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Modified exponential function method for the KP-BBM equation

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Abstract

In this study, the travelling wave solutions of the Kadomtsev-Petviashvili-Benjamin-Bona- Mahony equation were obtained by using modified exponential function method. This method provides the solution of nonlinear partial differential equation by using exponential function. The submitted solutions are implied in terms of the hyperbolic functions, trigonometric functions. The 2D and 3D graphics and contour simulations of these solution functions were obtained by using computational program.

Keywords: The nonlinear equations, the Kadomtsev-Petviashvili-Benjamin-Bona-Mahony equation (KP-BBM), the modified exponential function method (MEFM).

2010 MSC: 35Q99, 35C07.

1. Introduction

Solutions of nonlinear partial differential equations have an important place in various fields. There are various methods in the literature to find solutions to such equations. Some of these are as follows; Kudryashov's method [30, 35], the sine-Gordon expansion method [9, 19], the new function methods [1, 2, 13], the trial equation method [31, 34] the extended trial equation method [39, 40], the generalized Bernoulli sub-equation function method [7, 14, 44] and so on [3–6, 10, 11, 15–18, 20–25, 27, 29, 33, 36–38, 41, 42, 45–50] . In this study, we apply the modified exponential function method (MEFM) [8, 9, 26] to solve the Kadomtsov-Petviashvili-Benjamin-Bona-Mahony (KP- BBM) equation [12, 28, 32, 43, 51].

$$\left(u_t + u_x - a\left(u^2\right)_x - bu_{xxt}\right)_x + ku_{yy} = 0. \tag{1.1}$$

2. Analysis of the Method

The summary of the modified exponential function method is given in the following steps [26].
Consider the general form of nonlinear partial differential equation;

$$P\left(u, u_x, u_y, u_t, u_{xx}, u_{yy}, u_{tt}, \cdots\right) = 0, \tag{2.1}$$

where $u = u(x, y, t)$is unknown function, P is a polynomial that has $u(x, y, t)$ function and its partial

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derivatives respect to x, y and t.
Step 1. Suppose that the travelling wave transformation as,

$$u(x,y,t) = U(\zeta), \quad \zeta = x + y - ct, \tag{2.2}$$

where c is a nonzero constant that can be determined later. If the equation (2.1) is obtained by substituting the necessary derivatives Eq. (2.2), the following nonlinear ordinary differential equation is obtained.

$$N\left(U, U', U'', U''', \cdots\right) = 0. \tag{2.3}$$

Step 2: Let's consider the following solution function for the equation;

$$U(\zeta) = \frac{\sum_{i=0}^{N} A_i \left[\exp\left(-\Omega\left(\zeta\right)\right)\right]^i}{\sum_{j=0}^{M} B_j \left[\exp\left(-\Omega\left(\zeta\right)\right)\right]^j} = \frac{A_0 + A_1 \exp\left(-\Omega\right) + \cdots + A_N \exp\left(N\left(-\Omega\right)\right)}{B_0 + B_1 \exp\left(-\Omega\right) + \cdots + B_M \exp\left(M\left(-\Omega\right)\right)}, \tag{2.4}$$

where $A_i, B_j, (0 \leqslant i \leqslant N, 0 \leqslant j \leqslant M)$ are constants can be determined. $A_N \neq 0, B_M \neq 0$, and $\Omega = \Omega(\zeta)$ solves the following ordinary differential equation;

$$\Omega'(\zeta) = \exp\left(-\Omega\left(\zeta\right)\right) + \mu \exp\left(\Omega\left(\zeta\right)\right) + \lambda. \tag{2.5}$$

When we solve the Eq. (2.5), we get the solution families as follows [3, 41].
Family 1: When $\mu \neq 0,\ \lambda^2 - 4\mu > 0$,

$$\Omega(\zeta) = \ln\left(\frac{-\sqrt{\lambda^2 - 4\mu}}{2\mu} \tanh\left(\frac{\sqrt{\lambda^2 - 4\mu}}{2}(\zeta + E)\right) - \frac{\lambda}{2\mu}\right). \tag{2.6}$$

