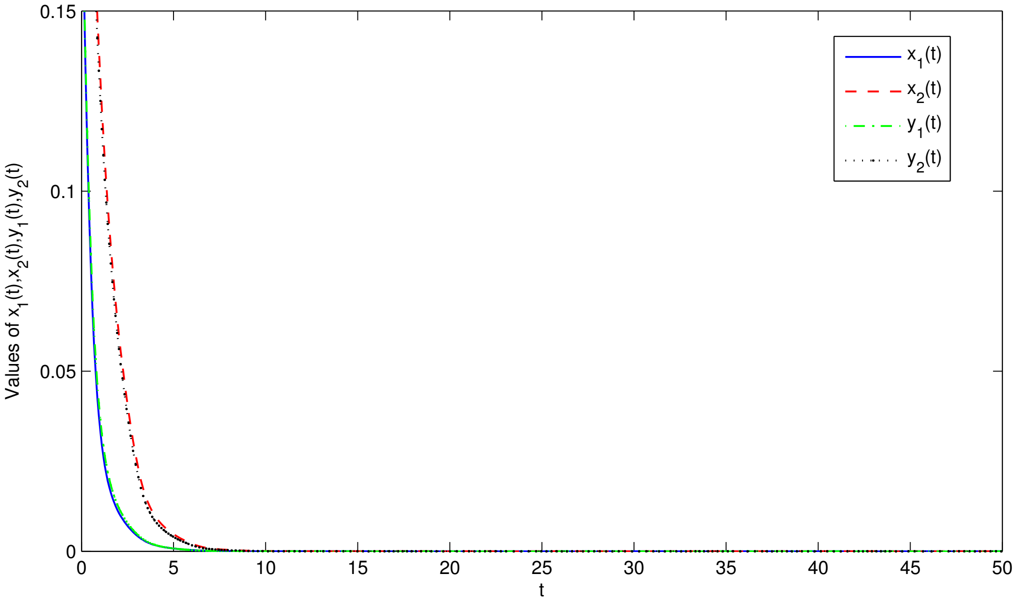


Figure 2: Dynamics of system (3.2).

From the Fig. 2. we can see, species x_1, x_2, y_1, and y_2 in system (3.2) are go to extinction.

Acknowledgment

This work was supported by the National Natural Science Foundation of Xinjiang under Grant No. 2016D01C075.

References

[1] L. Chen, *Models and Research Methods of Mathematical Ecology*, Science Press, Beijing, (1988). 1

[2] F. D. Chen, M. S. You, *Permanence, extinction and periodic solution of the predator-prey system with Beddington-DeAngelis functional response and stage structure for prey*, Nonlinear Anal. Real World Appl., **9** (2008), 207–221. 1

[3] J. G. Cui, L. S. Chen, W. D. Wang, *The effect of dispersal on population growth with stage-structure*, Comput. Math. Appl., **39** (2000), 91–102. 1, 1.2

[4] S. Hsu, T. Hwang, Y. Kuang, *Rich dynamics of a ratiodependent one prey two predators model*, J. Math. Biol., **43** (2001), 377–396. 1

[5] Z. X. Li, L. S. Chen, J. M. Huang, *Permanence and periodicity of a delayed ratio-dependent predator-prey model with Holling type functional response and stage structure*, J. Comput. Appl. Math., **233** (2009), 173–187. 1

[6] Z.-H. Ma, Z.-Z. Li, S.-F. Wang, T. Li, F.-P. Zhang, *Permanence of a predator-prey system with stage structure and time delay*, Appl. Math. Comput., **201** (2008), 65–71. 1

[7] R. M. May, *Theoretical Ecology, Principle and Applications*, Sounders Press, Philadelphia, (1976). 1

[8] A. Muhammadhaji, R. Mahemuti, Z. D. Teng, *Periodic solutions for n-species Lotka-Volterra competitive systems with pure delays*, Chin. J. Math. (N.Y.), **2015** (2015), 11 pages. 1, 1

[9] A. Muhammadhaji, Z. D. Teng, X. Abdurahman, *Permanence and extinction analysis for a delayed ratio-dependent cooperative system with stage structure*, Afr. Mat., **25** (2014), 897–909. 1, 1

[10] Z. Teng, L. Chen, *The positive periodic slotion in periodic Kolmogorov type systems with delays*, Acta. Math. Appl. Sinica., **22** (1999), 446–456. 1.3

[11] J. Y. Wang, Q. S. Lu, Z. S. Feng, *A nonautonomous predator-prey system with stage structure and double time delays*, J. Comput. Appl. Math., **230** (2009), 283–299. 1

[12] D. M. Xiao, W. X. Li, M. Han, *Dynamics in a ratio-dependent predator-prey model with predator harvesting*, J. Math. Anal. Appl., **324** (2006), 14–29. 1

[13] R. Xu, M. A. J. Chaplain, F. A. Davidson, *Permanence and periodicity of a delayed ratio-dependent predator-prey model with stage structure*, J. Math. Anal. Appl., **303** (2005), 602–621. 1

[14] R. Xu, Z. Ma, *The effect of stage-structure on the permanence of a predator-prey system with time delay*, Appl. Math. Comput., **189** (2007), 1164–1177. 1, 1

[15] R. Xu, Z. Q. Wang, *Periodic solutions of a nonautonomous predator-prey system with stage structure and time delays*, J. Comput. Appl. Math., **196** (2006), 70–86. 1

[16] Z. Q. Zhang, J. Wu, Z. C. Wang, *Periodic solutions of nonautonomous stage-structured cooperative system*, Comput. Math. Appl., **47** (2004), 699–706. 1, 1