Family 2: When $\mu \neq 0,\ \lambda^2 - 4\mu < 0$,

$$\Omega(\zeta) = \ln\left(\frac{\sqrt{-\lambda^2 + 4\mu}}{2\mu} \tan\left(\frac{\sqrt{-\lambda^2 + 4\mu}}{2}(\zeta + E)\right) - \frac{\lambda}{2\mu}\right). \tag{2.7}$$

Family 3: When $\mu = 0,\ \lambda \neq 0,$ and $\lambda^2 - 4\mu > 0$,

$$\Omega(\zeta) = -\ln\left(\frac{\lambda}{\exp\left(\lambda(\zeta + E)\right) - 1}\right). \tag{2.8}$$

Family 4: When $\mu \neq 0,\ \lambda \neq 0$ and $\lambda^2 - 4\mu = 0$,

$$\Omega(\zeta) = \ln\left(-\frac{2\lambda(\zeta + E) + 4}{\lambda^2(\zeta + E)}\right). \tag{2.9}$$

Family 5: When $\mu = 0,\ \lambda = 0$ and $\lambda^2 - 4\mu = 0$,

$$\Omega(\zeta) = \ln\left(\zeta + E\right). \tag{2.10}$$

where $A_0, A_1, \ldots, A_N, B_0, B_1, \ldots, B_M, E, \lambda, \mu$are constants. Using the balancing principle, the relationship between N and M, which is the upper boundary of the summation symbol in the equation (2.4).
Step 3: Substituting Eq. (2.5) along with solution families into Eq. (2.4) we have a polynomial of $\exp(\Omega(\xi))$. Algebraic equation system consisting of coefficients is obtained. Then, the coefficients of this system are solved and the solution function is obtained by substituting the equation (2.1).

3. Application

Let use Eq. (2.2) along with Eq. (2.4) into Eq. (1.1). In this case, we have nonlinear ordinary differential equation as follows;

$$(1+k-c)\,U - aU^2 + bcU'' = 0. \tag{3.1}$$

Using the balancing term $U''$and U^2, we get a relationship the following,

$$M+2=N. \tag{3.2}$$

Different cases can be obtained for suitable values of M and N. We have chosen $M=1$ and $N=3$ values, then some travelling wave solutions appeared as following submitted.

Case 1:

$$A_0=\frac{6bc\mu B_0}{a},\ A_1=\frac{6bc\left(\lambda B_0+\mu B_1\right)}{a},\ A_2=\frac{6bc\left(B_0+\lambda B_1\right)}{a},\ A_3=\frac{6bcB_1}{a},\ k=-1+c-bc\lambda^2+4bc\mu. \tag{3.3}$$

Family 1:

$$u_{1,1}(x,y,t)=\left(\frac{6bc\mu\left(-\lambda^2+4\mu\right)}{a\left(\lambda \mathrm{Cosh}\left[\frac{1}{2}\left(EE+\xi\right)\sqrt{\lambda^2-4\mu}\right]+\sqrt{\lambda^2-4\mu}\,\mathrm{Sinh}\left[\frac{1}{2}\left(EE+\xi\right)\sqrt{\lambda^2-4\mu}\right]\right)^2}\right), \tag{3.4}$$

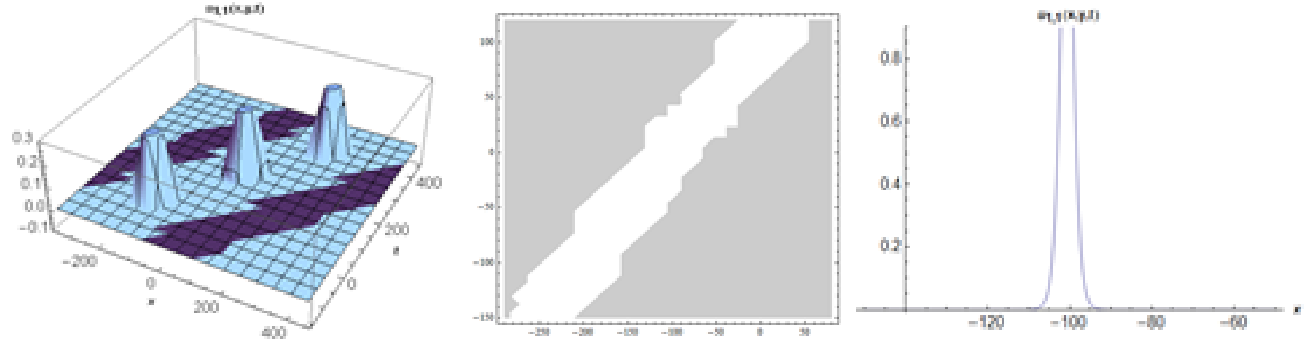

Figure 1: The 3D, contour graphic and 2D surfaces of Eq. (3.4) in$\lambda=3,\ \mu=2,\ c=1,\ a=-2,\ b=2,\ EE=100,\ t=1$.

Family 2:

$$u_{1,2}(x,y,t)=\left(\frac{6bc\mu\left(-\lambda^2+4\mu\right)}{a\left(\lambda \mathrm{Cos}\left[\frac{1}{2}\left(EE+\xi\right)\sqrt{-\lambda^2+4\mu}\right]-\sqrt{-\lambda^2+4\mu}\,\mathrm{Sin}\left[\frac{1}{2}\left(EE+\xi\right)\sqrt{-\lambda^2+4\mu}\right]\right)^2}\right), \tag{3.5}$$

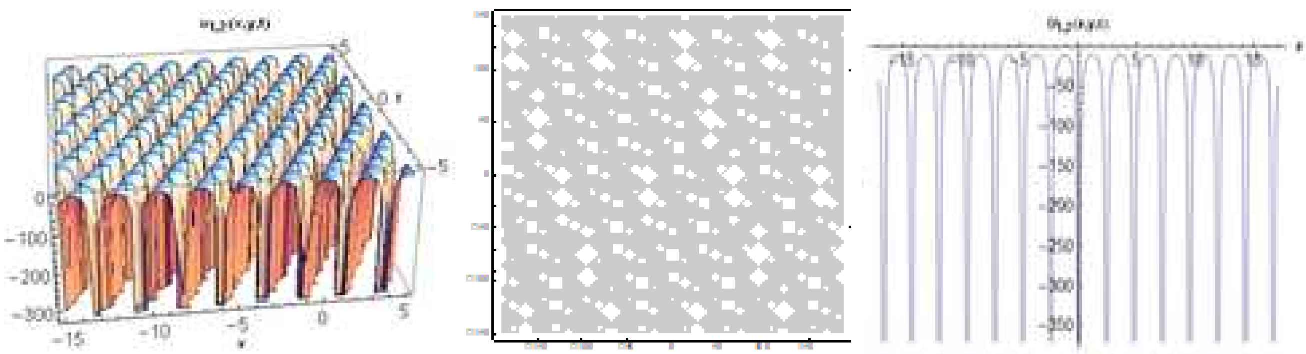

Figure 2: The 3D, contour graphic and 2D surfaces of Eq. (3.5) in$\lambda=1,\ \mu=2,\ c=1,\ a=-2,\ b=2,\ EE=100,\ t=1$.

Case 2:

$$\begin{aligned} &A_0 = \frac{(\lambda^2+2\mu)A_3B_0}{6B_1},\ A_1 = \frac{1}{6}A_3\left(\lambda^2+2\mu+\frac{6\lambda B_0}{B_1}\right),\ A_2 = A_3\left(\lambda+\frac{B_0}{B_1}\right), \\ &b = -\frac{aA_3}{a(\lambda^2-4\mu)A_3-6(1+k)B_1},\ c = 1+k-\frac{a(\lambda^2-4\mu)A_3}{6B_1}. \end{aligned} \tag{3.6}$$

Family 1:

$$u_{2,1}(x,y,t) = \frac{(\lambda^2-4\mu)\,\mathrm{Sech}\left[\frac{1}{2}\psi\right]^2\left(-4\mu+(\lambda^2-2\mu)\,\mathrm{Cosh}[\psi]+\lambda\sqrt{\lambda^2-4\mu}\,\mathrm{Sinh}[\psi]\right)A_3}{6B_1\left(\lambda+\sqrt{\lambda^2-4\mu}\,\mathrm{Tanh}\left[\frac{1}{2}\psi\right]\right)^2}, \tag{3.7}$$

where $\left(\psi = (EE+\xi)\sqrt{\lambda^2-4\mu}\right)$.

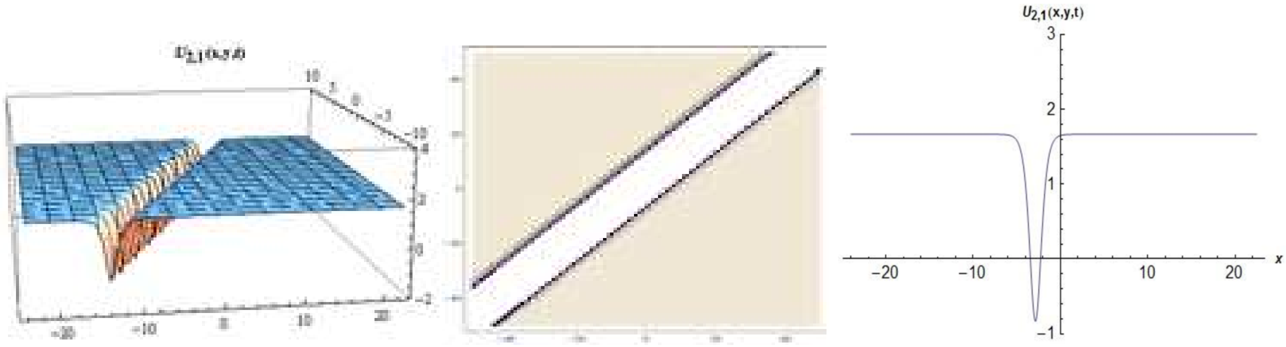

Figure 3: The 3D, contour graphic and 2D surfaces of Eq. (3.7) in $\lambda = 3,\ \mu = 1,\ c = 1,\ A_3 = 2,\ B_1 = 1,\ EE = 1,\ t = 1.$

Family 2:

$$u_{2,2}(x,y,t) = \frac{(\lambda^2-4\mu)\,\mathrm{Sec}\left[\frac{1}{2}\upsilon\right]^2\left(-4\mu+(\lambda^2-2\mu)\,\mathrm{Cos}[\upsilon]-\lambda\sqrt{-\lambda^2+4\mu}\,\mathrm{Sin}[\upsilon]\right)A_3}{6B_1\left(\lambda-\sqrt{-\lambda^2+4\mu}\,\mathrm{Tan}\left[\frac{1}{2}\upsilon\right]\right)^2}, \tag{3.8}$$

where $\left(\upsilon = (EE+\xi)\sqrt{-\lambda^2+4\mu}\right)$.

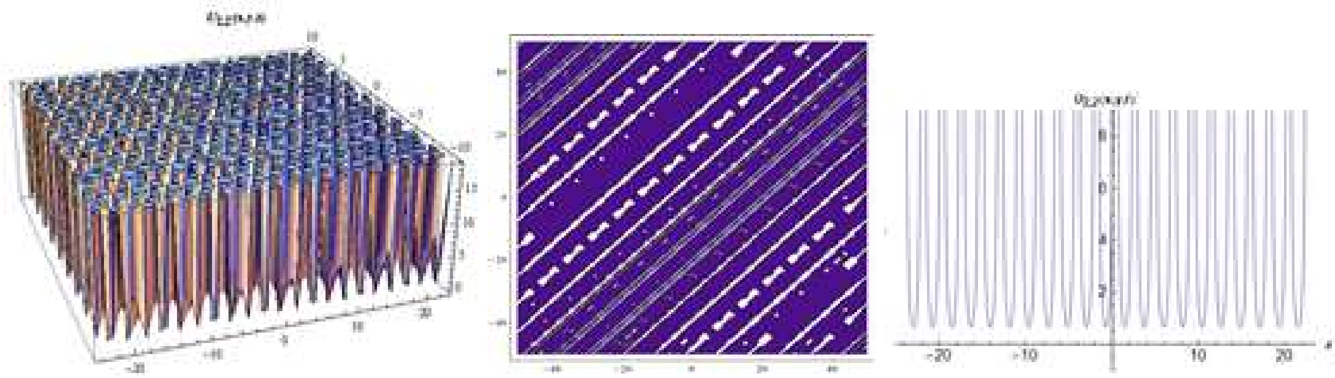

Figure 4: The 3D, contour graphic and 2D surfaces of Eq. (3.8) in $\lambda = 2,\ \mu = 3,\ c = 1,\ A_3 = 2,\ B_1 = 2,\ EE = 100,\ t = 1.$

Family 4:

$$u_{2,4}(x,y,t) = -\frac{\left(\lambda^2(-8+(EE+\xi)\lambda(4+(EE+\xi)\lambda))-4(2+(EE+\xi)\lambda)^2\mu\right)A_3}{12(2+(EE+\xi)\lambda)^2B_1}, \tag{3.9}$$

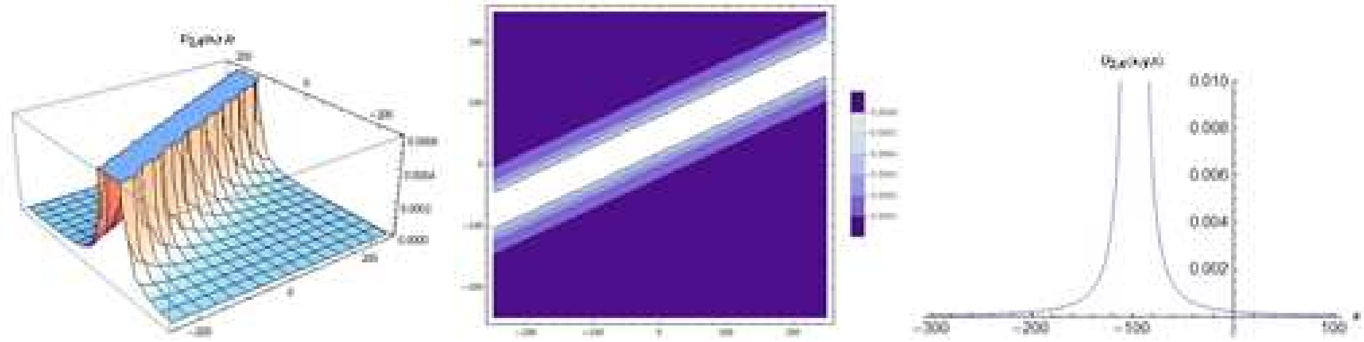

Figure 5: The 3D, contour graphic and 2D surfaces of Eq. (3.9) in $\lambda = -2,\ \mu = 1,\ c = 2, A_3 = 2,\ B_1 = 1,\ EE = 100,\ t = 1$.

4. Conclusion and Discussion

In this study, various travelling wave solutions of the KP-BBM equation were found using the modified exponential function method. When the obtained results are analyzed, it is seen that the modified exponential function method is a suitable mathematical method for solving nonlinear partial differential equations. The two and three dimensional and contour graphics of the solutions were drawn by using the Mathematica package program. When the obtained graphs are investigated, it is seen that periodic function curves appear. Finding such functions provides the advantage that the solution can be easily interpreted in the desired range.

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